PATTERNS OF DEBT USE IN SMALL BUSINESSES: A NON-PARAMETRIC ANALYSIS

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Abstract

This paper uses non-parametric techniques to examine patterns of debt use by small firms and how such patterns differ across firm categories. The methodological goal is to use the richness of the firm level data and allow convincing presentations with minimum of assumptions. The procedures used provide easily comprehensible graphical descriptions of the data. The procedures augment what can be discerned from descriptive statistics by accounting for differential weights and allowing for clustering that is a native feature of cross-sectional data. We also investigate how firms could benefit if credit availability improves. Though a model-based analysis would be required to provide a detailed analysis, our analysis suggests that greater credit availability will benefit all firms. Firms with low levels of equity will be better off as their credit constraints will be less binding, while firms with high levels of equity will benefit from acquiring more debt.

Keywords: Debt holding, Small business, Non-parametric, Credit constrained

JEL classification: G21 and C14

1. Introduction

Using data from the National Survey of Small Business Finances (NSSBF) (1993), this paper offers some insightful findings into the debt holdings of small businesses in the United States. The primary purpose of the paper is to examine

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²The opinions expressed here are those of the author and do not necessarily reflect the views and opinions of Amundi Investment Solutions Americas LLC.
patterns of use of debt by small firms and observe how those patterns differ across firm categories. Subsequently, firms’ debt pattern for varying levels of equity was examined. Firms’ propensity to hold debt and their likelihood of being credit constrained was also studied. Apart from observing debt patterns, we analyze the incidence of credit availability for small business.

Our methodological goal is to use the unusually rich and comprehensive firm level data collected by the NSSBF on small business finances and apply nonparametric tools to analyze it. The tools impose minimum of assumptions on the underlying distribution of the data and let the data speak for itself. In time-series data analyses, with thousands of observations, one has a luxury of being able to examine and make inferences, for example by plotting the series as a function of time. With the same number of observations, in a cross sectional setting it is less obvious what the appropriate graphical tools should be. Histograms for example, are useful for single variables, but the relationship between variables is harder to describe and discern. A two-dimensional scatter diagram on the other hand is too cluttered to be informative. Instead heavy reliance is placed on cross-tabulations and linear regressions as means of summarizing the data. While scatter diagrams do not transmit information clearly, cross-tabulations and regressions tend to over summarize, rarely doing justice to the amount of information available. This paper, highlights simple non-parametric techniques for density estimation and regression to summarize data and describe relationships. These methods provide easily comprehensible graphical descriptions of the data that are informative about the problem at hand. These techniques call for little more than presentation of the data relying less on economic or econometric assumptions. The results indicated that while the non-parametric techniques used may be limited in scope, they provide important insights by revealing natural clustering in the data.

This paper augments often used cross-sectional data analysis techniques with non-parametric methodology to understand the firms’ behavior toward using debt, debt-equity relationship and variations in debt usage. The paper examines firms’ probabilities of holding debt, being credit-constrained, applying for loans and incidence of loan approval. Descriptive statistics such as mean and variance capture the central tendency and dispersion of random variables, respectively. However, descriptive statistics do not incorporate the formula that accounts for differential weights and allows for clustering that may be a natural feature of cross-sectional data. Techniques like kernel density estimations provide simple mechanisms to incorporate such features of random variables in easily comprehensible visual representations.

3We define debt (or credit) as the combined amount of total loans including trade credit, mortgages, notes, bonds, and capital leases.

4 We focus on data visualization to this end as several studies have used different models to estimates demand for debt (Cole, 1998), incidence of loan (Cole, 2010) and have found statistically significant difference among different legal forms of small businesses.
Non-parametric methods were used to examine the relationship between debt use and levels of equity. Regressions are used to describe linear relationships between variables with some distributional assumptions. By contrast, kernel regressions are more akin to a cross-tabulation and devoid of causal significance. It is a descriptive device that is at best preliminary to a model-based analysis. Such description provides easily comprehensible ‘maps’ of the effects of differences across firms and some distributional characteristics on firms’ levels of debt without imposing the linearity and distributional assumptions. While kernel regression captures the relationship between variables, it provides no impression of the variability in debt use at each level of equity. Contour maps allow one to gauge the variability in debt use at each level of equity, and provide the flexibility to include observations on firms that have either no debt or equity or both. Exclusion of such firms renders the sub-sample non-random. Therefore, any model-based analyses of small firms’ debt that do not incorporate firms without debt either provides inconsistent results or their conclusions relate only to the sub-sample not to all small firms.

