A SYSTEMATIC METHOD OF PROJECT SELECTION BASED ON RISK AND RETURN CRITERIA AND ACCORDING TO THE MEAN-SEMIDEVIATION BEHAVIORAL HYPOTHESIS

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Abstract

The uncertain problem of Industrial project selection is the topic of discussion in this article. As the unrealistic assumption of certainty is relaxed in this problem, the decision maker is faced with a two-criterion decision model in which justifying between Risk and Return are the main concerns. The concept of Risk has been revised and the "Semi-Deviation" measure has been proposed to represent the risk of a project. Based on the new Mean-Semideviation Behavior, and according to Utility and Modern Portfolio theories, a more efficient method of project evaluation will be presented.

Keywords: Risk of Projects, Mean-Semideviation Framework, project selection.

1. Introduction

The process of investment decision making in today's major industries is under constant pressure by many environmental, technical, economical, and political factors. These factors include, but not limited to: technology advancements, new products and new techniques, volatility of oil and gas prices (as the main input to many processes), equipment costs, political risk, environmental limitations a and governmental regulations. Today, more technically advanced projects are being planned in the volatile political and economical climates. This trend in application of decision analysis is now encompassing many more fields and many other decision problems are now subject to strict decision analysis [Mian, 2002].

The objective of the firm has been the subject of research for many years and by many individuals. Many fields of science contribute to development of the knowledge about what the Goal of a firm should be. The objective (Goal) that is accepted here and by many financial authorities is based on the finance theory. This theory says that the goal of the firm should be to "to maximize the future value of the firm to its shareholders" [Neveu, 1989].

This statement concerns *Value of the Firm* and not the maximization of any kind of income. Since in our study of project evaluation and selection, measurement of the Value of the Firm is not practically achieved, an alternative goal is defined such that maximization of this goal will eventually lead to maximization of the firm's value. This alternative goal is the maximization of *present value* of the shareholders' future wealth. Future wealth is *directly* related to the future *cash flows* of the projects accepted by the firm and we can therefore conclude that by maximization of the "Net Present Value of a Project's Cash Flow" the primary objective of a firm is also achieved.

The real world situations are not simply deterministic and for years methods have been proposed to tackle with the probabilistic nature of these events. The most acceptable method of dealing with the problem of Probabilistic Project Evaluation and Selection has been the "two-criterion" decision analysis approach of justification between Risk (volatility) and Return (financial outcome of the project). There is also a theoretic consideration that as the alternative projects are non-repeatable future alternatives, the concept of probability (The limit of a mathematical ratio) for these alternatives should be reinvestigated. For this problem, an acceptable resolution has been proposed based on the concept of subjective probability.

The whole topic of subjectively determined probabilities rests upon rather good evidence and good knowledge of the events. James Bernoulli in his book *Ars Conjectandi*(1713) suggested that probability is a "degree of confidence" that an individual attaches to an uncertain event and this degree depends on the individual's knowledge and can vary from individual to individual. This theme was later developed by Lap Lace and DeMorgan but the formal concept of subjective probability as an operational theory was first formulated by Ramsey [Ramsey, 1926]. From this work, DeFinetti [DeFinetti, 1967] was able to demonstrate that a person's degree of belief (or, his subjective probability assignments) obeys the usual laws of objective probabilities.

According to the above findings we can use the subjectively determined probabilities of the occurrence and magnitude of cash flows in the future in the problem of project evaluation and selection. As cash flows of the projects are almost unique events that have not repeated in the past, it seems the only functional way is to estimate the probabilities subjectively. Also, Ackoff, Gupta, and Minas in their book "Scientific Method: Optimizing Applied research Decisions" mention that: the Decision-Maker possesses more information regarding the decision environment than an assumption of outright uncertainty would require, merely by being able to specify the subjectively probable outcomes of a prospective action.

2. Risk and Measurement of Risk

Most projects and investment opportunities have uncertain outcomes and because of the inability of human to predict the future, these projects and investment opportunities are said to be risky. In the project selection literature, risk is defined as the "probability of unwanted outcomes". The word "unwanted" is an important point in the definition, because the variation of conditions will inevitably generate outcomes that are detrimental to the goals of investment. The variation of the conditions in real world is an always present phenomenon, in which no outcome is indeed certainly predictable.

