

## REACTIVE MAX-MIN ANT SYSTEM: AN EXPERIMENTAL ANALYSIS OF THE COMBINATION WITH K-OPT LOCAL SEARCHES

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**ABSTRACT.** Ant colony optimization (ACO) is a stochastic search method for solving NP-hard problems. The exploration versus exploitation dilemma rises in ACO search. Reactive max-min ant system algorithm is a recent proposition to automate the exploration and exploitation. It memorizes the search regions in terms of reactive heuristics to be harnessed after restart, which is to avoid the arbitrary exploration later. This paper examined the assumption that local heuristics are useless when combined with local search especially when it applied for combinatorial optimization problems with rugged fitness landscape. Results showed that coupling reactive heuristics with k-Opt local search algorithms produces higher quality solutions and more robust search than max-min ant system algorithm. Well-known combinatorial optimization problems are used in experiments, i.e. traveling salesman and quadratic assignment problems. The benchmarking data for both problems are taken from TSPLIB and QAPLIB respectively.

**Keywords:** ant colony optimization, reactive search, quadratic assignment problem, traveling salesmen problem

### INTRODUCTION

Ant colony optimization (ACO) algorithms are multi-agent systems utilized for solving hard combinatorial optimization problems. Despite being one of the youngest metaheuristics, there is a large number of applications of ACO algorithms (Stützle, Lopez-Ibanez, & Dorigo, 2010; Talbi, 2009). The exploration versus exploitation dilemma exists in metaheuristics search not only ACO (Blum & Roli, 2003). It arises when promising regions of search space need to be quickly identified without spending too much time in poor regions (Dorigo & Stützle, 2010). Reactive search is an emergent approach for improving the internal behavior of metaheuristics (Battiti, Brunato, & Mascia, 2008). Restarting the search with the aid of memorizing the search history and parameter adaptation is the soul of reaction. It is to increase the exploration only when needed. Reactive max-min ant system (RMMAS) follows this approach and tries to improve the performance of ACO algorithms by proposing reactive heuristics to be considered as local heuristics for ants to find their way in search space (Sagban, Ku-Mahamud, & Shahbani, 2014). However, there is a proposition among ACO researchers that local heuristics becomes useless component when local searches is combined (Stützle, 1999). This hypothesis increased when ACO applied to combinatorial optimization problems with rugged search space such as quadratic assignment problem (QAP) (Dorigo &

Stützle, 2004). Apart from that, reactive search relies on parameter adaptation in addition to restarts strategy. Hence, the contribution of the present paper lies in the following aspect. Reactive heuristics are tested against the circumstance of combining local searches while applying to QAP where the fitness landscape is more rugged than the one in traveling salesman problem (TSP) where the induced RMMAS first applied.

The rest of the paper is organized as follows. The RMMAS algorithm is briefly described followed by the description of the combination with 3-Opt and 2-Opt local searches for TSP and QAP respectively (Johnson & McGeoch, 2007). Subsequence sections focus on the application to QAP and the experimental design. The results are then presented and the conclusion is drawn.

## REACTIVE MAX-MIN ANT SYSTEM

RMMAS is a recently proposed algorithm on top of max-min ant system (MMAS), the prominent ACO variant (Sagban et al., 2014). In this algorithm the artificial ants react to any stagnation behavior by redirecting their search toward unexplored regions in the search space. RMMAS relies on local heuristics called reactive heuristics to manage that reaction. A memory model, denoted by RH (refer to reactive heuristics), utilizes to memorize the history of search. The process of memorizing redefines the main exploration and exploitation components in ACO, the pheromone update and the probabilistic distribution. For the pheromone update, the definition of the evaporation rule is reformulated as follows.