Though a model-based analysis would be required to incorporate a hypothesis that some small firms are credit constrained, and what is the effect of such constraints on debt holdings of firms, a simple kernel regression with dichotomous dependent variable will allows one to discern whether firms with intrinsic preference for holding debt are more likely to be credit constrained. Such regressions are akin to Probit regressions and plot the probabilities. The probability plots allow one to draw simple conclusions about small businesses’ behavior toward holding incremental debt if credit availability increased due to some policy changes. In essence, non-parametric analyses contain a good deal less than what one would like to know. The methodology, nevertheless, allows the data to speak for itself.

This paper contributes to the existing body of literature on credit usage and small business in a several important ways. First, it documents that a large segment of small businesses in the United States are non-borrowers, i.e., they have no debt in their balance sheets. Empirical research indicated that small businesses with no debt grow slowly, hire fewer workers and invest less in productive capital (King and Levine, 1993; Rajan and Zingales, 1998). For policy makers, it raises questions such as why some small businesses use no debt and what would be the economic implications. For academicians, it would be relevant to be cognizant of a segment of small businesses that refrains from borrowing therefore skewing their empirical estimates about small business debt

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1As an example, Leece (1999) uses cluster analysis to segment market for risky financial assets and his analysis reveals that previous study may have missed some important interactions in the data.

2Kunz (2007) presents a rich application of choice model based on latent variable models and principal component analysis to visualize competitive market structures based in individual consumer choice data.
use. Second, our use of non-parametric methodology provides some compelling indications of how small businesses that do borrow will generally benefit from greater availability of credit.

There are important policy implications of understanding the patterns of debt usage by small business by different sub-categories of small business owners. First, a better understanding of the debt usage by different ownership types of small businesses is critical for targeting businesses that are more vulnerable to changing credit conditions. Since the debt usage differs across ownership types, policy intervention will be more effective when targeted toward groups of small businesses that are more vulnerable to changing credit availability. Any evidence on the differences on debt usage at firm level will help in drawing appropriate tax and transfer policies to aid such small business operators. Second, similar information on small businesses can also affect the outcome of a comprehensive monetary policy (Gertler and Gilchrist, 1994). For example, the “credit” or “lending” view stresses the ability of monetary policy to regulate the pool of funds available to bank-dependent borrowers. Our findings can be used to design monetary policies that may have a disproportionate impact on borrowers with limited or no access to capital markets.

The plan of the paper is as follows. The general organization follows the main objective of the paper and provides some methodological comments along the way. Section 2 brings up some salient features of small businesses that characterize small businesses’ debt and their equity holdings. Section 3 provides the analysis of debt and some distributional features of credit availability. Section 4 concludes.

2. Debt in Small Businesses

There are three parties to the choice of capital structure of a firm: a) the management, b) the equity holders and c) the creditors. The capital structure of a firm depends on the judgment that each of the groups has about the future cash flows. In small businesses, there could be just two parties: equity holders and banks. More precisely, managers and owners constitute the equity holders and banks are the creditors. In essence, owners sell financial claims when firms choose to borrow funds from banks. But, the absence of a well-formed and complete market for ownership stakes in small businesses affects firms’ financial decisions and differentiates them from those firms operating in mature capital markets. Small businesses are inevitably more dependent on the ability of a single individual or a small number of individuals to raise either debt or equity than in the case of large enterprises. In this section, the relationship between legal

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1See Kashyap and Stein (2000) for a survey.
2Romer and Romer (1990); Kashyap, Stein, and Wilcox (1993); and Bernanke and Blinder (1992) provide discussions.
forms of small firms and other categories of small businesses and their debt holdings are discussed and the firms’ debt and equity holdings and their relative positions in their financial growth cycles are analyzed.

To understand the amount of equity debt holdings, first look at the legal forms of ownership. Forms of ownership are related to agency considerations. The conditions for information asymmetry arise both due to less stringent auditing requirements and because a single owner can monopolize financial information more easily than a broad management group in a large corporation. Unlike large corporations, small businesses do not allow for separation of management and ownership. Small businesses such as proprietary and partnership firms can be seen as extensions of owners, especially in matters of their financial matters.

