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H-Statistic With Winsorized Modified One-Step *M*-Estimator For Two Independent Groups Design

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Abstract. Two-sample independent *t*-test is a classical method which is widely used to test the equality of two groups. However, this test is easily affected by any deviation in normality, more obvious when heterogeneity of variances and group sizes exist. It is well known that the violation in the assumption of these tests will lead to inflation in Type *I* error rate and depression in statistical test power. In mitigating the problem, robust methods can be used as alternatives. One such method is *H*-statistic. When used with modified one-step *M*-estimator (*MOM*), this test statistic (*MOM-H*) produce good control of Type *I* error even under small sample size but inconsistent across certain conditions investigated. Furthermore, power of the test is low which might be due to the trimming process. In this study, *MOM* is winsorized (*WMOM*) to sustain the original sample size. The *H*-statistic with *WMOM* as the central tendency measures (denoted as *WMOM-H*) showed better control of Type *I* error as compared to *MOM-H* especially under balance design regardless of the shapes of distribution investigated in the study. It also performed well under highly skewed and heavy tailed distribution for unbalanced design. In general, this study demonstrated that winsorization process (*WMOM*) could improve the performance of *H*-statistic in terms of Type *I* error rate control.

Keywords: Winsorize, Type *I* error, Robust Method.
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INTRODUCTION

The methods for testing the equality of two independent groups design is being continually improve since many years ago. The ultimate goal of the nonstop improvement made is to figure out a method that can control Type *I* error rate while simultaneously increasing the statistical power. The most popular method for testing the two independent groups is *t*-test. However, the test does not perform well when normality assumption is violated and worsens when heterogeneity of variance and sample size exist [1]. Violation from these assumption will eventually lead to inflation of Type *I* error rate and depression in statistical power [2][3][4][5][6]. As alternative, nonparametric test could be the choice when the issue of violation, however, this test seems to be less powerful and the sample size need to be large enough in order to achieve reliable results [7]. To alleviate the aforementioned problems, robust statistical methods are recommended. Robust methods are able to control Type *I* error and produce reasonable statistical power even when normality and homogeneity of variance are violated [8][9][10][11][12][13]. Trimming is one of the robust approaches to deal with non normality [14][15]. There are two type of trimming approaches namely symmetrical and asymmetrical trimming. The former approach is to trim symmetrically both left and right tail of the data based on the predetermined amount. On the other hand, the latter approach trimmed data based on the distribution of the data [16]. The amount of trimming is based on either predetermined or empirically determined. However, the predetermined trimming will be unnecessary when the data is normal, but trimming based on the former approach will trim the data regardless of the shape, but not in the case of the latter. One estimator using the latter approach is Modified one-step M-estimator (MOM). MOM is a location estimator. When used as the location measure for H-statistic [17] in testing for the equality of groups, the combination produced good control of Type I error but low in power [11][18]. This could be due to the loss of information caused by the trimming process.

To avoid loss of information and maintain the original sample size winsorization is another robust approach to deal with non-normal distribution. Winsorization was proposed by Charles P. Winsor (1895-1951), who was a biostatistician. This procedure focuses around the center of the data rather than the tails at both sides to avoid bias [8]. Basically, the procedure in winsorizing is almost similar with trimming but the data that are supposed to be trimmed will be replaced with the highest and lowest end of the remaining data respectively [19][20]. According to Dixon [21] and Rivest [22], this approach is able to control Type *I* error even under skewed distribution. Ahmad Mahir and Al-Khazaleh [23] applied adaptive winsorize in estimating missing values in time series data and found

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that the method performed consistently better than other classical methods. In a recent study, winsorized *MOM* by Haddad, Syed Yahaya and Alfaro [24] was used in as the centre measures in Hotelling's T^2 control chart. The chart was found to be robust (in control of Type I error) regardless of the conditions investigated and attained desirable value of statistical power.

In this study, we used winsorized MOM as the center measures of H statistics which was denoted as WMOM-H to test for two independent groups. The performance of WMOM-H was appraised using Type I error rates (robustness). The results of *WMOM-H* in this study were compared with other methods namely *MOM-H*, *t*-test and Mann-Whitney.

Winsorized Modified One-step *M*-estimator *(WMOM)*

Let $Y_{ij} = (Y_{1j}, Y_{2j}, ..., Y_{n_j j})$ be a sample from an unknown distribution F_j winsorized Modified one-step *M*-estimator (*WMOM*) proposed by Haddad et al. (2012) is given by;

$$\hat{\theta}_j = \sum_j^{n_j} \frac{Y_{new(i)j}}{n_j} \tag{1}$$

where
j = number of groups
$Y_{new(i)j}$ = the i^{th} ordered observations in group j (after replacement of extreme value)
n_j = # of observations for group j.

$$Y_{new(i)j} = \begin{cases} Y_{(i_1+1)j}, & (Y_{ij} - \hat{M}_j) < -2.24(MADn_j) \\ Y_{(i)j}, & -2.24(MADn_j) \le (Y_{ij} - \hat{M}_j) \le 2.24(MADn_j) \\ Y_{(n_j - i_2)j}, & (Y_{ij} - \hat{M}_j) > 2.24(MADn_j) \end{cases} \tag{2}$$

$$i_1 = \text{\# of observations } Y_{ij} \text{ such that } (Y_{ij} - \hat{M}_j) < -2.24(MADn_j), \tag{3}$$

$$i_2 = \text{\# of observations } Y_{ij} \text{ such that } (Y_{ij} - \hat{M}_j) > 2.24(MADn_j), \tag{4}$$

$(Y_{ij} - \hat{M}_j) < -2.24(MADn_j)$ and $(Y_{ij} - \hat{M}_j) > 2.24(MADn_j)$ are the formulas to determine the extreme value in the given data set. After replacing those extreme values, the *WMOM* value, $\hat{\theta}_j$, is estimated by averaging the entire new data.

