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A Short Survey on the Usage of Choquet Integral and Its Associated Fuzzy Measure in Multiple Attribute Analysis

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Abstract

Choquet integral operator is currently making inroads into many real multiple attribute analysis due to its ability on modeling the usual interactions held by the attributes during the aggregation process. Unfortunately, the process of identifying 2^n values of fuzzy measure prior to employing Choquet integral normally turns into a very complex one with the increasing number of attributes, n. On that note, this paper mainly reviews on some of the methods that have been proposed in reducing the complexity of identifying fuzzy measure values together with their pros and cons. The paper begins with a discussion on the aggregation process in multiple attribute analysis which then focuses on the usage of Choquet integral and its associated fuzzy measure before investigating some of the fuzzy measure identification methods. A simple numerical example to demonstrate the merit of using Choquet integral and the indications for future research are provided as well. The paper to some extent would be helpful in stimulating new ideas for developing simpler or enhanced versions of fuzzy measure identification methods.

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Keywords: Choquet integral; fuzzy measure; multiple attribute analysis

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1. Introduction

In the context of multiple attribute analysis, aggregation can be defined as a process of composing the performance scores (scores with respect to a set attributes) of each alternative under evaluation into a single or global score where based on these single scores, the alternatives are then classified or ranked up [1]. A function which synthesizes the attribute scores into a global score is usually referred as an aggregation operator. Effective aggregation operators are expected to satisfy several mathematical properties including three fundamental properties namely identity when unary, boundary condition and monotonicity [2], as well as several behavioral properties such as having the ability to express the interactions shared by evaluation attributes [3].

Normally, additive operators such as simple weighted average (SWA), quasi arithmetic means, ordered weighted average, weighted min and weighted max are used for aggregation purpose. Unfortunately, these operators presume that the attributes are always independent to each other [4]. This assumption is inapt with real scenario where in many cases, the attributes hold interactive characteristics [5]. Therefore, aggregation should not be always carried out using additive operators instead, Sugeno or Choquet integral operator could be used to deal with these interactive attributes [6].

Although both integrals are capable in capturing the usual interactions that exist between the attributes, the application of Choquet integral is widening across many disciplines with greater extent than the other one due to following two reasons. Firstly, as affirmed by Iourinski and Modave [7], Choquet integral is better suited for numerical or quantitative based problems whereas the Sugeno integral is more ideal for qualitative problems. In other words, the application of Choquet integral can generate more practical outcomes as most of the multiple attribute problems involve numbers which have a real meaning (interval or ratio level of measurement) where cardinal aggregation is required, unlike Sugeno integral which is more suitable for ordinal aggregation where only the order of the elements is important. Secondly, Choquet integral has the merit in producing unique solution in contrast to Sugeno integral.

2. Choquet integral and its associated fuzzy measure

The usage of Choquet integral usually requires a prior identification of fuzzy measure values, g. These values not only represent the importance of each attribute, but also the importance of all possible combinations or subsets of attributes [8]. Let $C = (c_1, c_2, \ldots, c_n)$ be a finite set of criteria. A set function $g(.)$ defined on the set of the subsets of C, $P(C)$, is called a fuzzy measure if it satisfies the following conditions:

- $g: P(C) \to [0,1]$, and $g(\emptyset) = 0, g(C) = 1$(boundary condition)
- $\forall A, B \in P(C)$, if $A \subseteq B$, then $g(A) \leq g(B)$ (monotonic condition)

The boundary condition interprets that an empty set, with the absence of any attributes, has no importance where $g\{\emptyset\} = 0$ and the maximal set, with the presence of all attributes, has maximal importance where $g\{C\} = 1$. Meanwhile, monotonicity condition implies that adding a new attribute to a combination or subset cannot decrease its importance. A fuzzy measure can express three types of interactions that could be shared by the attributes. Suppose A and B are two subsets of attributes where $A \cap B = \emptyset$, then the interaction shared by these two subsets can be described as follows [9]:

- If the value or importance of the combination of A and B is equal to the sum of respective importance assigned to A and B such that $g\{A \cup B\} = g\{A\} + g\{B\}$, then it can be claimed that A and B are sharing additive effect or in other words, being independent to each other.
- If the importance of the combination of A and B is lesser than or equal to the sum of respective importance assigned to A and B such that $g\{A \cup B\} \leq g\{A\} + g\{B\}$, then it can be claimed that A and B are sharing sub-additive effect or being redundant to each other.
- If the importance of the combination of A and B is greater than or equal to the sum of respective importance assigned to A and B such that $g\{A \cup B\} \geq g\{A\} + g\{B\}$, then it can be claimed that A and B are expressing super-additive or synergistic effect.