Proprietorship provides the simplest structure with a few legal requirements. The business and the owner are treated as one entity for legal and tax purposes. The main advantage is its simplicity and flexibility, but the lack of distinction puts owners’ personal assets at risk in the event of default or failure of the business. Therefore, banks judge proprietary firms not only by firm characteristics, but also with those of the owners. The reliance of a proprietary firm on its owner makes it vulnerable and, therefore, not a highly regarded prospect for a potential lender. In many instances, a firm’s creditworthiness is as good as that of its owner. While proprietary firms provide a better recourse in case of default or failure, opacity of business activity makes them a less attractive prospect for a lender. Partnership firms have a wider financial base than that of proprietary firms. Though the sources of finance available to partnership firms are similar to those available to proprietary firms, there is a greater pool to draw from by the virtue of multiple partners. To the extent that a partnership reduces the vulnerability and riskiness of the business by providing a broader range of managerial skills and greater financial stability, partnership firms may be in a better position to raise capital than a proprietary firm.

The ease with which corporations acquire capital depends on their size, stability, quality of management, nature of the industry, track record and their investment opportunity sets. The nature of limited liability to shareholders is perceived as a disadvantage by potential lenders, and in many cases lenders will require additional collateral both from shareholders as private individuals and from the company. It is hypothesized that bank-firm relationships matter more in credit extension to C- and S- corporations than to proprietary and partnership firms. When firms default, banks have greater recourse with proprietary or partnership firms than with corporations.

Looking beyond the legal form of ownership, the study extended the analysis to some of the owner characteristics, such as race and gender. Research on small business lending indicates that some owners face racial discrimination in the credit market (Cavalluzo and Cavalluzzo, 1998). Impact of such actions would be seen either in low loan approval rate for certain groups of owners or such groups may self-select not to apply for loans. Dummy variables for the race of owner captures the extent of discrimination, while the selection methodology used in other studies took into account the lower incidence for loan application in
general (Cole, 1998; Cavalluzzo, Cavalluzzo and Wolken, 2002; Blanchflower, Levine and Zimmerman, 2003). To account for sexual discrimination, we could include a dummy variable for female operators, though the literature indicated that female owners are generally not discriminated in the credit market (Cavalluzzo and Cavalluzzo, 1998). While ownership structure explains the legal framework and role of some owner characteristics on debt in small business, it ignores relative positions of firms in their financial growth cycle and relationship between equity and debt.

The study started with a basic presumption that assets of firms are highly correlated with the stages of growth. Financial growth cycle explains firms’ debt holdings relative to firms’ financial growth cycle. Firms go through five stages of growth, namely: formation, rapid growth, growth to maturity, maturity and decline (Weston and Brigham, 1981). Despite limited agreement on the terminology, financial growth cycle theory provides a reasonable framework to analyze the debt of small businesses.

Major sources of finance for start-up businesses are insider finance and particularly owners’ personal resources. So this stage of growth does not see much bank debt as defined earlier, except in some instances usage of personal credit cards. Insufficient resources at this stage may result in under-capitalization of firms. As firms grow, they gain access to venture capital on the equity side and bank finance on the debt side. Continued growth beyond formation is very likely to be financed by retained earnings, trade credit and bank loans – some of the only and critical sources of finance small businesses have (Sahlman, 1988; Wetzel Jr., 1994). Rapid growth outstrips financial resources leading to liquidity crises and the outcome is a greater reliance on short-term finance due to lack of long-term finance.

The unavailability of long-term debt and equity, which is defined as financial gap, will have direct effect on the financial characteristics of small businesses. In the presence of financial gap, firms are presented with very different situations. Firms have to rely more on internally generated funds. Lack of long-term debt finance forces reliance on short-term sources and reduces liquidity. Similarly, lack of equity finance results in higher levels of leverage. Thus, short-term debt substitutes not only long-term debt but also equity.

Apart from issues of ownership and stages of growth, other factors may affect the financial characteristics of small businesses. Neck (1977) grouped such factors as hosts, agents and environment. The host are the owner-manager, agents are various financial environment institutions, and the environment constitutes legal, tax and economic institutions and market conditions. Hosts matter, because different owners using the same amount of finance could produce very different

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9This classification does not imply that firms necessarily go through all growth phases. Maturity and decline of a firm can take place at any point after the formation. Incidentally, some of the small businesses avoid some stages by forming and floating in the stock market at the same time. In some instances, the term growth is misleading, because growth can continue to occur during the maturity stage.
results. Agents, particularly creditors, often turn out to be a major source of external funds. The environment is so broad that it is beyond the scope of this paper. But what does come up very often is the transparency of small business activities and the extent to which small businesses are legally required to disclose their financial statements publicly.