The incorporation of risk into the investment decisions in mostly due to the pioneering works of Harry M. Markowitz, in which he proposed that although expected return of an investment is the most important criterion in investment analysis, maximization of expected (or mean) return from the investments is not a wise decision in the real world situations. He notes that this would be generally an unwise decision because the typical investor, although wanting returns to be high, also wants returns to be "as certain as possible". Thus the investor in seeking to both maximize expected return and minimize uncertainty (that is *risk*, in our discussion), has two conflicting objectives that must be balanced against each other at the time of investment.

In virtually all literature about investment, risk is defined as the volatility of returns, measured by standard deviation (or variance) of the probability distribution of return of the project or portfolio of projects. This dominance of definition in all scientific resources reflects the common belief of the academics and practitioners. On the other hand, if we refer to the basic definition of risk, "the probability of unwanted outcomes", some inconsistency would seem apparent. The standard deviation and variance consider risk as variations in both upward and downward directions, and rational investors (risk averse decision makers) would prefer lower risk to higher risk. This statement implies that for a rational investor, achieving higher than expected returns are also as undesirable as achieving lower than expected returns. This problem questions the use of variance and standard deviation as proper measures of risk.

All the concerns about inappropriateness of standard deviation would disappear if we deal with symmetrically distributed project returns, particularly in the form of normal distributions. In that case, the chances of positive outcome that is a certain distance away from the center of the distribution are just as great as the chances of negative outcome that is an equal distance in the opposite direction. In these situations, the standard deviation successfully describes the "bad" part of the project's return distribution.

But what if the project's return distribution is not normally distributed? In many project appraisal applications there is good reason that the bad outcomes are not exactly the mirror image of the good outcomes, i.e. we will be dealing with asymmetrical distributions of project return. If for example, we use standard deviation to measure risk in a right-skewed distribution of project *NPV*, we would ignore the fact that most of the project *NPV* is on the "good" side of the project's expected return.

If we wish to explicitly focus only on the likelihood of undesirable projects results in defining and measuring risk, the use of semi-variance is recommended. Semivariance is analogous to variance, but only those potential outcomes below the expected value are used in its calculation. Because semivariance is the average squared deviation below the expected return, it penalizes projects with relatively large potential shortfalls. The semivariance is more useful than the variance when the underlying distribution is asymmetric and just as useful when the underlying distribution is symmetric; in other words, the semivariance is at least as useful a measure of risk as variance. Moreover, the semivariance combines the information provided by two statistics, variance and skewness, into one measure[1].

In the alternative *MSB* framework, the investor's utility is given by

$$U = U(\mu_p, \sum_p), \qquad (1)$$

where $\sum_p$ denotes the downside deviation of returns (semideviation for short) of the investor's project or portfolio.

In this framework, the risk of project *i* taken individually is measured by the project's downside standard deviation of NPV. We will have:

$$\sum_i = \sqrt{(E\{ \min[(R_i - \mu_i),0]^2\})} \qquad (2)$$

[1] the semivariance of return (or of NPV) can be used to generate an alternative behavioral hypothesis, mean-semivariance behavior [Estrada, 2004]. This behavior is perfectly correlated with the expected utility (and with the utility of expected compounded return) and can therefore be defended along the same lines used by Levy and Markowitz [Levy and Markowitz, 1979], [Markowitz 1991].

The above expression is in fact a special case of the semideviation, which can be generally expressed with respect to any benchmark NPV, called *B*. this $\sum_{Bi}$ can be formulated as:

$$\sum_{Bi} = \sqrt{(E\{ \min[(R_i - B),0]^2\})} \qquad (3)$$

Throughout this document we will use as the only benchmark for project *i* the arithmetic mean of the distribution of NPV and thus we will denote the semideviation of project *i* as $\sum_i$.

The above definitions may cause computational complexity for calculation of semideviation of projects' NPV. As there is the famous rule of thumb for estimating variance of a symmetric and close to normal distribution, a similar rule of thumb can also be derived for estimating semivariance and semideviation of a skewed distribution. The rule of thumb for variance estimation is;

$$v_p = [1/6(N_{99\%} - N_{1\%})]^2 \qquad (4)$$

Where; v_p = variance of the project NPV.