To put it differently, the importance of an alliance of attributes can be actually estimated by understanding the interaction shared by the attributes. For instance, consider a multiple attribute problem comprising three attributes, $C = (c_1, c_2, c_3)$. Assume the individual importance or contribution of the attributes towards the performance of a target are $g\{c_1\} = 0.3$, $g\{c_2\} = 0.2$ and $g\{c_3\} = 0.1$ respectively, then the importance of fuzzy measure consisting c_1 and c_2, $g\{c_1, c_2\}$ can be estimated as follows:

- If c_1 and c_2 are being redundant, the presence or combination of both attributes does not significantly the performance of the target as both of them share some similar information. Therefore, too much importance should not be given on the combination of these attributes. Thus, the importance assigned on the combination of these two attributes should be at most 0.5; $g\{c_1, c_2\} \leq 0.3 + 0.2$ (sub-additive effect).
- If the synergy between c_1 and c_2 can significantly enhance the performance of the target, then more importance should be given on the combination of these attributes. Therefore, the importance assigned on these two attributes when considered jointly should be at least 0.5; $g\{c_1, c_2\} \geq 0.3 + 0.2$ (super-additive effect).
- If c_1 and c_2 are independent to each other, then the importance assigned on the combination of these two attributes should be equal to 0.5; $g\{c_1, c_2\} = 0.3 + 0.2$ (additive effect).

With the complete set of fuzzy measure values and available performance scores, the Choquet integral operator can be then applied to compute the aggregated or global score of each alternative. Let g be a fuzzy measure on $C = (c_1, c_2, \ldots, c_n)$ and $X = (x_1, x_2, \ldots, x_n)$ be the performance scores of an alternative with respect to the attributes in C. Suppose $x_1 \geq x_2 \geq \cdots \geq x_n$, then $T_n = (c_1, c_2, \ldots, c_n)$ and the aggregated score using Choquet integral can be identified using (1) [10].

$$\begin{aligned} &Choquet_g(x_1, x_2, \ldots, x_n) \\ &= x_n . g\{T_n\} + [x_{n-1} - x_n]. g\{T_{n-1}\} + \ \ldots + [x_1 - x_2]. g\{T_1\} \\ &= x_n . g\{c_1, c_2, \ldots, c_n\} + [x_{n-1} - x_n]. g\{c_1, c_2, \ldots, c_{n-1}\} + \cdots + [x_1 - x_2]. g\{c_1\} \end{aligned} \tag{1}$$

where T_n is determined based on the descending order of the performance scores . For better understanding, presume that the scores of a student, x in three subjects (attributes), Mathematics (x_M), Physic (x_P), Biology (x_B) are 75, 80, and 50 respectively. Hence, $x_P \geq x_M \geq x_B$, and so $T_n = \{\ P, M, B\ \}$. As a result, the aggregated score of the student based on Choquet integral model (1), $Choquet_g(x_M, x_P, x_B) = x_B . g\{\ P, M, B\ \} + (x_M - x_B). g\{\ P, M\} + (x_P - x_M). g\{\ P\}$.

3. An additive operator versus Choquet integral: a numerical example

Consider a student evaluation problem borrowed from [11]. Assume there are three students (A, B, and C) and we want to identify the best student who has no any weak points. Further assume that the overall performance scores of the students are assessed based on three subjects namely Mathematics (M), Statistics (S) and Literature (L) and their scores with respect to each subject, ranging from 0 to 20, are as shown in Table 1.

Table 1. Decision matrix for student evaluation problem

Student/subject	$\boldsymbol{M}$	$\boldsymbol{S}$	$\boldsymbol{L}$
$\boldsymbol{A}$	18	17	10
$\boldsymbol{B}$	10	12	18
$\boldsymbol{C}$	14	15	15

Suppose the overall performance score of each student is measured using SWA operator with an equal importance is assigned on each subject as follows: $w_M = \frac{1}{3}$, $w_S = \frac{1}{3}$ and $w_L = \frac{1}{3}$, then the computed overall scores and final ranking of each student can be summarized as presented in Table 2.