3. Debt Patterns of Small Businesses

The study used the data from NSSBF (1993) to describe debt patterns of small businesses. Non-parametric methods were applied to examine the relationship between debt and equity, and how does this relationship vary across legal forms of firms and owners’ gender and ethnicity. The study examines a general hypothesis that some small firms are credit constrained and some small firms do not have any debt in their capital structure. Using a simple kernel regression with a dichotomous dependent variable, the investigation attempted to predict the effect of greater credit availability on firms that are credit constrained and those that do not have any debt. A similar analysis was conducted to understand the effect on firms’ incidence of credit application and credit approval. This paper begins with the description of firms’ financial data, and this forms the basis for Tables 1 and 2.

Table 1: Structure of the Sample: Summary Statistics

<table>
<thead>
<tr>
<th>Firm Categories</th>
<th>Debt</th>
<th>Equity</th>
<th>Assets</th>
<th>Firm age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proprietary</td>
<td>92,349</td>
<td>109,821</td>
<td>188,403</td>
<td>13.99</td>
</tr>
<tr>
<td>Partnership</td>
<td>2,038,759</td>
<td>963,067</td>
<td>2,852,831</td>
<td>14.78</td>
</tr>
<tr>
<td>S-corporations</td>
<td>856,214</td>
<td>782,826</td>
<td>1,981,783</td>
<td>13.75</td>
</tr>
<tr>
<td>C-corporations</td>
<td>1,121,264</td>
<td>995,394</td>
<td>2,596,871</td>
<td>17.58</td>
</tr>
<tr>
<td>Female owned Firms</td>
<td>335,334</td>
<td>255,808</td>
<td>651,813</td>
<td>12.92</td>
</tr>
<tr>
<td>Male owned Firms</td>
<td>938,362</td>
<td>746,582</td>
<td>1,925,309</td>
<td>15.84</td>
</tr>
<tr>
<td>Minority operated Firms</td>
<td>200,129</td>
<td>216,991</td>
<td>449,578</td>
<td>11.27</td>
</tr>
<tr>
<td>Majority operated Firms</td>
<td>954,104</td>
<td>746,385</td>
<td>1,945,232</td>
<td>16.13</td>
</tr>
</tbody>
</table>

Table 1 shows some firm characteristics and their descriptive statistics for different ownership categories. There are 4,637 firms in the NSSBF, and the study chose four firm characteristics – debt, equity, assets and age, and mapped them for different legal forms of the firms and owners’ gender and ethnicity. The NSSBF has proportional representation of each of the categories and throughout this paper debt is used as a measure of leverage; either the absolute amount of debt or the debt-asset ratio. Judging by the average amount of debt, C– and
S- corporations and proprietary firms are less levered than partnership firms. When it comes to equity, proprietary firms have the highest equity followed by C- and S-corporations, and partnership firms have the least amount of equity. C-corporations are some of the oldest firms in the sample, while other firms’ average age is about 14 years.\textsuperscript{10} Minority and female owned firms are the youngest firms in the sample. Table 1 ranks partnership firms the largest with the highest average amount of assets, followed by C and S-corporations. Lower panels of Table 1 present similar statistics for firms by female and minority owned organizations. Minority owned firms average about 92% debt-equity or 44% debt-asset compared to 127% debt-equity and 49% debt-asset ratios for majorities. However, such disparity is not observed among business ownership differentiated by gender. Such inferences drawn from Table 1 point to the central tendencies and can be misleading when we compare similar statistics at greater granularity. It is expected that non-parametric methods that were used in this paper will improve upon inferences that were generally gathered from descriptive statistics.

The study also examined the incidence of firms’ holding debt and the probability of firms’ being credit constrained. Some small firms chose not to have any debt in their capital mix, and it is easy to identify them in the present dataset. However, it was difficult to identify firms that are credit constrained due to the lack of any direct measure of a credit constraint. The NSSBF of 1988-89 and 1993 provides information to identify credit constraint firms. A firm is defined be credit constrained if the firm replied affirmatively to one of the two following questions: a) “With the most recent loan application did a bank turn down the loan application or has the firm been unable to get as much as it applied for?” and b) “During the past three years, were there times when the firm needed credit, but did not apply because it thought the application would be turned down?”