$N_{99\%}$ = the value of NPV, where 99% of the NPV values are below, and 1% of the values are above that value.

$N_{1\%}$ = the value of NPV, where 1% of the NPV values are below, and 99% of the values are above that value.

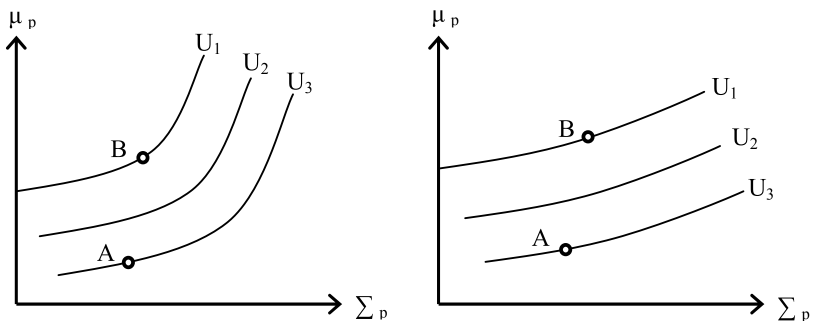
The rule for estimating semivariance and semideviation is derived directly from the assumption that skewed distributions obtained as the results of simulation consist of *3* semideviations between their mean value and their lowest value.

$$\sum_i = 1/3\ [\mu_i - N_{1\%}] \qquad (5)$$

3. Mean-Semideviation Framework

As mentioned before, the Mean-Semideviation behavior (*MSB*) introduced in [Estrada, 2004] will eventually generate a different Portfolio Theory based on a different Utility Theory that are consistent with this alternative behavioral hypothesis [Estrada, 2004]. This behavior is perfectly correlated with the expected utility (and with the utility of expected compounded return) and can therefore be defended along the same lines used by Levy and Markowitz [Levy and Markowitz, 1979], [Markowitz 1991]. In these new theories, all rules and relations are the same as before except that "Deviation" is replaced with "Semideviation".

The proposed method of project selection is based primarily on the concept of indifference curves in *MSB*. An indifference curve represents a set of risk and expected return combinations that provide the decision maker with the same amount of utility. The investor is said to be indifferent between any of the risk-expected return combinations of the same indifference curves. Because indifference curves represent an investor's preferences for risk and expected return, they can be drawn on a two dimensional system, where the horizontal axis indicates risk (Semi-deviation) of a project denoted by $\sum_p$, and vertical axis indicates reward as measured by expected return of project and is denoted by μ_p. Figure 1 illustrates indifference curves that typical risk averse investors have.



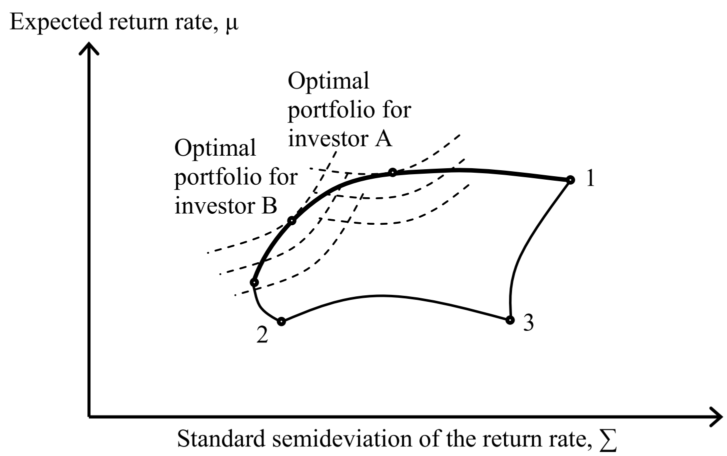
notes: Typical indifference curves that belong to risk averse investors. The investor that his indifference curve is depicted in the left diagram is more risk averse than the investor whose indifference curve is depicted on the right diagram.

Figure 1. Indifference curves of typical risk averse investors

The indifference curves on the left belong to an investor that is more risk averse than the investor whose indifference curves are depicted in the right diagram. In both cases, the indifference curves that are located on the right and above the others entail higher utility levels. In both cases, point *B* is more desirable than the point *A* because it is located on indifference curves that entail higher utility (In both cases, regardless of the degree of risk aversion, $U_1 \succ U_2 \succ U_3$ in which "›" means "has higher utility than").

