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Numerical Solution of Third Order Ordinary Differential Equations Using a Seven-Step Block Method

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Abstract

This paper aims to provide a direct solution to third order initial value problems of ordinary differential equations. Multistep collocation approach is adopted in the derivation of the method. The new block method is zero-stable, consistent and convergent. The application of the new method to solving differential equations gives better results when compared with the existing methods.

Mathematics Subject Classification: 65L06, 65L05

Keywords: Interpolation, Block Method, Third Order Ordinary Differential Equations

1 Introduction

This paper considers the general third order initial value problems of ordinary differential equations of the form

$$y''' = f(x, y, y', y'') \quad y(a) = y_0, y'(a) = w_1, y''(a) = w_2, a, y, f \in \Re \tag{1}$$

The numerical solution of higher order ordinary differential equations through the reduction method was majorly used in the past in such a way that the differential equation will be reduced to its equivalent system of first order and suitable numerical methods designed for first order were used to solve the resulting systems (Lambert [1], Brown [2], Jeltsch [3]).

Direct methods of solving higher order ordinary differential equations had been examined by some authors like Awoyemi [4], Mohammed [5] and Omar and Suleiman [6–8] .One of the methods of solving higher order ordinary differential equations directly is predictor-corrected method and this is discussed extensively in Awoyemi [4], Adesanya et al. [9], Odekunle et al. [10], Kayode and Adeyeye [11] and Kayode and Obarhua [12]. It is observed that the method has a lot of setbacks and this includes:

- Too many functions to be evaluated per step due to the involvement of predictors which always result to computational burden that affects the accuracy of the method.

- The computer program designed to examine the accuracy of the method are always found to be complicated

- A lot of computer time and human effort are involved.

The use of method without predictors was adopted by Omar & Suleiman [6–8], Ehigie et al. [13] and Adesanya et al. [14]. These scholars independently developed methods for solving higher order ordinary differential equations to proffer solution to the setbacks in predictor-corrector method. The development of block methods for solving second order ordinary differential equations have been carried out by many researchers but few literatures are found on block method for solving (1) directly. Yap, Ismail and Senu [15] developed accurate block hybrid collocation method with order six for solving third order ordinary differential equations. Furthermore, an accurate scheme by block method having an order seven for solving (1) directly was developed by Olabode [16] but the method is of lower accuracy.

This paper proposes a block method with uniform order eight for solving (1) directly. The use of approximated power series as an interpolation equation and its derivative as a collocation equation is adopted in the development of the method.

2 Derivation of the Method

We consider power series approximate solution of the form

$$y(x)=\sum_{j=0}^{k+3} a_j x^j \tag{2}$$

as an interpolation equation. Where k=7. The first, second and third derivatives of (2) give

$$y'(x)=\sum_{j=0}^{k+3} j a_j x^{j-1} \tag{3}$$

$$y''(x)=\sum_{j=0}^{k+3} j(j-1) a_j x^{j-2} \tag{4}$$

$$y'''(x)=\sum_{j=0}^{k+3} j(j-1)(j-2) a_j x^{j-3} \tag{5}$$

We interpolate equation (2) at $x=x_{n+i}, i=3(1)5$ and equation (5) is collocated at $x=x_{n+i}, i=0(1)7$. The interpolation and collocation equations give