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} \quad \forall \tau_{ij} \in T \quad (1)$$

$$Evaporate(RH, T, threshold) = \begin{cases} RH \leftarrow rh_1 & \text{if } \tau_{ij} < threshold \\ RH \leftarrow rh_0 & \text{if } \tau_{ij} \geq threshold \end{cases} \quad (2)$$

Where  $\rho$  is the evaporation rate,  $T$  is the pheromone matrix,  $RH$  is the reactive heuristics matrix,  $rh_x$  is either zero (0) or one (1) while its value relies on the value of *threshold*, i.e. the minimum level of pheromone. This new formula differs from the standard evaporation rule of MMAS.

When the optimization process starts the entries of matrix RH set to zero (0). With continuing the process the pheromone density accumulates on some pheromone trails and disappears in some others because of the pheromone update dynamism. Together with changing  $RH$  values to zero (0) when the trails below the threshold and to one (1) otherwise, the said memory model records the history of search.

After some run time, solution components associated with high pheromone density will attract more artificial ants. To reduce the risk of stagnation in which all ants follow the same path, the restarts strategies are used. Sagban et al. (2014) identified the trigger to determine the restart point effectively. Once the restart trigger the reactive heuristics will be considered as local heuristics in the probabilistic distribution as follows.

$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^\alpha \cdot \mu_{ij}^\beta \cdot rh_{ij}}{\sum_{c_{il} \in N(S)} \tau_{il}^\alpha \cdot \mu_{il}^\beta \cdot rh_{il}} & \text{if } c_{il} \in N(S) \text{ and restart is active} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Where  $k$  is the index of current active ant,  $\mu$  is the pre-heuristic associated with traveling salesman problem,  $N$  is the list of unvisited components ( $c_{il}$ ),  $\alpha$  and  $\beta$  are parameters to adjust the weights of pheromone and pre-heuristics.

Although the RMMAS algorithm is successfully applied to TSP, it does not yet examined when coupled with local search routines. The impression in ACO community is that local heuristics become useless when hybridized with local search. However, the results in this paper proved that this is not the case with reactive heuristics.

The solutions' quality is greatly improved if it is extended to include local search. The question is which local search? Earlier experiments (Dorigo & Stützle, 2004) showed that coupling ACO with K-Opt algorithms (Johnson & McGeoch, 2007) is much more effective than coupling with others. In the TSP case, the 3-Opt algorithm moves three (3) components of a solution with a different set of three components to improve the quality of solutions. In this kind of local search the neighborhood structure is larger than other k-Opt algorithms. This results in time consuming process. One speedup technique is using *don't look bit* data structure in which a bit is associated with each node of the sequence. At the beginning of the search all bits are turned off. The bit associated with node  $h$  is turned on when a search for an improving move starting from  $h$  fails. The bit associated with node  $h$  is turned off again when an improving exchange involving  $h$  is executed. The use of *don't look bits* favors the exploration of nodes that have been involved in a profitable exchange. Another technique is the nearest neighborhood list which in RMMAS was twenty (20) nodes length. In the QAP case, the 2-Opt algorithm moves two components instead of three. Yet, first improving move is immediately performed can be used as a speedup technique. RMMAS is using a truncated first-improvement 2-opt. In particular, at most two complete scans of the neighborhood are done. The local search is fast even it is not necessarily returns a locally optimal solution.

#### APPLICATION OF RMMAS TO QAP

This section discusses the circumstance of applying reactive heuristics to QAP the hardest NP-hard problem. It concerns the situation when local search routines are coupled with RMMAS so that artificial ants can traverse such fitness landscape without trapping in local optima. In QAP implementation application the pre-heuristics information is omitted to show the role of reactive heuristics.

The QAP can best be described as the problem of assigning a set of facilities ( $n$ ) to a set of locations ( $n$ ) with given distances between the locations and given flows between the facilities. The flows and locations are two  $n \times n$  matrices denoted by A and B respectively, where  $a_{ij}$  is the flow between facility  $i$  and  $j$  and  $b_{rs}$  is the distance between location  $r$  and  $s$ . The objective is to place the facilities on locations in such a way that the sum of the product between flows and distances is minimal. The problem can be formulated as follow.

$$f(\emptyset) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} b_{\emptyset i \emptyset j} \quad (4)$$

When RMMAS is applied to QAP the way the solutions are constructed after restart has to be defined. It is by assigning facilities in some order to locations. So that the pheromone trails  $\tau_{ij}$  refers to specific location for facilities, that is,  $\tau_{ij}$  represent the desirability of assigning facility  $i$  to location  $j$ . The ants are used to construct valid solutions for the QAP assigning every facility to exactly one location and not using a location by more than one facility. In this way, a facility is randomly chosen among unassigned ones. Then, this facility is placed in free location according to the following probabilistic distribution rule.