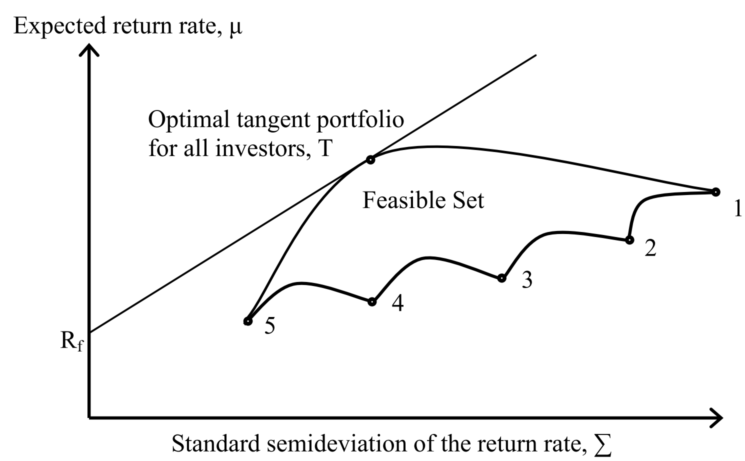
Portfolio theory is based on two attributes, the expect return and standard deviation of return. Based on these two attributes all the useful results of the portfolio theory have been developed and applied in various situations. As Estrada [Estrada, 2004] mentions, the new system of mean-semivariance is perfectly parallel with the previous system of mean-variance. According to these findings we can construct portfolios of risky securities similar to those constructed by Markowitz and use semideviation instead of standard deviation. The result will be a similar umbrella shaped region comprising of infinite number of portfolios.

All securities (and projects also) can be depicted in the μ-$\sum$ system in a similar manner, and all discussions about market portfolio and capital market line will be still correct. The market portfolio will be comprised of all risky securities in the market (their risk designated by their semideviation of return) and will be the efficient portfolio that all investors in the market will choose to invest. Also the risk free asset in the market will be defined the same way as before; a security with zero risk, zero deviation, and of course zero semideviation.



Notes: The highlighted line is the *efficient frontier* that dominates all other points of the region. Each investor according to his or her preferences (shape of his/her indifference curves) will choose a portfolio on the efficient frontier. Although semideviation is being used as the measure of risk instead of standard deviation, the relations have not changed.

Figure 2. Schematic representation of 3 correlated risky securities combined to make risky portfolios.



Notes: The risky securities can be combined with the risk free security to make a straight line of combined portfolios. The line R_f-T is comprised of portfolios that are partly risky and partly risk free. As the diagram illustrates, the line R_f-T dominates all other feasible portfolios.

Figure 3. The combination of risky security and risk free security.

4. Risk Equalization and Project Ranking

Based on the theoretic findings presented in Figure 3, it can be concluded that any single project can also be combined with the Risk-free security (in our discussion, the risk free security can be lending/borrowing money to the bank at the risk-free interest rate) and form a unique portfolio. All points on the line between a project's coordinates on μ-$\sum$ system and the Risk-Free security (called the risk equalization line) represent a two-security portfolio. This special portfolio is composed of *x*% Risk-Free security and (100-*x*)% project. The term (100-*x*)% is also referred to venture participation and can be determined at the tangent point of the investor's indifference curve and the risk equalization line. Although determination of the optimum venture participation is a very interesting problem in this field, we will not study it in this article and will try to stick to the project evaluation problem.

As reported in [Bussey, 1978], Tuttle and Litzenberger developed a proper model of risk adjustment by combining borrowing and lending with equity capital to finance the proposed project. Our approach is based on a similar logic in which we compare two projects by equalizing their risk through changing their financing ratio.

Consider the alternative projects *K* and *M*, with known expected return rates and known semideviations of return rate. We will equalize the risk of project *K* to the risk of the Project *M*. Let:

μ_z = the expected return rate to equity from the project.
$\sum_z$ = the estimated semideviation of the return rate to equity.
R_f = the riskfree borrowing and lending rate.
α = the financing ratio of project *K* (unity plus the debt-to-equity ratio).
μ_K = expected return rate from the project *K*.
$\sum_K$ = expected semideviation of return from project *K*.