$$AX=B \tag{6}$$

where,

$$A=\begin{pmatrix}
1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 & x_{n+3}^9 & x_{n+3}^{10}\\
1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 & x_{n+4}^8 & x_{n+4}^9 & x_{n+4}^{10}\\
1 & x_{n+5} & x_{n+5}^2 & x_{n+5}^3 & x_{n+5}^4 & x_{n+5}^5 & x_{n+5}^6 & x_{n+5}^7 & x_{n+5}^8 & x_{n+5}^9 & x_{n+5}^{10}\\
0 & 0 & 0 & 6 & 24x_n & 60x_n^2 & 120x_n^3 & 210x_n^4 & 336x_n^5 & 504x_n^6 & 720x_n^7\\
0 & 0 & 0 & 6 & 24x_{n+1} & 60x_{n+1}^2 & 120x_{n+1}^3 & 210x_{n+1}^4 & 336x_{n+1}^5 & 504x_{n+1}^6 & 720x_{n+1}^7\\
0 & 0 & 0 & 6 & 24x_{n+2} & 60x_{n+2}^2 & 120x_{n+2}^3 & 210x_{n+2}^4 & 336x_{n+2}^5 & 504x_{n+2}^6 & 720x_{n+2}^7\\
0 & 0 & 0 & 6 & 24x_{n+3} & 60x_{n+3}^2 & 120x_{n+3}^3 & 210x_{n+3}^4 & 336x_{n+3}^5 & 504x_{n+3}^6 & 720x_{n+3}^7\\
0 & 0 & 0 & 6 & 24x_{n+4} & 60x_{n+4}^2 & 120x_{n+4}^3 & 210x_{n+4}^4 & 336x_{n+4}^5 & 504x_{n+4}^6 & 720x_{n+4}^7\\
0 & 0 & 0 & 6 & 24x_{n+5} & 60x_{n+5}^2 & 120x_{n+5}^3 & 210x_{n+5}^4 & 336x_{n+5}^5 & 504x_{n+5}^6 & 720x_{n+5}^7\\
0 & 0 & 0 & 6 & 24x_{n+6} & 60x_{n+6}^2 & 120x_{n+6}^3 & 210x_{n+6}^4 & 336x_{n+6}^5 & 504x_{n+6}^6 & 720x_{n+6}^7\\
0 & 0 & 0 & 6 & 24x_{n+7} & 60x_{n+7}^2 & 120x_{n+7}^3 & 210x_{n+7}^4 & 336x_{n+7}^5 & 504x_{n+7}^6 & 720x_{n+7}^7
\end{pmatrix}$$

$X=[a_0,a_1,a_2,a_3,a_4, a_5,a_6, \cdots a_{k+3}]^T$ and $B=[y_{n+3}, y_{n+4}, y_{n+5}, f_n, f_{n+1}, f_{n+2}, \cdots, f_{n+k}]^T$.

Solving for the unknown variables a_j using Gaussian elimination method which are then substituted into the interpolation equation (2) produces a continuous implicit scheme of the form

$$y(z) = \sum_{j=3}^{k=2} \alpha_j(z) y_{n+j} + h^3 \sum_{j=0}^{k} \beta_j(z) f_{n+j} \tag{7}$$

The coefficients of $\alpha_j(z)$ and $\beta_j(z)$ are given as

$$\begin{pmatrix} \alpha_3(z) \\ \alpha_4(z) \\ \alpha_5(z) \end{pmatrix} = \begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 3 & \frac{5}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} z^0 \\ z^1 \\ z^2 \end{pmatrix},$$

$$\begin{pmatrix} \beta_0(z) \\ \beta_1(z) \\ \beta_2(z) \\ \beta_3(z) \\ \beta_4(z) \\ \beta_5(z) \\ \beta_6(z) \\ \beta_7(z) \end{pmatrix} = L \begin{pmatrix} z^0 \\ z^1 \\ z^2 \\ z^3 \\ z^4 \\ z^5 \\ z^6 \\ z^7 \\ z^8 \\ z^9 \\ z^{10} \end{pmatrix}$$