On average C- and S-corporations are more likely to be credit constrained compared to proprietary and partnership firms, and also more likely to have debt (see Table 2). Proprietary and partnership firms are less likely to be credit constrained and to hold debt. It was noted that there are similar proportions of credit-constrained firms irrespective of owners’ gender or race. An economy wide credit crunch may affect corporations more adversely because of their greater reliance on debt than proprietary and partnership firms. The final two columns of Table 2 show the percentage of firms that applied for a loan in last three years and the percentage of firms that had their loans approved. The incidence of holding debt and being credit-constrained separately were analyzed from the probability of applying for a loan and the application being approved for all firms in the survey. This was done because firms’ decision to hold debt can be seen as a separate process than their decision to apply for credit.\textsuperscript{11}

\textsuperscript{10}Berger and Udell (1995) have shown that longer relationship does impact availability of credit as it lowers the likelihood of collateral being required for lines of credit.  
\textsuperscript{11}See Cole (2010) for an analysis of why some small businesses do not use credit.
Averages such as those in Table 1 and 2 conceal as much as they reveal. Let us look at examples from each table. Table 1 ranks partnership firms as the largest with the highest mean value of assets over C- and S- corporation firms. More large firms in C- and S-corporation categories than in partnership category were found when examined their asset values at different percentile levels. It could be true for any descriptive statistics that we stated in Table 1, and it is hoped that non-parametric methods will improve upon them. Similarly, in Table 2 the broad inter-categorical patterns of distribution tell us which group of firms will be more impacted by a poor lending scenario due to their greater debt exposure. It may be concluded that corporations are more likely to be credit constrained looking at the statistic in Table 2, when compared to partnership and proprietary firms. However, any direct effects of credit crunch will be difficult to uncover unless the methodology allows us to examine firms at different levels of debt.

**Table 2: Small Businesses and their Bank Activities**

<table>
<thead>
<tr>
<th>Firm Categories</th>
<th>Debt</th>
<th>Credit-constrained</th>
<th>Equity*</th>
<th>Applied**</th>
<th>Approved***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent of firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proprietary</td>
<td>0.65</td>
<td>0.44</td>
<td>0.88</td>
<td>0.27</td>
<td>0.76</td>
</tr>
<tr>
<td>Partnership</td>
<td>0.75</td>
<td>0.53</td>
<td>0.86</td>
<td>0.43</td>
<td>0.88</td>
</tr>
<tr>
<td>S-corporations</td>
<td>0.83</td>
<td>0.61</td>
<td>0.82</td>
<td>0.52</td>
<td>0.84</td>
</tr>
<tr>
<td>C-corporations</td>
<td>0.83</td>
<td>0.62</td>
<td>0.84</td>
<td>0.52</td>
<td>0.88</td>
</tr>
<tr>
<td>Female owner</td>
<td>0.70</td>
<td>0.52</td>
<td>0.83</td>
<td>0.35</td>
<td>0.79</td>
</tr>
<tr>
<td>Male owner</td>
<td>0.78</td>
<td>0.56</td>
<td>0.85</td>
<td>0.45</td>
<td>0.85</td>
</tr>
<tr>
<td>Minority operated</td>
<td>0.70</td>
<td>0.58</td>
<td>0.83</td>
<td>0.33</td>
<td>0.60</td>
</tr>
<tr>
<td>Majority operated</td>
<td>0.78</td>
<td>0.55</td>
<td>0.85</td>
<td>0.45</td>
<td>0.88</td>
</tr>
</tbody>
</table>

*Percentage of firms that have positive amount of reported equity.
**Percentage of firms had applied for a loan in last three years.
***Percentage of firms that had loan applications approved.

Figure 1 shows estimates of the distribution of total debt for four legal forms of firms at different levels of debt, unlike the statistics of Table 1. Institutional features indicate that lenders have better recourse against funds lent to proprietary and partnership firms than to S- or C-corporations. Corporations are characterized by limited owner liability unlike proprietary and partnership firms.
It stands out that corporations have generally been in business for longer and are at later stages in their financial growth cycle than proprietary and partnership firms. We expect to see different distributional attributes of debt across the firms of these four legal categories. Density functions allow us to examine distribution attributes of debt at different levels. For estimation purposes, the logarithmic transformation for most variables was chosen because the distribution of debt like most of other variables is skewed, and log transformation yields a distribution that is more symmetric and closer to the normal distribution.\footnote{We provide a brief note in Appendix 1 on bandwidth selection in kernel estimation.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{Distribution of Debt Across Legal Forms of Firms}
\end{figure}