Since $\sum_f = 0$ (The risk free asset has no semideviation since its return is constant during our study period), the estimate of return rate to equity is:

$$\textstyle\sum_z = \alpha \sum_K \qquad (6)$$

When α dollars per dollar of equity are invested in the project *K* and $(1 - \alpha)$ dollars are borrowed per dollar equity, the expected return on equity from the project is $\alpha\mu_k$, and the cost of borrowing is $(\alpha - 1)R_f$. Because the total return is composed of the return to equity plus the return to borrowed capital, the expected return to equity is:

$$\mu_z = R_K + (\alpha - 1)\mu_K - (\alpha - 1)R_f \qquad (7)$$

$$\mu_z = R_f + \alpha(\mu_K - R_f) \qquad (8)$$

Recalling that $\alpha = \sum_z / \sum_i$, by differentiating the expected return rate with respect to semideviation, $\sum_z$, we will have:

$$d\mu_z/\sum_z = (\mu_k - R_f)/\sum_K \quad (9)$$

We can define $\alpha' = \sum_M/\sum_K$,in which $\sum_M$ is the current risk of the first project, project *M*. The risk effect of a project on the firm's return rate to equity can be equalized either through long-term lending of an amount equal to $[(1/\alpha') - 1]$ of the cost of investment project if $\sum_K > \sum_M$, or the long-term borrowing of $[1 - (1/\alpha')]$ of the cost of the project if $\sum_K < \sum_M$. The risk adjusted expected return rate on the project *K* is:

$$\mu'_i = R_f + \alpha'(\mu_i - R_f) \quad (10)$$

Which is a *linear relationship* depicted in figure 4.

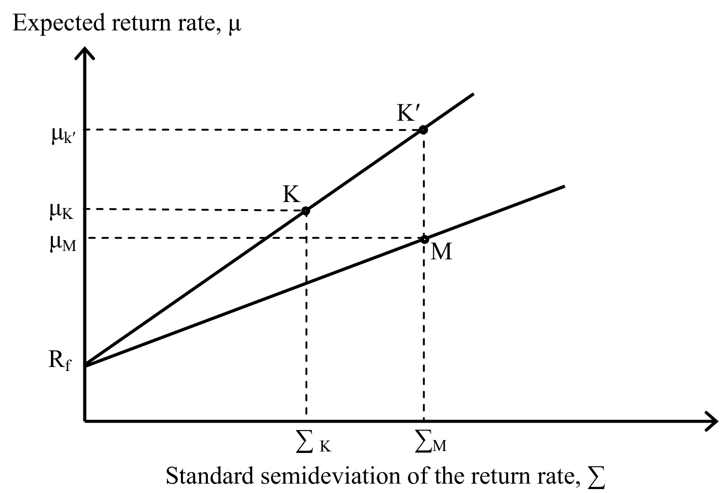


Figure 4. Risk adjustment of a project

Project *K′* is the risk equalized state of project *K* in which its risk has been equalized to the risk of the project *M*. It is apparent that the rational risk averse investor would prefer project *K′* to project *M* because it offers higher expected return in the same risk level. This statement is also true for all points (all portfolios, in fact) on the project *K*'s risk equalization line in comparison to the points on the project *M*'s risk equalization line.