$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^\alpha \cdot r h_{ij}}{\sum_{l \in U(k)} \tau_{il}^\alpha \cdot \tau_{il}}, & \text{if location } j \text{ is free and restart is active} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where  $U(K)$  denotes the set of unassigned items. The intuition behind this rule is to prefer the visible  $\tau_{ij} \times rh_{ij}$  values which are the unvisited locations  $j$  for facility  $i$ . RMMAS utilizes the pseudo-random proportional rule (one of the important features in one of the best ACO variants, namely ant colony system).

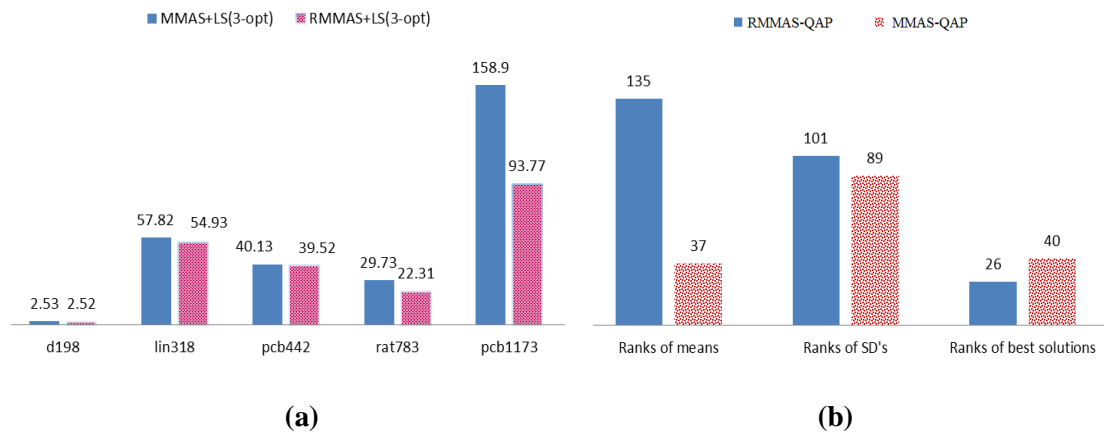
## EXPERIMENTAL DESIGN

The goal of the conducted experiments is to test whether the reactive heuristics useful or not when RMMAS is combined with two of k-Opt local searches for TSP and QAP. To achieve these goals, comparisons with MMAS, the best ACO variants in solving TSP and QAP, are conducted. Each comparison is conducted ten (10) times to avoid any stochastic effect. The experiments were conducted on Windows 8 64-bit operating system, processor Intel Core i3-3217U with CPU @ 1.80GHz, RAM 4GB. The proposed algorithm was coded in C language. The QAP and TSP instances are selected from QAPLIB (Burkard, Cela, Karisch, & Rendl, 1997) and TSPLIB (Reinelt, 1991) repositories respectively. For QAP case, selected instances were *bur26a*, *bur26b*, *bur26c*, *bur26d*, *bur26e*, *bur26f*, *bur26g*, *bur26h*, *chr25a*, *els19*, *kra30a*, *kra30b*, *tai20b*, *tai30b*, *tai35b*, *tai40b*, *tai50b*, *tai60b* and *tai80b* while for the TSP case they were *d198*, *lin318*, *pcb442*, *rat783* and *pcb1173*. Running time for TSP is set to 10 seconds while for QAP it was proportional to the size and the structure of the instance as in Gambardella, Taillard, and Dorigo (1999). The parameter settings are as follows. The number of ants ( $m$ ) is equal to five (5). The pheromone intensity ( $\alpha$ ) and pre-heuristic distance ( $\beta$ ) are equal to one (1) and two (2) respectively. Evaporation rate ( $\rho$ ) is 0.5. The initial pheromone ( $\tau_0$ ) is set to  $1/\rho * C^{nn}$ . The exploration/exploitation parameter  $q_0$  is equal to 0.98. The metrics that we need to test are the average, standard deviation and the best quality of solutions. The non-parametric statistical tests Wilcoxon and Chi-square are used to verify the significance of the improvement in quality. Wilcoxon signed-ranks test is based on the positive and negative ranks of each of the said matrices in the comparison. The test performed with 0.05 significance level and one-tailed hypothesis.