where,

$z = \dfrac{x - x_{n+6}}{h}$ and

$$L = \begin{pmatrix}
\frac{3720}{7257600} & \frac{904}{7257600} & \frac{-6430}{7257600} & 0 & \frac{7200}{7257600} & \frac{3696}{7257600} & \frac{-588}{7257600} & \frac{-960}{7257600} & \frac{-300}{7257600} & \frac{-40}{7257600} & \frac{-2}{7257600} \\
\frac{-16740}{3628800} & \frac{-5250}{3628800} & \frac{26840}{3628800} & 0 & \frac{30240}{3628800} & \frac{-15120}{3628800} & \frac{2688}{3628800} & \frac{3960}{3628800} & \frac{1185}{3628800} & \frac{150}{3628800} & \frac{7}{3628800} \\
\frac{68940}{3628800} & \frac{27768}{3628800} & \frac{-99737}{3628800} & 0 & \frac{113400}{3628800} & \frac{54432}{3628800} & \frac{-11214}{3628800} & \frac{-14400}{3628800} & \frac{-4050}{3628800} & \frac{-480}{3628800} & \frac{21}{3628800} \\
\frac{-143580}{3628800} & \frac{-43870}{3628800} & \frac{233800}{3628800} & 0 & \frac{-252000}{3628800} & \frac{-112560}{3628800} & \frac{28560}{3628800} & \frac{30120}{3628800} & \frac{7725}{3628800} & \frac{850}{3628800} & \frac{35}{3628800} \\
\frac{1969020}{3628800} & \frac{2708580}{3628800} & \frac{524915}{3628800} & 0 & \frac{378000}{3628800} & \frac{143640}{3628800} & \frac{-50610}{3628800} & \frac{-38880}{3628800} & \frac{-8850}{3628800} & \frac{-900}{3628800} & \frac{-35}{3628800} \\
\frac{1734660}{3628800} & \frac{3692418}{3628800} & \frac{2296320}{3628800} & 0 & \frac{-453600}{3628800} & \frac{-81648}{3628800} & \frac{58464}{3628800} & \frac{30600}{3628800} & \frac{6075}{3628800} & \frac{570}{3628800} & \frac{21}{3628800} \\
\frac{12660}{3628800} & \frac{279200}{3628800} & \frac{666355}{3628800} & \frac{604800}{3628800} & \frac{219240}{3628800} & \frac{-11760}{3628800} & \frac{-37338}{3628800} & \frac{-13440}{3628800} & \frac{-2310}{3628800} & \frac{-200}{3628800} & \frac{-7}{3628800} \\
\frac{1980}{3628800} & \frac{-6498}{3628800} & \frac{-16480}{3628800} & 0 & \frac{21600}{3628800} & \frac{21168}{3628800} & \frac{9744}{3628800} & \frac{2520}{3628800} & \frac{375}{3628800} & \frac{30}{3628800} & \frac{1}{3628800}
\end{pmatrix}.$$

In order to find the discrete schemes and its derivatives, equation (7) is evaluated at $x = x_{n+i}, i = 0,1,2,6,7.$ The first and second derivatives are evaluated at $x = x_{n+i}, i = 0(1)7$. These discrete schemes and its derivatives at x_n are combined in a matrix form to produce a block of the form:

$$EY_{N+1} = FY_N + hGY'_N + h^2HY''_N + h^3(IF_{N+1} + JF_N) \qquad (8)$$

where

$$Y_{N+1} = [y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+5}, y_{n+6}, y_{n+7}]^T, Y_N = [y_{n-6}, y_{n-5}, y_{n-4}, y_{n-3}, y_{n-2}, y_{n-1}, y_n]^T,$$
$$Y'_N = [y'_{n-6}, y'_{n-5}, y'_{n-4}, y'_{n-3}, y'_{n-2}, y'_{n-1}, y'_n]^T, Y''_N = [y''_{n-6}, y''_{n-5}, y''_{n-4}, y''_{n-3}, y''_{n-2}, y''_{n-1}, y''_n]^T,$$
$$F_{N+1} = [f_{n+1}, f_{n+2}, f_{n+3}, f_{n+4}, f_{n+5}, f_{n+6}, f_{n+7}]^T, F_N = [f_{n-6}, f_{n-5}, f_{n-4}, f_{n-3}, f_{n-2}, f_{n-1}, f_n]^T,$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} G = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{pmatrix},$$

