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Image enlargement using biharmonic Said-Ball surface

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Abstract. This paper discusses the use of biharmonic cubic Said-Ball surfaces in image enlargement area. Resizing an image through up sampling or down sampling is generally common for making smaller image fit a bigger screen in full screen mode or reducing a higher resolution image to a smaller resolution. However due to some limitation, this paper will focus on image enlargement based on scaling factor of two. We use biharmonic cubic Said-Ball subject to a given four boundary curves condition respectively. We implement and evaluate the performance of the proposed method based on peak signal to noise ratio (PSNR) indicator using well-known gray-scale test images.

1. Introduction

In computer graphics, the process of resizing a digital image is called as image scaling. which is involves a trade-off between smoothness, sharpness and efficiency ([1]). With bitmap graphics, as the size of an image is reduced or enlarged, the pixels that form the image become increasingly visible, making the image appear soft if pixels are averaged, or jagged if not. With vector graphic, the trade-off may be in processing power for re-rendering the image, which may be noticeable as slow re-rendering with still graphics, or slower frame rate and frame skipping in computer animation. Resizing an image through up sampling or down sampling is generally common for making smaller imagery fit a bigger screen in full screen mode or reducing a higher resolution image to a smaller resolution. For example, in zooming a bitmap image, it is difficult to discover any more information in the image than already exists, and this will affect its quality.

Thus, to overcome the aforementioned problems, there are number of techniques to enlarging and reducing an input image especially by using interpolation methods such as nearest neighbor interpolation, bilinear interpolation, bicubic and B-spline interpolation ([2]). As mention by Han ([2]), the bicubic interpolation algorithm is the commonly used in image processing software produce relatively clear picture quality, but it needs larger amount of calculation due to the needs of 16 adjacent pixels to interpolate the missing information of pixel in output image. In this paper, we will discuss the use of biharmonic partial differential equation bounded by the given boundary curves of Said-Ball ([3]; [4]) surface known as biharmonic Said-Ball surface to fill the gap of missing pixels information in output image. The main idea is to find Said-Ball solutions to some natural PDEs which can only be controlled through the boundary control points and those adjacent to them. For this project, we shall use a lower dimension of biharmonic Said-Ball surface ([5]) of degree 3 by 3 which consists of 16 control points where twelve of them are the boundary control points. Using the biharmonic PDE, we derive a relationship between inner control points and the given boundary control points.

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2. Image scaling concept

Let an input image is given by m by n pixel, (x, y) and (x', y') are the arbitrary input and output pixels respectively. If input pixels are resizing by a factor of s_x and s_y respectively at pixel (1, 1), then the new output of $2m$ by $2n$ pixels are obtained by the following transformation.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \dfrac{ms_x - 1}{m-1} & 0 \\ 0 & \dfrac{ns_y - 1}{n-1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \dfrac{m(1-s_x)}{m-1} \\ \dfrac{n(1-s_y)}{n-1} \end{pmatrix} \tag{1}$$

As an example, we use $m = n = 2$ and scaling factor $s_x = s_y = 2$ as shown in Figure 1 where scaling up the 2 by 2 pixels in input image by a factor of 2, produces 4 by 4 pixels in output image. Note that, the circles in Figure 1(b) represented the original pixels in input image while the squares representing misssing pixels information in output image.

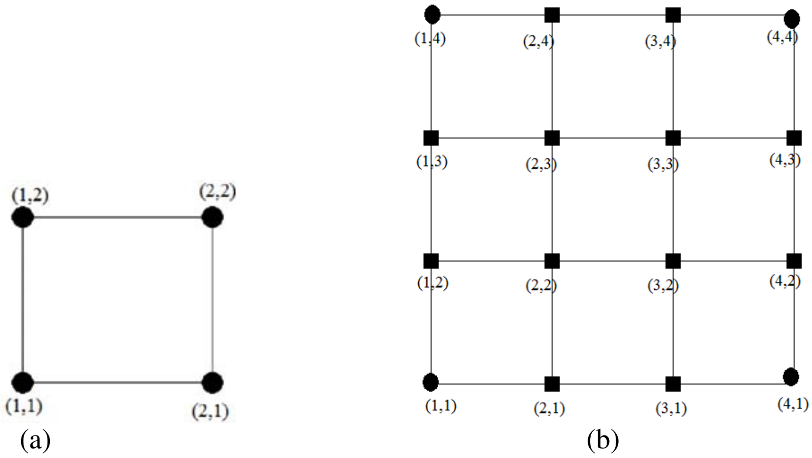

Figure 1. (a) 2 by 2 input pixels (b) 4 by 4 output pixels by scaling factor 2 of input pixel.