The most obvious feature of Figure 1 is the relative positions of debt holdings of four types of the firms. Partnership firms and S- and C- corporations have higher level of average debt and some of them have very large amount of debt as evident by fat right tail of the distribution. Proprietary firms tend to hold less debt. The kernel density plot shows their debt holdings and suggests how it is different from other three categories of legal forms of firms. The mean level of debt of S-corporations is about 75\% of that of the partnership firms and 40\% C-corporations. The density plots indicate that S- and C- corporations tend to have much similar distributions of debt holdings. Partnership firms have greater number of outliers evident by the fat right tail and a significant mass of firms around its median value.
Univariate plots of Figure 1 reveal little about the relationship between two or more variables. While organizational structure theory of small firms predicts that partnership firms may have greater amount of debt among four legal categories of firms because they provide a greater recourse against loan default and failure than corporations and proprietary firms. The firm growth cycle theory sheds light on role of relationship on borrowing as such relationships forms very gradually over extended period of time.\textsuperscript{13} We want to gauge how debt levels differ with equity for different categories of firms using kernel regressions.

Debt-asset ratios for different levels of equity for each category of firms are plotted in Figure 2. For each value of equity, Figure 2 shows a weighted average value of debt-asset ratios nearby. The weights are the same weights used to construct the density, but are scaled by the estimate of the density at the point. The plots of estimated regressions look similar. But, the advantage of the technique used is that the underlying data determines the shape of the function. No assumptions were made about how the data should look like – for example lie along a straight line, or along a low-order polynomial. Non-parametric regression, therefore, reveals the true relationship between debt-asset ratio and level of equity.

![Figure 2. Debt-Asset Regressions, Bandwidths are 0.88, 0.69, 0.59 and .91](image)

The downward sloping curves in Figure 2 indicate that firms with less proportion of debt will have greater equity, and regression plots affirm that the role of debt in the capital mix declines with increasing equity.\textsuperscript{14}

\textsuperscript{13} A similar model is employed by Koutmos and Booth (1995).
\textsuperscript{14} The data are available from the authors upon request.
It was noted that at low equity levels the debt-asset ratio is high for all types of firms, and then tends to decline in a two-step fashion. Over the whole distribution, the curve is steeper at low levels of equity then flattens out for firms with larger amounts of equity. Debt-asset ratios could be as high as 75%, before it declines precipitously. It can be concluded that firms that borrow tend to have at the minimum about one-third to one-half debt as a share of total assets.

The impact of credit crunch would be very different for firms if we incorporate two obvious aspects that are not being captured in Figure 2: some firms choose not to have any debt or equity in their capital structure, and some of them self-select not to apply for loans fearing denial. If all firms have equal access to bank credit and there is no precipitous difference in the interest rates charged to firms, then in case of a general credit crunch firms with high levels of debt will be more adversely affected. Apart from obvious omission of the firms without debt and/or equity, the curves in Figure 2 can also be misleading for giving no impression of the variability in debt holdings at each level of equity. On average firms with low levels of equity have as much as 60 to 80% debt in their capital structure as evidenced by the regression lines in Figure 2.

Contour maps bring to surface the variability in debt at each level of equity and vice-versa while incorporating observations with no equity and/or no debt (see Figures 3a & b).

**Figure 3a.** All firms: Bivariate Density Contour Maps; Log of Debt, Bandwidths are 0.60 and 0.72

The graphs present estimates of the joint density of the logarithm of debt (on x-axis) and the logarithm of equity (on y-axis). While the x and y-axis are the same as in the kernel regression graphs, and the height of the graph represents the fraction of firms at the levels of the log value of equity and log value of debt.
Patterns of Debt Use in Small Businesses: A Non-Parametric Analysis: 59-78

represented by the co-ordinates along the base. Figure 3b provides the joint density surface map also known as the net-map that presents a visual impression of the surface of the joint density. The joint density map gives a better picture of relative height and the concentration of mass. The visual superiority of the contour maps is the following. First, the density surface provides information about the tails of the distribution where there may be very few observations. Second, one also observe that whether firms have just debt or equity or both. The density does not fall to zero, and the presence of an open ‘hole’ or ‘cave’ indicates either firms with some equity and no debt or vice-versa. The joint density graphs also bring to surface any clustering of firms. It was noted that for the most part Figures 3a and b are similar, and that no new caveat emerges.