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{25}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 18 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{49}{2} \end{pmatrix},$$

$$I=\begin{pmatrix}
\frac{562618}{362800} & \frac{-662757}{362800} & \frac{694230}{362800} & \frac{-506675}{362800} & \frac{239406}{362800} & \frac{-65823}{362800} & \frac{8002}{362800}\\
\frac{38762}{28350} & \frac{-32823}{28350} & \frac{34730}{28350} & \frac{-25300}{28350} & \frac{11934}{28350} & \frac{-3277}{28350} & \frac{398}{28350}\\
\frac{183654}{44800} & \frac{-105381}{44800} & \frac{135450}{44800} & \frac{-98955}{44800} & \frac{46818}{44800} & \frac{-12879}{44800} & \frac{1566}{44800}\\
\frac{118432}{14175} & \frac{-46608}{14175} & \frac{86880}{14175} & \frac{-58520}{14175} & \frac{27744}{14175} & \frac{-7632}{14175} & \frac{928}{14175}\\
\frac{2052250}{145152} & \frac{-577125}{145152} & \frac{1603750}{145152} & \frac{-891875}{145152} & \frac{456750}{145152} & \frac{-125375}{145152} & \frac{15250}{145152}\\
\frac{30024}{1400} & \frac{-6156}{1400} & \frac{24840}{1400} & \frac{-10800}{1400} & \frac{7128}{1400} & \frac{-1764}{1400} & \frac{216}{1400}\\
\frac{15697738}{518400} & \frac{-2369787}{518400} & \frac{13613670}{518400} & \frac{-4621925}{518400} & \frac{4336206}{518400} & \frac{-655473}{518400} & \frac{114562}{518400}
\end{pmatrix}$$

and

$$J=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \frac{335799}{3628800}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{13376}{28350}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{51327}{44800}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{29976}{14175}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{2451875}{145152}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{6912}{1400}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{3520209}{518400}
\end{pmatrix}.$$

The first and second derivatives of (8) give

$$\begin{pmatrix} y'_{n+1}\\ y'_{n+2}\\ y'_{n+3}\\ y'_{n+4}\\ y'_{n+5}\\ y'_{n+6}\\ y'_{n+7}\end{pmatrix}=\begin{pmatrix}1\\1\\1\\1\\1\\1\\1\end{pmatrix}y'_n+\begin{pmatrix}1\\2\\3\\4\\5\\6\\7\end{pmatrix}hy''_n+h^2\begin{pmatrix}
\frac{308455}{1344780} & \frac{704619}{1344780} & \frac{-759771}{1344780} & \frac{785218}{1344780} & \frac{-569816}{1344780} & \frac{268387}{1344780} & \frac{-73642}{1344780} & \frac{8940}{1344780}\\
\frac{14939}{28350} & \frac{55642}{28350} & \frac{-34986}{28350} & \frac{39950}{28350} & \frac{-29405}{28350} & \frac{13926}{28350} & \frac{-3832}{28350} & \frac{466}{28350}\\
\frac{496773}{604800} & \frac{2113614}{604800} & \frac{-650997}{604800} & \frac{1394820}{604800} & \frac{-985365}{604800} & \frac{465102}{604800} & \frac{-127899}{604800} & \frac{15552}{604800}\\
\frac{15824}{14175} & \frac{71152}{14175} & \frac{-11496}{14175} & \frac{56720}{14175} & \frac{-29960}{14175} & \frac{14736}{14175} & \frac{-4072}{14175} & \frac{496}{14175}\\
\frac{102425}{72576} & \frac{475000}{72576} & \frac{-40125}{72576} & \frac{421250}{72576} & \frac{-130625}{72576} & \frac{102900}{72576} & \frac{-26875}{72576} & \frac{3250}{72576}\\
\frac{597}{350} & \frac{2826}{350} & \frac{-108}{350} & \frac{2670}{350} & \frac{-495}{350} & \frac{918}{350} & \frac{-126}{350} & \frac{18}{350}\\
\frac{86506}{43182} & \frac{413600}{43182} & \frac{1200}{43182} & \frac{400000}{43182} & \frac{-34000}{43182} & \frac{160800}{43182} & \frac{24400}{43182} & \frac{5453}{43182}
\end{pmatrix}\begin{pmatrix} f_n\\ f_{n+1}\\ f_{n+2}\\ f_{n+3}\\ f_{n+4}\\ f_{n+5}\\ f_{n+6}\\ f_{n+7}\end{pmatrix}$$