Table 2. Result for student evaluation problem using SWA operator

Student	$\boldsymbol{M}$	$\boldsymbol{S}$	$\boldsymbol{L}$	Overall performance score	Ranking
A	18	17	10	$\frac{1}{3}(18) + \frac{1}{3}(17) + \frac{1}{3}(10) = 15$	1
B	10	12	18	13.33	3
C	14	15	15	14.67	2

The result derived using SWA shows that student A has the highest rank followed by C and B. However, this result is somewhat counterintuitive because if the school is in search for a well-balanced student without any weak points, student C should be considered as the best one. The crux of this counterintuitive result is due to the usage of SWA operator which assumes that the subjects independently contribute to a student's overall performance. In other words, the subjects are assumed to express additive effect. Nevertheless, appropriate fuzzy measure values and Choquet integral operator can be used as a solution for this undesirable result.

Suppose the individual importance of each subject is assigned by adhering to the initial ratio (1:1:1) where $g\{M\} = g\{S\} = g\{L\} = 0.4$, and since mathematics and statistics are redundant to each other, the importance on the combination of these two subjects should be lesser than or equal to the sum of their individual importance. Therefore, assume $g\{M,S\} = 0.5 \leq g\{M\} + g\{S\}$. Besides, since it is believed that a student's overall performance increases drastically if he or she is being good at both mathematics and literature (or statistics and literature) or in other words, since mathematics (or statistics) shares super-additive effect with literature, then the importance assigned on the combination of L, M and L, S should be greater than or equal to the sum of their individual importance. Hence, assume $g\{L,M\} = 0.9 \geq g\{L\} + g\{M\}$ and $g\{L,S\} = 0.9 \geq g\{L\} + g\{S\}$. Not to mention, as per the two axioms of fuzzy measure, $g\{\emptyset\} = 0$ and $g\{M,S,L\} = 1$. The estimated fuzzy measure values and the scores as in Table 1 are then precisely substituted into Choqeut integral model to compute the overall performance score of each student. The result can be summarized as shown in Table 3.

Table 3: Result for student evaluation problem using Choquet Integral

Student	Overall performance score	Ranking
$\boldsymbol{A}$	Step 1: scores are ranked in descending order where $x_M \geq x_S \geq x_L$, and so $T_n = \{M,S,L\}$ Step 2: estimated fuzzy measure values and scores are replaced accordingly into Choquet integral model (1) $Choquet_\mu(x_M, x_S, x_L)$ $= x_L.\, g\{M,S,L\} + (x_S - x_L).\, g\{M,S\} + (x_M - x_S).\, g\{M\}$ $= (10)(1) + (17 - 10)(0.5) + (18 - 17)(0.4) = 13.9$	2
$\boldsymbol{B}$	14.2	3
$\boldsymbol{C}$	14.9	1

Based on the result in Table 3, it can be concluded that by applying Choquet integral which captures the interactions between the subjects, the expected student (student C), who has no any weak points is identified as the best student.

4. Fuzzy measure identification methods

As mentioned formerly, before employing Choquet integral, it is essential to identify the importance of all subsets of attributes or in other words, the fuzzy measure values. However, it is rather impossible or burdensome for the decision makers to subjectively estimate 2^n values of fuzzy measure especially when the number of attributes, n is sufficiently large [12, 13]. As a result, some identification methods such as minimization of squared error based method and constraint satisfaction based method were introduced [14] to aid the decision makers in estimating these values.

However, both methods came with several inconveniences. The usage of the former method requires some initial inputs or information on the desired overall score of each alternative which actually cannot be easily or accurately offered by the decision makers [15]. Meanwhile, the later method requires various types of initial inputs such as partial ranking of the alternatives, partial ranking of the attributes, intuitions about the importance of the attributes and interaction among attributes which are also could not be easily offered by the decision makers especially when they are ill-informed on the existing problem [16]. Besides, since both methods were developed based on optimization models, finding a solution via these methods still remains as a bottleneck for the decision makers especially when the analysis involves huge number of attributes.

Kojadinovic [17] formulated an unsupervised identification method in order to assist those decision makers who are unknowledgeable on the existing problem and facing difficulties in providing the necessary initial inputs. Via this method, the fuzzy measure values can be simply estimated based on the available performance scores by means of information-theoretic functions. However, the major shortcoming of this method is that it normally requires a large set of performance scores to estimate the fuzzy measure values precisely.

With the intention to further reduce the complexity allied with the process of estimating general fuzzy measure values, several patterns or subfamilies of fuzzy measure were proposed. Among many types of fuzzy measures, λ-measure which was introduced by Sugeno [18] emerges as one of the widely applied fuzzy measures due to its ease of usage, mathematical soundness and modest degree of freedom [19].