H Statistic

H-statistic is originally proposed by Schrader and Hettmansperger [17] which is readily adaptable to any central tendency measure. The statistic is defined as

$$H = \frac{1}{N} \sum_{j=1}^{J} n_j (\hat{\theta}_j - \hat{\theta}_.)^2 \tag{5}$$

$$H = \sum_j n_j \tag{6}$$

$$\hat{\theta}_{.} = \sum_j \hat{\theta}_j / J \tag{7}$$

In this study, $\hat{\theta}_j$ is the WMOM.

EMPIRICAL INVESTIGATION

In order to highlight the strength and weakness of *WMOM-H*, a few variables were manipulated to create the desired conditions for testing *WMOM-H*. This study will focus on two groups with small sample size for both balanced and unbalanced design. For balanced design, a sample of n = 20 was assigned to each group with group variance of 1. On the other hand, for unbalanced design, the groups were divided into $n_1 = 15$ and $n_2 = 25$ with group variance of 1:36 for both positive and negative pairings. Positive pairing is the pairing of largest group size with largest variance or smallest group size with smallest variance and vice versa for negative pairing.

Each of the design was then tested with three different types of distributions which included normal, Chi-Square with three degrees of freedom and g-and-h distribution with $g = h = 0.5$ to represent perfect condition, mild departure condition and extreme departure condition respectively. These distributions are transformations of the standard normal distribution. For each design, 5000 datasets were simulated and 599 bootstrap samples were generated. The random samples were drawn using SAS generator RANNOR [25].

RESULTS AND CONCLUSION

Results of Type *I* error rate for the simulation are shown in TABLE (1) and (2) for balanced and unbalanced design respectively. According to Bradley [26], a procedure is considered as robust when empirical Type *I* error, $\hat{\alpha}$ falls between $0.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$ Thus, for a nominal level of $\alpha = 0.05$, the Type *I* error rate should fall between 0.025 and 0.075.

TABLE(1). Type *I* error rate for balance design [N = 40 (20, 20) and variances (1:1)]

Distributions	***WMOM-H***	***MOM-H***	***t*-test**	**Mann-Whitney**
Normal	0.0526	0.0410	0.0528	0.0526
Chi-Square, χ^2_3	0.0526	0.0422	0.0500	0.0566
g = 0.5, h = 0.5	0.0396	0.0244	0.0288	0.0526
Grand Average	**0.0483**	**0.0359**	**0.0439**	**0.0539**

TABLE(2). Type *I* error rate for unbalance design [N = 40 (15, 25) and variances (1:36)]

Distributions	**Pairing**	***WMOM-H***	***MOM-H***	***t*-test**	**Mann-Whitney**
Normal	Positive	0.0628	0.0496	0.0198	0.0448
	Negative	0.0570	0.0470	0.1268	0.1086
	Average	**0.0599**	**0.0483**	**0.0733**	**0.0767**
Chi-Square, χ^2_3	Positive	0.0684	0.0626	0.0238	0.0666
	Negative	0.0674	0.0642	0.1678	0.1312
	Average	**0.0679**	**0.0634**	**0.0958**	**0.0989**
g = 0.5, h = 0.5	Positive	0.0532	0.0328	0.0118	0.0426
	Negative	0.0436	0.0324	0.1048	0.0976
	Average	**0.0484**	**0.0326**	**0.0583**	**0.0701**
Grand Average		**0.0587**	**0.0481**	**0.0758**	**0.0819**

For balanced design as in TABLE(1), Type *I* error rates for *WMOM-H, MOM-H, t*-test and Mann-Whitney are within the 0.025 and 0.075 interval for all distributions and are considered robust according to Bradley's robust criteria. Based on the overall performance as represented by the grand average, *WMOM-H* procedure produced the closest average Type I error to the nominal level, but the most consistent rates could be observed under Mann Whitney column.

On the other hand, for unbalanced design as in TABLE (2), only *WMOM-H* and *MOM-H* produced Type *I* error rates within the Bradley's robust criteria regardless of the distributions and pairings. For Mann Whitney procedure, only Type I error rates under positive pairing are robust while t-test failed to produce any robust condition.

Comparing between *WMOM-H* and *MOM-H* procedures, we observed that the Type I error rates for *WMOM-H* are more consistent than *MOM-H* and *WMOM-H* are more robust under extreme condition.

In a nut shell, *WMOM-H* is proven to perform well in both balanced and unbalanced design. This can be verified by the results in TABLE(1) and (2) which shows that the Type *I* error rates produced are consistent and close to the nominal level, 0.05, regardless of the distributions pairings and designs. This robust method which is based on winsorized *MOM* can suggested as an alternative for classical methods is comparing the equality of two groups as it can perform consistently and has good control of Type *I* error (robust) even under the influence of non-normality and variance heterogeneity.

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