Figure 3b. All firms: Bivariate Density Contour Maps; Debt-Asset, Bandwidths are 0.60 and 0.72

Our next step is to incorporate a general hypothesis prevalent in the literature that credit constraints do exist in small businesses; as small businesses have varying levels of debt therefore they face varying degrees of credit constraints. The NSSBF dataset provides an additional caveat and that is small businesses demonstrate different willingness to hold debt and some carry no debt at all. Though a model-based analysis is required to provide estimates of overall impact of credit constraints on the level of debt held by firms, the data in

15More details on contour maps can be found in Härdle (1987) and Silverman (1986).
the survey allows us to visualize probability of holding debt and being credit constrained for different levels of equity. We are able to shed light on whether firms with intrinsic preference for holding debt are more likely to be credit constrained. Figure (4) shows estimates of proportion of small firms that chose to have debt in their capital mix and firms that are credit constrained. This is a non-parametric regression using a dependent variable which takes one for firms with debt and zero otherwise, and one for firms that are credit constrained and zero otherwise. The regression is akin to a Probit regression, and regression lines plot the probability of holding debt and firms being credit constrained, respectively.

![Figure 4. All Firms: Incidence of Debt and Probability of being Credit Constrained, Bandwidths are 0.31 and 0.39](image)

The general inference from Figure 4 is that among small firms the likelihood of holding debt is greater than the likelihood of being credit constrained. As expected at low levels of equity, small businesses are more likely to have debt. The probability of holding debt reduces more rapidly as level their equity share increases. Over 60% of firms are credit constrained with no or limited equity, and in excess of 40% of them continue to be credit-constrained at high levels of equity. The probability of a firm being credit constrained decreases with greater levels of equity and so does the probability of holding debt; the upper line is downward sloping at higher levels of equity-asset ratios. The positioning of curves also suggests that for any equity-assets ratio the probability of holding debt is higher than the probability of being credit constrained. Therefore firms that are more likely to have debt are more likely to
be credit-constrained. Figures 3a and 3b where it was included both borrowers and non-borrowers confirms an upward sloping relationship between equity and debt. Figure 4 shows that firms with equity-asset ratio greater than 0.90 have probability of being credit constrained at 40% or greater. Therefore, most small businesses will benefit from better credit availability. First, for firms with lower equity-assets ratios, better conditions will lower their probability of being credit constrained. Second, firms with higher equity-assets ratios can presumably lower the weighted average cost of capital by acquiring more debt for equity.17

Figure 5. All Firms: Incidence of Loan Application and Approval, Bandwidths are 0.43 and 0.29

It is emphasized that better credit conditions may encourage more request for debt. The analysis of incidence of loan application and approval reveals that better credit environment may rarely improve the probability of loan approval, however, may increase the incidence of loan application. The process of obtaining a loan is sequential in nature. The first step in the process of applying for one is critical. Most factors that influence the incidence of loan application also affect the probability of loan approval, and NSSBF (1993) cites very similar reasons firms gave for not applying for loans and banks gave for declining loan applications.

17 The distribution at the tails could be misleading. At the tails the estimated density becomes smaller, and since their estimate enters into the denominator of the conditional mean, the regression function is less precisely estimated.
However, while examining the probability of loan approval, the probability of applying for a loan should be taken into consideration.\textsuperscript{18} Figure (5) suggests that if credit conditions turn favorable, the increase in loan application approval may not be as large as the increase in number of firms that apply for loans.

4. Conclusions

The main conclusion of this paper is that there are some marked differences among small businesses in their debt holdings across different ownership types. The contour maps uncover an important detail - some small businesses do not carry any debt in their balance sheet and some have no equity. This becomes an important distributional feature to be recognized in any empirical work. The often-used practice of excluding such firms may provide results that cannot be generalized. Cole (2010) noted that this sub-sample of small businesses has received limited attention in academic work. It is important to recognize that several empirical strategies can be used to incorporate these firms with no debt or equity. For example, Heckman’s two stage estimation technique can ameliorate the selection bias of leaving out a subsample of data.