λ-measure can be defined as follows. Let $C = (c_1, c_2, \dots, c_n)$ be a finite set. A set function $g_\lambda(.)$ defined on the set of the subsets of C, $P(C)$, is called a λ- measure if it satisfies the following conditions:

- $g_\lambda: P(C) \to [0,1]$ and $g_\lambda(\emptyset) = 0, g_\lambda(C) = 1$ (boundary condition)
- $\forall A, B \in P(C)$, if $A \subseteq B$, then implies $g_\lambda(A) \ \leq g_\lambda(B)$ (monotonic condition)
- $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B)$, for all $A, B \in P(C)$ where $A \cap B = \emptyset$ and $\lambda \in \ [-1, +\infty]$.

Note that (a) and (b) are fundamental properties for any types of fuzzy measure and (c) is an additional property of λ-measure. λ-measure is constrained by a parameter λ, which describes the degree of additivity the attributes hold. According to Gürbüz, Alptekin and Alptekin [20] and Hu and Chen [21]:

- If λ < 0, then it implies that the attributes are sharing sub-additive (redundancy) effect. This means a significant increase in the performance of the target can be achieved by only enhancing some attributes in C which are having higher individual importance.
- If λ > 0, then it interprets that the attributes are sharing super-additive (synergy support) effect. This means a significant increase in the performance of the target only can be achieved by simultaneously enhancing all the attributes in C regardless of their individual importance.
- If λ = 0, then it indicates that the attributes are non-interactive.

As $C = c_j = \{c_1, c_2, \dots, c_n\}$ is finite, then the complete set of λ- measure values can be identified using equation (2).

$$g_\lambda\{c_1, c_2, \dots, c_n\} = \frac{1}{\lambda}\left| \ \prod_{j=1}^{n}(1 + \lambda g_j) - 1\right| \ for -1 < \lambda < +\infty \tag{2}$$

With regards to equation (2), $g_j = g_\lambda(c_j), j = 1, \dots, n$ denotes the fuzzy density or individual importance of each attribute. If $\sum_{j=1}^{n} g_j = 1, \ \lambda = 0$, but in the case of $\sum_{j=1}^{n} g_j \neq 1$, the λ value can be determined by solving equation (3).

$$1 + \lambda = \prod_{j=1}^{n}(1 + \lambda g_j) \tag{3}$$

In the early years, in order to simplify the process of identifying λ-measure values, various methods have been formulated by LeszczySnski, Penczek, and Grochulski [22], Sekita and Tabata [23], Tahani and Keller [24], and

Wierzchon [25], but these methods still suffer from large inputs or information requirement as the decision makers need to subjectively estimate the importance of all subsets of attributes to employ them.

As a result, Lee and Leekwang [26] developed a genetic algorithm (GA) based identification method which is computationally simpler and at the same time, it does not require complete subjective estimations for all subsets of attributes. A few years later, Chen and Wang [27] proposed another method based on sampling design and GA which is also simple, fast, easily programmable, and most importantly, it only requires a few data to run the solution procedure. Nevertheless, there are a few disadvantages incorporated with these methods. Firstly, it is claimed that the more inputs or estimations offered by decision makers on the values of subsets, the better would be the solution generated by these methods or in other words, the method failed to have a scheme to control the amount of information lost on the basis of generating a satisfactory solution. Secondly, since these two methods were developed based on GA, they do have many intrinsic flaws such as slow convergence speed and uncertainty of extreme position.

Takahagi [28] proposed a method based on diamond pair-wise comparisons that only require two types of inputs from the decision makers where on the horizontal axis of the diamond, they need to indicate the relative importance of attributes whereas the vertical axis is used to express the interaction shared by each pair of attributes. In nutshell, the method only requires $n(n-1)$ inputs from the decision makers. However, the method needs refinements on few aspects. Firstly, in order to ensure the decision makers are able to answer the diamond pair-wise comparisons without any complications, an understandable instruction is needed. Secondly, suitable interpretations on the axis, especially vertical axis should be provided. Thirdly, unlike analytical hierarchy process (AHP), where consistency index is defined, this method has not proposed any means to measure the consistency on the interaction comparisons.

Larbani, Huang, and Tzeng [29] proposed a novel and simple method for identifying λ^0-measure values; a type of fuzzy measure which does not hold the typical λ-measure characteristics. Using this method, the decision makers only need to provide $n(n-1)/2$ pair-wise comparisons of interdependence between attributes, and if possible a fuzzy evaluation on the individual importance of n attributes. However, unlike λ-measure, the method fails in providing clear indications on which attributes can be to be improved in order to significantly enhance the performance of the target.