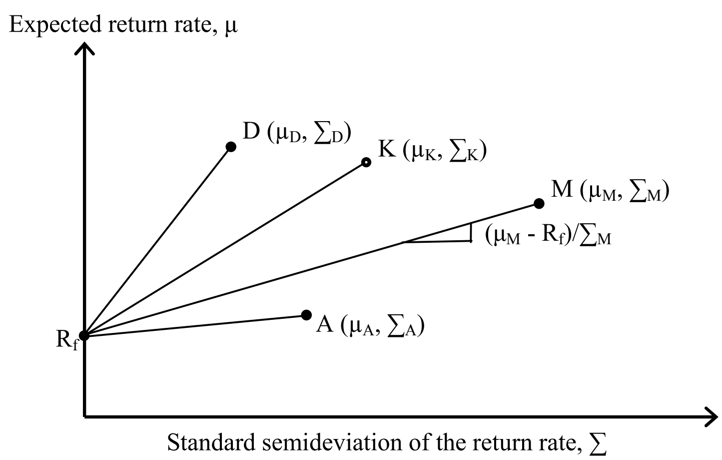
If we consider Project *M* as the company's own existing project (or portfolio of existing projects), then the risk equalization line of project *M* becomes a company established decision criterion. All projects such as *K* having values $(\mu_K, \sum_K)$ lying above the company's existing projects line would yield expected return rates greater than those required by the company's existing projects, and acceptance of such projects would increase the value (wealth) of the firm. On the other hand, if project *A* with values of $(\mu_A, \sum_A)$ lying below the firm's market line is considered, it would tend to reduce the value of the firm and should be rejected.

Figure 4 illustrates that, a project will be accepted if the slope of its risk equalization line, connecting the coordinate point *(μ_k, $\sum_k$)* of the project with the riskless asset point *(R_f, 0)*, exceeds the slope of the firm's equity line. Such projects, even in their unequalized state, are accepted by the firm as good projects because they always lie on a higher indifference curve. The decision criterion would become:

Accept project *K*, if:

$$(\mu_k - R_f)/\sum_K > (\mu_M - R_f)/\sum_M \qquad (11)$$

Some projects may be so close to the firm's equity line that it may not add much to the value of the firm. Acceptance of these projects may not be economical. To solve this problem, a constant is added to μ_M in order to express a safety margin for project selection. However, the reader should note that the above decision criterion can also be used to rank projects or exclusive groups of the projects.



Notes: This criterion can also be used to rank alternative investment projects. Project *D* appears to be the most attractive project, following by *K*. project *A* is not accepted in any ways because it may reduce the value of the firm.

Figure 5. The company's line as the acceptance/rejection decision criterion.

According to the example presented in Figure 5, since

$$(\mu_D - R_f)/\sum_D > (\mu_K - R_f)/\sum_K > (\mu_M - R_f)/\sum_M \qquad (12)$$

Projects D and K are accepted. And since

$$(\mu_A - R_f)/\Sigma_A < (\mu_M - R_f)/\Sigma_M \quad (13)$$

Project A is rejected. According to the proposed model it can be easily shown that projects D and K would add to the value of the company and create wealth if they are undertaken. On the other hand, project A would reduce the value of the company and should not be accepted.

5. Conclusion

The project selection methodology offered in this article has a fundamental advantage over previous methodologies because it implicitly takes into account the recognition of *financial risk* of both projects and the company itself (reflected in the existing projects of the company). Also the concept of risk is revised and a more appropriate measure for risk in industrial projects has been introduced. The model takes advantage of the findings in Modern Portfolio Theory and Utility Theory to select projects more efficiently.

The underlying financial theory also demonstrates that the proper selection criterion is the reward-to-risk ratio of the company itself since projects that provide ratios greater than that of the company will, in general, increase the present net worth of the firm

References

Ackoff, R. L., and Gupta, S., and Minas J. S., (1962) "*Scientific Method: Optimizing Applied Research Decisions*", John Wiley and Sons Inc.

Bussey, Lynn E., (1978) "*The Economic Analysis of Industrial Projects*", Prentice-Hall Inc.

Clemen, R. T., (1996) "*Making Hard Decisions-An Introduction to Decision Analysis*" Duxbury Press, Wads Worth Publishing Co.

Cozzolino, J., (March 1978) "*A New Method For Measurement And Control Of Exploration Risk*", SPE Paper 6632

Cozzolino, J., (Feb. 1977) "*A Simplified Utility Framework for the Analysis of Financial Risk*", SPE Paper 6359 presented at the SPE Hydrocarbon Economics and Evaluation Symposium, Dallas, Texas

DeGarmo, E. P., and Canada, J. R., (1973) "*Engineering Economy*", McMillan Co.,

Elton, E. J., and Gruber, M. J., (1995) "*Modern Portfolio Theory and Investment Analysis*", John Wiley and Sons Inc.