## RESULTS

This section presents the results of the combination with 3-Opt local search and the results of the application to QAP. In the first part Figure 1(a) visualizes (in the y-axis) the quality of solutions measured by the standard deviation of the best solutions found during ten (10) runs in TSP problem. The x-axis presents for each TSP instance the performance of each of the competitive algorithms, i.e.  $MMAS_{3-opt}$  and  $RMMAS_{3-opt}$  algorithms. The results showed that the proposed  $RMMAS_{3-op}$  produce good solutions for all TSP instances. The fact that pre-heuristics is useless with local search cannot be imposed for reactive heuristics. The experiments showed that the enhanced algorithm outperformed the standard one because of its ability to avoid the premature convergence which helps in increasing the quality of solutions.

In the second part of results Figure 1(b) presents the statistical results of applying RMMAS to QAP with comparison with MMAS. Wilcoxon signed-ranks statistical test showed that the search of RMMAS is more robust as it outperforms the original one in the number of ranks for standard deviation and mean of the quality of solutions. In the comparison of means, the MMAS algorithm collected (37) ranks while the proposed algorithm collected (135) ranks. The result was significant at  $p \leq 0.05$  while p-value was equal to (0.001659). In the comparison of standard deviations, the MMAS algorithm collects (89) ranks while RMMAS collects (101) ranks. The p-value was (0.40517) and the result was not significant at  $p \leq 0.05$ . In the comparison of best solutions, the MMAS collected (40) ranks while RMMAS collects (26) ranks. The p-value was 0.26763 and the result was not significant at  $p \leq 0.05$ .



**Figure 5 (a).The Performance of RMMAS (3-Opt) versus MMAS (3-Opt) for TSP, (b).Bar Chart for Sum of Ranks Obtained from Applying RMMAS versus MMAS in QAP**

The results are inconclusive. To verify the significance of the enhancement, the statistical Chi-square test for frequencies is performed with significance level equal to 0.05. The result was significant at  $p < 0.05$  because the  $p$ -value is  $< 0.00001$ . Table 1 provides information about the observed cell totals and (the expected cell totals).

**Table 1. Frequencies of Ranks Obtained from Applying RMMAS versus MMAS in QAP**

Categories	Ranks of RMMAS	Ranks of MMAS	Row totals
Ranks of means	135 (105.29)	37 (66.71)	172
Ranks of the SD's	101 (116.31)	89 (73.69)	190
Ranks of the best solution	26 (40.40)	40 (25.60)	66
Column totals	262	166	428 ( <b>Grand Total</b> )

## CONCLUSION AND FUTURE WORK

This paper has presented an experimental analysis of the reactive heuristics choice made in RMMAS algorithm when coupled with k-Opt local searches. A new application to the QAP has also been included. The analysis shows that combining reactive heuristics with k-Opt local searches has a profound impact on the quality of solutions obtained for TSP. For the QAP application, the quality improvement was inconclusive. Although the RMMAS search was more robust than MMAS, it needs more adaptive features to automate the exploration and exploitation before triggering restart.

The experimental analysis presented in this paper can be further explored by using robust exploration indicators to adjust the exploration and exploitation in RMMAS. This can be done through the on-line parameter controllers.

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