and

$$\begin{pmatrix} y''_{n+1} \\ y''_{n+2} \\ y''_{n+3} \\ y''_{n+4} \\ y''_{n+5} \\ y''_{n+6} \\ y''_{n+7} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} y''_n + h \begin{pmatrix} \frac{36799}{120960} & \frac{139849}{120960} & \frac{-121797}{120960} & \frac{123133}{120960} & \frac{-88547}{120960} & \frac{41499}{120960} & \frac{-11351}{120960} & \frac{1375}{120960} \\ \frac{5535}{18900} & \frac{29320}{18900} & \frac{-3195}{18900} & \frac{12240}{18900} & \frac{-9635}{18900} & \frac{4680}{18900} & \frac{-1305}{18900} & \frac{160}{18900} \\ \frac{6625}{22400} & \frac{33975}{22400} & \frac{6885}{22400} & \frac{29635}{22400} & \frac{-15165}{22400} & \frac{6885}{22400} & \frac{-1865}{22400} & \frac{225}{22400} \\ \frac{278}{945} & \frac{1448}{945} & \frac{216}{945} & \frac{1784}{945} & \frac{-106}{945} & \frac{216}{945} & \frac{-64}{945} & \frac{8}{945} \\ \frac{7155}{24192} & \frac{36725}{24192} & \frac{6975}{24192} & \frac{41625}{24192} & \frac{13625}{24192} & \frac{17055}{24192} & \frac{-2475}{24192} & \frac{275}{24192} \\ \frac{41}{140} & \frac{216}{140} & \frac{27}{140} & \frac{272}{140} & \frac{27}{140} & \frac{216}{140} & \frac{41}{140} & 0 \\ \frac{5257}{17280} & \frac{25039}{17280} & \frac{9261}{17280} & \frac{20923}{17280} & \frac{20923}{17280} & \frac{9261}{17280} & \frac{25039}{17280} & \frac{5257}{17280} \end{pmatrix} \begin{pmatrix} f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \end{pmatrix}$$

3 Analysis of the Method

3.1 Order of the Method

Our method (8) has a uniform order $[8,8,8,8,8,8,8]^T$ together with error constants $[\frac{-165}{89141}, \frac{-179}{15283}, \frac{-2889}{98560}, \frac{-256}{4661}, \frac{-460}{5209}, \frac{-999}{7700}, \frac{-2402}{13423}]^T$ as defined by Lambert [1].

3.2. Zero Stability

The method (8) is said to be zero-stable as $h \to 0$ if the roots $z_p, p = 1,2,....., N$ of the first characteristic polynomial $\rho(z) = 0$, that is, $\rho(z) = \det(EA' - F)$ satisfies $|z_p| \le 1$ and for the root with $|z_p| = 1$ the multiplicity must not exceed the order of differential equation under consideration. This is demonstrated below

$$\rho(z) = \left\| \begin{pmatrix} z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & z & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & z \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\| = 0$$

This gives $\rho(z) = z^6(z-1)$ which implies $z = 0,0,0,0,0,0,1$. Therefore, our method

(8) is convergent since it is zero-stable and consistent (Henrici [17]).

3.3. Numerical Experiments

The following differential problems are considered for comparing purposes

1: $y''' = -y \quad y(0) = 1, y'(0) = -1, y''(0) = 1, \; 0 \le x \le 1, h = 0.1$

Exact solution: $y(x) = e^{-x}$

Problem (1) stated above was solved by Olabode (2007) having order *P*=9. Our new method with order *P*=8 was also applied to solve the same problem. The results are shown below in Table 1.

Table 1: Results of the new method are compared with Olabode [18]

x	Exact Solution	Numerical Result	Error in our new method P=8	Olabode [18], *P*=9
0.1	0.904837418035959520	0.904837418033821120	2.138401E-12	2.1760E-12
0.2	0.818730753077981820	0.818730753077376310	6.055156E-13	1.3935E-11
0.3	0.740818220681717770	0.740818220674322010	7.395751E-12	3.4443E-11
0.4	0.670320046035639330	0.670320046037797490	2.158163E-12	6.4477E-11
0.5	0.606530659712633420	0.606530659727479220	1.484579E-11	1.0316E-10
0.6	0.548811636094026390	0.548811636105011600	1.098521E-11	1.4979E-10
0.7	0.496585303791409470	0.496585303822838330	3.142886E-11	2.0486E-10
0.8	0.449328964117221560	0.449328964140316870	2.309530E-11	2.6756E-10
0.9	0.406569659740599110	0.406569659792140600	5.154149E-11	6.9382E-10
1.0	0.367879441171442330	0.367879441253447680	8.200535E-11	1.4224E-10