Estrada, J., (2004) "*Mean-Semivariance Behavior: An Alternative Behavioral Model*", Journal of Emerging Market Finance

Estrada, J., (2005) "*Mean-Semivariance Behavior: Downside Risk and Capital Asset Pricing*", International Review of Economics & Finance

Estrada, J., (2000)"*The Cost of Equity in Emerging Markets: A Downside Risk Approach*", Emerging Markets Quarterly,

Graves, S. B., and Rinquest, J. L., (2003) "*Models and Methods for Project Selection: Concepts from Management Science, Finance and Information Technology*", Kluwer Academic Publishers

Larche, I., and MacKay, J. A., (1999) "*Economic Risk in Hydrocarbon Exploration*", Academic Press,

Markowitz, H. M., (1959) "*Portfolio Selection: Efficient Diversification of Investments*" Cowles Foundation Monograph 16, John Wiley and Sons Inc.,

Mian, M. A., (2002) "*Project Economics and Decision Analysis, Volume 1: Deterministic Models*", Penn Well Publishing Corp.

Neufvile, Richard de, (1990) "*Applied Systems Analysis, Engineering Planning and Technology Management*" McGraw-Hill Co.

Ramsey, F. P., (1931)"*Truth and Probability*", Foundations of Mathematics and Other Logical Essays, also printed in [Klyeberg, 1964]

Ross, Stephen A., Westerfield, Randolph W., and Jordan, Bradford D., (2004)"*Essentials Of Corporate Finance*", McGraw-Hill Co. 2004

Savvides, S. C., (March 1994) "*Risk Analysis in Investment Appraisal*", Project Appraisal

Sharpe, W. F., and Alexander, G. J., and Bailey, J. V., (1999) "*Investments*", Prentice-Hall Inc.

Stermole, F. J., and Stermole, J. M., (2000) "*Economic Evaluation and Investment Decision Methods*", Investment Evaluations Corp.

Walls, M. R., (2004) "*Combining Decision Analysis And Portfolio Management To Improve Project Selection In The Exploration And Production Firm*", Journal of Petroleum Science and Engineering

Webb, Alan, (2003) "*The Project Manager's Guide to Handling Risk*", Gower Publishing Ltd.

Appendix. Determination of Risk and Return for Single Projects

A project under conditions of uncertainty, as discussed, is supposed to be consisted of random cash flows rather than known constant cash flows. A project can be supposed to be a sequence of random cash flows Y_t s ($t = 0, 1, 2, \ldots, n$) , each occurring at the end of the *n* time periods. So the project is composed of *n* random cash flows from present time, to the future.

If Y_t is considered a random variable, we must suppose that the values of *Yt* are governed by a random process. Thus the relative frequencies of the random vales taken by Y_t can be represented by probability or density functions that are either *discrete* functions or *continuous* functions.

The question is: how can the probability distribution of each cash flow increment, Y_t, be determined? In general, there is no other way that cash flow data for future projects must be estimated by analysts processing the necessary expertise.

The analyst's job is to examine various schemes or checklists to determine the source elements that contribute to cash outflows and inflows. By doing these estimates the analyst should develop a mean (expected value) of Y_t and variance of Y_t, or alternatively determine the *probability distribution function* of each net cash flow increment, Y_t.

A1. Assuming the Incremental Cash Flows to have β Density Function

One way of determining the mean and variance of random cash flows is based on the use of Beta distribution used in PERT methodology. This method requires the analyst to make an optimistic estimate (the upper bound of Beta distribution), a pessimistic estimate (the lower bound of Beta distribution) and a most likely estimate (the mode of Beta distribution) for each cash flow increment, Y_t.

A β-distribution resembles a normal distribution with the exceptions that:

1- the β-distribution function is truncated in tails.

2- It may be skewed to right or left.

In estimating the Beta distribution it should be noted that approximately six standard deviations should exist between the pessimistic and optimistic cash flow estimates. The mean and variance of the cash flow increment, Y_t, in any period t can be determined by:

$$E[Y_t] = 1/6\ [\text{Est}\ (Y_p) + 4\ \text{Est}\ (\tilde{Y}) + \text{Est}\ (Y_o)] \qquad (1a)$$

$$V[Y_t] = \{1/6[\text{Est}\ (Y_o) - \text{Est}\ (Y_p)]\}^2 \qquad (2a)$$

Where,

$E[Y_t]$ = mean of cash flow increment for time period *t*.
$V[Y_t]$ = variance of the cash flow increment for period *t*.
Est ($\tilde{Y}$) = most likely estimate of cash flow in period *t*.
Est (Y_p) = pessimistic estimate of cash flow increment in period *t*.
Est (Y_o) = optimistic estimate of cash flow increment in period *t*.