Pixels (1, 1), (4,1), (1, 4) and (4,4) in output image will preserve the information's of the pixels in input image while the remaining pixels intensity needs to be filled. Transformation of pixels from input window to output window can be done using (1). In order to obtain the missing pixels information in an output image, these points should be transform to the pixels in original input image by using inverse transform of (1). Thus the points (2,1), (3,1), (1,2), (2,2), (3,2), (4,2), (1,3), (2,3), (3,3), (4,3), (2, 4) and (3,4) of output window can be represented as (4/3, 1), (5/3, 1), (1,4/3), (4/3,4/3), (5/3, 4/3), (2,4/3), (1, 5/3), (4/3,5/3), (5/3,5/3), (2, 5/3) and (5/3,2) in input window respectively, and these points need to be interpolated using suitable interpolation techniques to obtained the respective intensities of output pixels. In this study, we will propose the interpolation technique based on biharmonic cubic Said-Ball surface.

3. Said-Ball rectangular surface

Degree m by n Said-Ball patch S is defined as, [3]

$$S(u,v) = \sum_{i,=0}^{m} \sum_{j=0}^{n} b_{ij} S_i^3(u) S_j^3(v),\ 0 \le u, v \le 1$$

with b_{ij} are the control points and

$$S_i^n(t) = \begin{cases} \binom{\lfloor n/2 \rfloor + i}{i} t^i (1-t)^{\lfloor n/2 \rfloor + 1}, & 0 \le i \le \lceil n/2 \rceil - 1 \\ \binom{n}{n/2} t^{n/2} (1-t)^{n/2}, & i = n/2, \\ S_{n-i}^n (1-t), & \lfloor n/2 \rfloor + 1 \le i \le n. \end{cases} \tag{2}$$

where $\lfloor n/2 \rfloor$ and $\lceil n/2 \rceil$ denote the greatest integer less than or equal to $n/2$ and the least integer greater than or equal to $n/2$, respectively.

For the purpose of our research, we will consider the degree three Said-Ball surface. From (2), a cubic Said-Ball rectangular surface (m=n=3) is given by matrix form as

$$X(u, v) = UBV \tag{3}$$

where $U = [(1-u)^2 \;\; 2u(1-u)^2 \;\; 2u^2(1-u) \;\; u^2]$, $V = [(1-v)^2 \;\; -v(1-v)^2 \;\; 2v^2(1-v) \;\; v^2]^T$ and $B = (b_{ij})_{3x3}$, $i, j = 0,1\ldots,3$ is the control point matrix.

4. Biharmonic Said-Ball surface

Biharmonic equation for parametric surface $X(u,v)$ is defined as the differential equation obtained by applying the biharmonic operator also known as the bilaplacian that is the differential operator defined by

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right)^2 X(u,v) = 0, \text{ u and v are the parameters.} \tag{4}$$

The function X from (3) which satisfy the harmonic condition in (4) is given by the following theorem ([7]).

Theorem 1. Given a control net in $R^{3,}$ $\{q_{ij}\}_{i,j=0}^n$, the associated $X(u, v)$ surface $X : [0,1] \text{ x } [0,1] \to R^3$ is biharmonic $\nabla^4 X = 0$ iff $\forall i \in \{1,2,\ldots,m\}$ and $j \in \{1,2,\ldots,n\}$,

$$\begin{aligned} 0 = & \sum_{k=0}^{4} \sum_{r,s=0}^{m,n} b_{m,i-k,k} \left(\left(f_{i-k+4,r} - 4 f_{i-k+3,r} + 6 f_{i-k+2,r} - 4 f_{i-k+1,r} + f_{i-k,r} \right) q_{rs} h_{sj} \right) + \\ & 2 \sum_{k,l=0}^{2} \sum_{r,s=0}^{m,n} a_{m,i-k,k}\, a_{n,j-l,l} \left(\left(f_{i-k+2,r} - 2 f_{i-k+1,r} + f_{i-k,r} \right) q_{rs} (h_{r,j-l+2} - 2h_{r,j-l+1} + h_{r,j-l}) \right) + \\ & \sum_{k=0}^{4} \sum_{r,s=0}^{m,n} b_{n,j-l,l} \left(f_{i-k,r} q_{rs} (h_{r,j+4,r} - 4h_{r,\, j+3} + 6h_{r,j+2} - 4h_{r,j+1} + h_{rj}) \right) \end{aligned} \tag{5}$$

where for $i \in \{0,1,..,m\}$

$a_{mi0} = (m-i)(m-i-1)$, $a_{mi1} = 2(i+1)(m-i-1)$, $a_{mi2} = (i+1)(i+2)$, and $a_{mik} = 0$ otherwise, and

for $j \in \{0,1,..,m-4\}$

$b_{mi0} = (m-i)(m-i-1)(m-i-2)(m-i-3)$, $b_{mi1} = 4(i+1)(m-i-1)(m-i-2)(m-i-3)$,

$b_{mi2} = 6(i+1)(i+2)(m-i-2)(m-i-3)$, $b_{mi3} = 4(i+1)(i+2)(i+3)(m-i-3)$,

$b_{mi4} = 4(i+1)(i+2)(i+3)$ and $b_{mik} = 0$ otherwise.