2. $y''' = e^{x} \quad y(0) = 3, y'(0) = 1, y''(0) = 5 \;\; 0 \le x \le 1, h = 0.1$

Exact solution: $y(x) = 2 + 2x^2 + e^{x}$

This problem 2 above was considered by Awoyemi, Kayode and Adoghe (2014). We applied our new method to solve the same problem and the results are demonstrated in Table 2 below.

Table 2: Results of the new method are compared with Awoyemi, Kayode and Adoghe [19]

x	Exact Solution	Numerical Result	Error in our new method P=8	Awoyemi, Kayode and Adoghe [19], P=9
0.1	3.125170918075647700	3.125170918075673000	2.531308E-14	00000E+00
0.2	3.301402758160169700	3.301402758160330900	1.612044E-13	2.8422E-13
0.3	3.529858807576003300	3.529858807576405700	4.023448E-13	1.6729E-12
0.4	3.811824697641270600	3.811824697642024300	7.536194E-13	2.9983E-11
0.5	4.148721270700128200	4.148721270701340600	1.212364E-12	3.1673E-11
0.6	4.542118800390509700	4.542118800392290500	1.780798E-12	9.1899E-11
0.7	4.993752707470477500	4.993752707472934200	2.456702E-12	8.9531E-11
0.8	5.505540928492468600	5.505540928470347600	2.212097E-11	1.9168E-10
0.9	6.079603111156950800	6.079603111104630900	5.231993E-11	2.1110E-10
1.0	6.718281828459045500	6.718281828370444400	8.860113E-11	4.9398E-10
1.1	7.424166023946433800	7.424166023814952300	1.314815E-10	8.6728E-10
1.2	8.200116922736548000	8.200116922555027400	1.815206E-10	2.3764E-09

3. $y''' = 3\sin x \quad y(0) = 1, y'(0) = 0, y''(0) = -2 \;, h = 0.1$

Exact solution: $y(x) = 3\cos x + \dfrac{x^2}{2} - 2$

Olabode (2009) solved the above differential problem. We considered the same problem to test the accuracy of our method and the results are shown in Table 3 below.

Table 3: Comparison of the results of the new method with Olabode [16]

x	Exact Solution	Numerical Result	Error in our new method *P*=8	Olabode [16], *P*=7
0.1	0.990012495834077020	0.990012495834094450	1.743050e-14	3.4077519E-11
0.2	0.960199733523725120	0.960199733523833370	1.082467e-13	1.2372514E-10
0.3	0.911009467376818090	0.911009467377089210	2.711165e-13	1.7681812E-10
0.4	0.843182982008655380	0.843182982009163310	5.079270e-13	4.0865533E-10
0.5	0.757747685671118280	0.757747685671934730	8.164580e-13	3.7111825E-10
0.6	0.656006844729034810	0.656006844730234520	1.199707e-12	7.0964790E-10
0.7	0.539526561853465480	0.539526561855119820	1.654343e-12	7.4653450E-10
0.8	0.410120128041496560	0.410120128208960490	1.674639e-10	1.9585035E-09
0.9	0.269829904811993430	0.269829905145632600	3.336392e-10	3.8880070E-09
1.0	0.120906917604419300	0.120906918104591640	5.001723e-10	6.3955807E-09
1.1	-0.03421163572326779	-0.034211635056196628	6.670712e-10	9.5232678E-09
1.2	-0.19292673656997961	-0.192926735735650250	8.343294e-10	1.3169979E-08

4 Conclusion

We have developed a block method of order *P*=8 for direct solution of general third order ordinary differential equations. The results generated when the new method was applied to third order initial value problems show a better performance over the existing methods.

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