The net present value for a project is simply the sum of the discounted periodic cash flow increments. In the probabilistic case, it is assumed the cash flow increments are random variables and the summation of random variable cash flow increments results in a project's net present value that is itself a random variable.

$$P = Y_0 + Y_1/(1+i) + Y_2/(1+i)^2 + Y_3/(1+i)^3 + \ldots + Y_n/(1+i)^n \quad (3a)$$

Where *P* is the Random net present value for the project under investigation, Y_t is the random cash flow increment in the *t* th period, and *i* is the known rate of discount.

The expected net present value of the project is simply the sum of discounted mean cash flow increments;

$$E[P] = \sum_{t=0}^{n} E[Y_t]/(1+i)^t \quad (4a)$$

The variance of net present value, assuming that the project is composed of correlated random cash flows, can be calculated as;

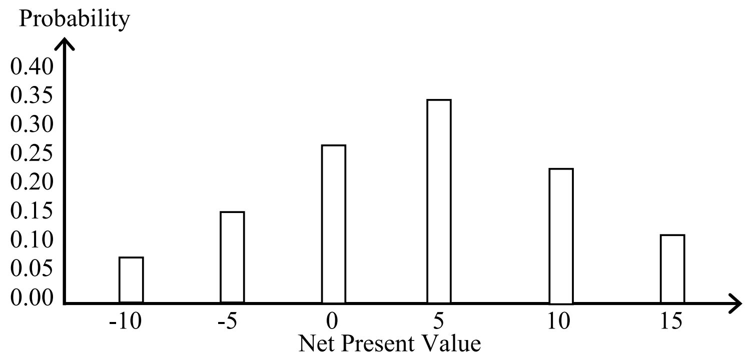
$$V[P]=\sum_{t=0}^{n} \sigma_t^2/(1+i)^{2t} + 2\sum_{\tau=0}^{n+1}\sum_{\theta=1}^{n} \mathrm{Cov}\,(Y_\theta\, Y_\tau)/(1+i)^{\tau+\theta} \quad (5a)$$

$$\mathrm{Cov}\,(Y_\tau\, Y_\theta) = \rho_{\theta\tau}\sigma_\tau\sigma_\theta \quad (\tau \neq \theta) \quad (6a)$$

A2. Assuming the Incremental Cash Flows to have Multinomial Distribution

In an essentially similar approach the cash flow increments are assumed to have multinomial probability distribution. When multinomial random variables are discounted to present time and form the net present value, one can say that the net present value is also a random variable with a multinomial distribution. Here we can present the net present value as below;

Figure 5. Net present value having multinomial distribution



The measure of central tendency (expected net present value) is simply;

$$E[P] = \sum_{i=1}^{n} p_i X_i \qquad (7a)$$

Where;

X_i = the *i*th possible value for the random variable.
P_i = the probability of *i*th value occurring.
N = number of possible values that the random variable might take.

The measure of dispersion (variance of the net present value) can be calculated as;

$$\sigma^2 = \sum_{i=1}^{N} p_i (X_i - E[P])^2 \qquad (8a)$$

$$\sigma_{XY} = \sum_{i=1}^{N} p_i (X_i - E[P]_X)(Yi - E[P]_Y) \qquad (9a)$$

A.3 Simulation Approaches

It is also possible to assume interdependencies among the whole input variables and neglect the interdependencies among the incremental cash flows. This assumption is also more consistent with the causative logic (Since correlations among input variables can be described according to systematic rules) rather than the regression logic (pure statistical relations among incremental cash flows). Input variables such as labor cost, Investment Cost, Operational Costs, Salvage value of the project and Product Prices can then be entered into a simulation model that gives the variance and expected return of each alternative project at the end. Recent advances in computational capacities of microcomputers have made it possible to simulate the outcomes of projects more efficiently and gain more realistic results.