The f and h are the conversion matrices from Said-Ball curve $X(u)$, $X(v)$ into Bezier curve ([3]; [4]). See [7] for further details and proof.

Using Theorem 1 with $m = n = 3$ (cubic case), we get the following corollary.

Corollary 1. A cubic Said-Ball surface as given by (3) is biharmonic if and only if

$$\begin{aligned} b_{11} &= -(b_{00} - 2b_{01} + b_{03} - 2b_{10} - 2b_{13} + b_{30} - 2b_{31} + b_{33})/4 \\ b_{12} &= -(b_{00} - 2b_{02} + b_{03} - 2b_{10} - 2b_{13} + b_{30} - 2b_{32} + b_{33})/4 \\ b_{21} &= -(b_{00} - 2b_{01} + b_{03} - 2b_{20} - 2b_{23} + b_{30} - 2b_{31} + b_{33})/4 \\ b_{22} &= -(b_{00} - 2b_{02} + b_{03} - 2b_{20} - 2b_{23} + b_{30} - 2b_{32} + b_{33})/4 \end{aligned} \tag{6}$$

Remark: The inner control points of harmonic Said-Ball surface can be obtained by using the four lines of boundary control points.

5. Image interpolation using biharmonic rectangular patch

Given m by n pixels of grayscale input image represent by a $(m-1)$ $(n-1)$ rectangular patches and being scaled up by a factor of α and β using (1), resulting an output image of αm by βn pixels of higher resolution represented by (α_m-1) (β_n-1) rectangular patches. Let (i, j), $i = 1, 2, \ldots m$, $j = 1,2, \ldots, n$ represented input pixels and z_{ij} (0-255) be its corresponding grayscale intensity. Our aim is to find the function $z = F(I, J)$ which interpolate the given input pixels (Vertices of Rectangular patches) that is $F(i, j) = z_{ij}$. We start with construction of rectangulation for a given input pixels using our rectangulation algorithm ([9]). The rectangle where the missing pixels information occurs will be identified. The pixels ant its intensity values at corner of this rectangle will be used as vertex control points of harmonic cubic Said-Ball patch.

To generate a function that interpolates a given grayscale intensity value at the vertices of the rectangular, we also need to estimate the partial derivative with respect to horizontal and vertical directions at these vertices. Partial derivatives are estimated by using well-known method for uniform grid that is forward, central and backward difference methods. Furthermore, from these estimated derivatives, the Said-Ball coefficients adjacent to the vertex of all rectangular shall be calculated using the similar approach as discuss in [8] and from (6), the remaining coefficients can be calculated.

6. Experimental result

In this simulation result, we use six well-known test grayscale images such as Lena (512 x 512), Rice (640 x 640), Cameraman (256 x 256), Pout (480 x 480), Mandrill (512 x 512) and Thumb Print (250 x 250). To evaluate the performance of our proposed methods the test images were zoom out to half of its original size by using the simple image interpolation in MATLAB (bilinear method) and this image will be scaled up by factor of two to to get an original size with the proposed methods. We compare the performance of our methods with the most better built-in interpolation method in MATLAB namely bicubic convolution method by using peak signal to noise ratio (PSNR) indicator. The value of PSNR will reflect the quality of image that is the larger PSNR means that the higher quality of images (Han, 2013). The comparison of the PSNR for the methods is given in Table 1.

Table 1. Comparison of PSNR for different methods.

Test Image	Bicubic Convolution	Biharmonic Said-Ball
Rice	41.83085	40.05488
Cameraman	39.86494	40.14994
Pout	49.92441	49.69483
Mandrill	36.74323	36.31523
Lena	36.99528	36.86232
Thumb	35.33343	35.81028

The following observations can be obtained from Table 1. For the small size images such as Cameraman (256 x 256) and Thumb (250 x 250) our proposed methods of Biharmonic Said-Ball outperformed the bicubic convolution method. For the large images Rice (640 x 640), Pout (480 x 480), Mandrill (512 x 512) and Lena (512 x 512) out methods can be considered as comparable accuracy compare to bicubic method. This is because the values of PSNR of our proposed methods are not much difference compared to bicubic method. An example of image enlargement using scaling factor 2 is given in Fig.2. Our proposed biharmonic cubic Said-Ball and bicubic interpolation methods produce almost similar in smoothness of image edges and both method is comparable in preserving an image quality.

7. Conclusion

In this work, we have proposed the use of biharmonic Said-Ball surface of degree three in image enlargement area. Base on PSNR indicator, we show that the performance of our methods for resizing of sample grayscale images with scaling factor of two produces comparable accuracy in term of image quality compared to the well-known existing bicubic convolution method.

(a) (b) (c) (d)

Figure 2. Visualization of test image 2 (cameraman) using interpolation algorithm
a) Original Image (256x256) b) Shrunk image (128 x 128)
c) Bicubic d) Biharmonic Said-Ball

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