Krishnan [30] introduced a hybrid multiple attribute method that uses factor analysis as one of its components to extract the large set of attributes into fewer independent factors as a means to reduce the actual number of fuzzy measure values that need to be identified before employing Choquet integral from 2^n to $\sum_{p=1}^{q} 2^{|f_p|}$ where $f_p = (f_1, f_2, \ldots, f_q)$ represents the set of extracted factors, q denotes the total number of factors, and $|f_p|$ represent number of attributes within factor p. However, the process of collecting data to perform a meaningful factor analysis could be time consuming as it may require the involvement of large number of respondents.

Table 4 recaps the fuzzy measure identification methods discussed in this section together with their advantages and disadvantages.

5. Conclusion and recommendations

Employing additive operators in multiple attribute analysis to aggregate the performance scores of an alternative could lead to faulty results or decisions as these operators assume independencies among attributes, which is completely fallacious in reality. This paper suggests that instead of using the conventional additive operators, Choquet integral should be utilized for the aggregation purpose as it has the ability to deal with interactive attributes. The only drawback of Choquet integral is it requires a prior identification of 2^n values of fuzzy measure where the complexity of identifying these values hikes up with the increasing number of evaluation attributes, n.

Many methods have been proposed to simplify the process of identifying fuzzy measure values and each of these methods has its own pros and cons. Through this paper, it can be noticed that the usability of each identification method can be actually measured based on three aspects: types of inputs required by the methods, number of inputs required by the methods and number of fuzzy measure values that need to be identified through each method. Future research can focus to further enhance the methods highlighted in this paper or formulate new versions of methods which:

- only require types of inputs or information that can be easily offered by decision makers and/or,
- only demand minimal number of inputs from decision makers and/or,
- significantly reduce the actual number of fuzzy measure values that need to be identified.

It can be claimed that the simpler is the fuzzy measure identification procedure, the more motivated will be the decision makers in utilizing the advantageous Choquet integral.

Table 4. Fuzzy measure identification methods

Methods	Advantages (A)/ Disadvantages(D)
Minimization of squared error and constraint satisfaction based method *(used to identify general fuzzy measure values)*	(D) former method requires inputs or information on the desired overall score of each alternative which cannot be easily or accurately offered by the decision makers (D) later method requires various types of initial inputs or information such as partial ranking of the alternatives, partial ranking of the attributes, intuitions about the importance of the attributes and interaction among attributes (D) finding the solution via these methods is somewhat complex for the analysis involving huge number of attributes, n
Unsupervised identification method *(used to identify general fuzzy measure values)*	(A) helpful for those decision makers who are ill-informed on the existing problem and having the complications in providing the necessary initial inputs or information (D) normally requires a large number of performance scores to estimate the fuzzy measure values precisely
Methods by [22-25] *(used to identify λ- measure values)*	(D) large inputs or information requirement as the decision makers need to subjectively estimate the importance of all subsets of attributes
GA based method/ sampling design and GA based method *(used to identify λ- measure values)*	(A) simple , fast , and easy to be programmed (A) only requires a few data to run the solution procedure (D) failed to have a scheme to control the amount of information lost (D) many intrinsic flaws such as slow convergence speed and uncertainty of extreme position
Diamond pair-wise comparisons method *(used to identify λ- measure values)*	(A) only requires two types of inputs from the decision makers; relative importance of attributes and interaction shared by each pair of attributes (A) only requires $n(n-1)$ inputs from the decision makers (D) the instruction for the decision makers to answer the diamond pair-wise comparisons is somewhat vague (D) requires suitable interpretations on the vertical axis (D) a method to measure the consistency of interaction comparisons is not available
λ^0-measure identification method *(used to identify λ^0-measure values)*	(A) decision makers only need to provide $n(n-1)/2$ pair-wise comparisons of interdependence between attributes, and if possible a fuzzy evaluation on the individual importance of n attributes; total number of inputs required is $n[(n-1)/2+1]$ (D) fails in providing clear indications on which attributes can be to be improved in order to significantly enhance the performance of alternatives
Hybrid MAA model using factor analysis *(used to identify λ- measure values)*	(A) reduces the actual number of fuzzy measure values that need to be identified before employing Choquet integral from 2^n to $\sum_{p=1}^{q} 2^{\lvert f_p \rvert}$ (D) data collection for the purpose of performing factor analysis could be time consuming and may involve large number of respondents

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