It was also observed that greater credit availability is likely to benefit all small businesses. However, the benefit would come from reducing the probability of being credit constrained for firms with no or low equity, allowing firms to switch to less expensive debt or by increasing the probability of application for loans. On the methodological front, the study was able to view some distributional features of debt in small businesses using a large data set. Though some of the substantive conclusions will require rigorous model based analyses, the study was able to use the data to draw conclusions about the impact of favorable credit conditions. By analyzing the sub-samples for different categories of firms, we are able to shed light on how different is allocation of debt within the small business sector. It was observed that larger corporations have more debt and more equity, but are also more likely to be credit constrained.

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\textsuperscript{18} Cole (2009) suggests that a large number of firms that do not apply for credit have substantial amount of debt in their books.
Appendix 1

Bandwidth Selection

Our purpose behind bandwidth selection is how near observations have to be to contribute to the weighted average at each point. If the bandwidth is too wide, then all observations will fall in one band, and the histogram or the regression will be an uninteresting single rectangle or one regression point. If the bandwidth is too narrow, then few of them will contain more than one reading, and the histogram will consist of a large number of same size narrow rectangles or the regression curve will be very rough.

The simplest method would be to experiment with different bandwidths, say l, at each value of log of equity to calculate the average of the debt-asset ratio of firms whose equity is within l of the value. This procedure gives equal weight to all observations near and close to the value of log of equity being considered. We can improve on this procedure using a method that gives greater weight to firms the closer is their value to log of equity that is being considered. Therefore, the estimate of the regression corresponding to a point X, m(X) is:

\[
m(X) = \sum w_i(X, X_i)Y_i,
\]

where n is the sample size, X and Y are the x and y values of observation i, and i runs from 1 to n. In the method described above, the weights will be zero for X_i far from X and close to one otherwise. Some other methods do allow all observations to contribute and simply let the weights decline with the distance between X and X_i.

The estimator (1) is a very general one, and is described as a kernel estimator with the following weight scheme:

\[
w_i(X, X_i) = K_h(X - X_i)/\sum K_h(X - X_i),
\]

where K_h is the kernel, h is the bandwidth. K_h is a symmetric monotone decreasing function that integrates to unity. Most statistical software provide a menu of kernels to choose from. We use the Epanechnikov kernel that is described by:

\[
K_h(X - X_i) = \frac{3}{4h} \left[ 1 - \left( \frac{X - X_i}{h} \right)^2 \right] I(|X, X_i| \leq h),
\]

where I is an indicator function such that I = 1 if X and X_i are within h of one another, I = 0 otherwise.
The equations (1) - (3) are used for non-parametric regressions in the paper. In some figures, the dependent variable $y$ is either logarithm of debt, or debt-asset ratio or simply one or zero depending on whether the firm does or does not have debt or is credit constrained. The graphs such as in Figure 2 of text are constructed by calculating (1) for 20 equally spaced values of the log of equity and plotting the results. To select bandwidth, we start with trial and error. The basic criterion behind this procedure is to select a bandwidth that appears to give enough smoothness without obscuring detail. We do experiment with cross-validation, one of the procedures generally followed in the literature. However, we find that the informal methods were unlikely to be misleading, at least for the graphical purposes of this paper.

For the density estimations, such as Figure 1 in the text, we follow a similar procedure. At each point on the x-axis, we take into account 50 nearby observations. We obtain an estimate of density by taking a ratio of the count and of the sample size. Like kernel regression, it is sensible to use a weighing arrangement that gives closer firms greater weights. We use the following kernel function to achieve this.

$$f_h(X) = n^{-1} \sum k_h(X_i - X),$$

where $K_h(.)$ integrates to unity; a condition required to generate a proper estimate of the density. Equation (4) is used for both the univariate densities and also for the kernel regressions.

Contour maps and surface plots are calculated using similar general principles. We first construct a grid over the range of the two variables. Second, at each point on the grid, a weighted count is made of the observation within a neighborhood of the point. We use a kernel weighting function just as we did for kernel regressions and densities. The bivariate Epanechinov is given by,

$$K(d_i) = \frac{2}{\pi h^2} (1 - d_i'^d_i) I(d_i'^d_i \leq 1),$$

where $d_i$ is a two element vector of deviations of $X_i - X$ and $Y_i - Y$ each divided by the bandwidth $h$. An observation for estimation is included only if it is within a circular region centered at the current point and with radius $h$. The density estimate at a point is given by,

$$f_h(Z) = \frac{(\det S)^{-1/2}}{n} \sum k[h^{-2}(Z - Z_i)'S^{-1}(Z - Z_i)],$$

where $k(d'd) = K(d)$, and $S$ is the sample variance covariance matrix of the two variables.
References


