Journal of Mathematics Research; Vol. 10, No. 1; February 2018
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Korovkin Approximation Theorem with Ω Striped

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Received: September 29, 2017 Accepted: October 17, 2017 Online Published: November 18, 2017

doi:10.5539/jmr.v10n1p20 URL: https://doi.org/10.5539/jmr.v10n1p20

Abstract

In this paper, we discuss some theorem reached M. Mursaleen, there are several properties of statistical lacunary summability presented (Mursaleen, M. & Alotaibi, A., 2011; Mursaleen, M. & Alotaibi, A., 2011; Edely, O. H. & Mursaleen, M., 2009). This is concerned the motivate to narrowly delineated context denoted by Ω striped usage in prove our theorem (theorem A). We introduce some piecewise polynomial functions (Kopotun, K. A., 2006) and some results Korovkin theorem.

Keywords: Piecewise polynomial functions, Statistical lacunary summability, Strongly θ_q-convergent, Korovkin type theorem

2010 Math. Sub. Classification: 41A10, 41A25, 41A36, 40A30

1. Introduction and Main Results

The aim of this paper a completed the striped used in many area of Korovkin theorem (Mursaleen, M. & Alotaibi, A., 2011; Al-Muhja, M., 2015).

We will need accept the following:

Let $K \subseteq \mathbb{N}$. Then $\delta_\theta(K) = \lim_r \frac{1}{h_r} \left|\{k_{r-1} < i < k_r : i \in K\}\right|$ is said to be θ–density of K.

Definition 1.1 (Fridy, J. A. & Orhan, C., 1993) A sequence $x = (x_k)$ is said to be lacunary statistically convergent to L, if for every $\epsilon > 0$, the set $K_\epsilon := \{k \in \mathbb{N}: |x_k - L| \geq \epsilon\}$ has θ–density zero, i.e. $\delta_\theta(K_\epsilon) = 0$. In this case we write $S_\theta - \lim x = L$. That is, $\lim_r \left|\{k_{r-1} < i < k_r : |x_k - L| \geq \epsilon\}\right| = 0$. In this case we write $S_\theta - \lim_i x_i = L$, and we denote the set of all lacunary statistically convergent sequence by S_θ.

Definition 1.2 (Mursaleen, M. & Alotaibi, A., 2011) Let $h = (h_r)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $h_{r+1} \leq h_r + 1$, $x_1 = 0$. The generalized de la Vallee-Poussin mean is defined by $t_r(x) = \frac{1}{h_r}\sum_{j \in I_r} x_j$ where $I_r = [r - h_r + 1, r]$.

Definition 1.3 (Mursaleen, M. & Alotaibi, A., 2011) A sequence $x = (x_k)$ is said to be θ-summable to L, if $\lim_r t_r(x) = L$.

Definition 1.4 (Mursaleen, M. & Alotaibi, A., 2011) A sequence $x = (x_k)$ is said to be statistically lacunary summable (or statistically θ-summable) to L, if for every $\epsilon > 0$, the set $K_\epsilon := \{r \in \mathbb{N}: |t_r(x) - L| \geq \epsilon\}$ has natural density zero, i.e., $\delta\big(K_\epsilon(\theta)\big) = 0$. That is, $\lim_n \frac{1}{n}\ |\{r \le n : |t_r(x) - L| \ge \epsilon\}| = 0$. In this case we write $S_\theta - \lim x = L$. We denote the set of all statistically lacunary summable sequences by θ_S.

Definition 1.5 (Mursaleen, M. & Alotaibi, A., 2011) A sequence $x = (x_k)$ is said to be strongly θ_q-convergent $(0 < q < \infty)$ to the limit L, if $\lim_r \frac{1}{h_r} \sum_{j \in I_r} \left|x_j - L\right|^q = 0$, and we write it as $x_k \to L[C_\theta]_q$. In this case L is called the $[C_\theta]_q$-limit of x. We denote the set of all strongly θ_q-convergent sequences by $L[C_\theta]_q$.

Let $\mathcal{S}_r(z_n)$ be the space of all piecewise polynomial functions of degree r (order $r+1$), with the knots

$$z_n = (z_i)_{i=0}^{n}, -1 = z_0 < z_1 < \cdots < z_{n-1} < z_n = 1. \quad (1)$$

Definition 1.6 (Al-Muhja, M., 2015) A spline s in $G_s/\{I_e\}$ is said to be homogeneous of degree $\lambda \in \mathcal{R}$ if $s \circ \gamma_\tau = \tau^\lambda s$ for $\tau > 0$.

Definition 1.7 (Al-Muhja, M., 2015) A distribution σ on G_s is said to be homogeneous of degree λ if $\langle \sigma, \left(\tau^{-\sum_{i=1}^{\eta} d_i}\right) s \circ \gamma_{\tau-1} \rangle = \tau^\lambda \langle \sigma, s \rangle$, for $s \in G_s$, $d_i \in \mathcal{R}$, $\eta \in \mathbb{N}$ and $\tau > 0$.

Definition 1.8 (Al-Muhja, M., 2015) A linear differential operator Υ on G_s is said to be homogeneous of degree λ if $\Upsilon(s \circ \gamma_\tau) = \tau^\lambda (\Upsilon s) \circ \gamma_\tau$, for any $s \in G_s$ and $\tau > 0$.

Lemma 1.9 (Al-Muhja, M., 2015) Let $A = (a_{jn})$ be nonnegative regular summability matrix. For all $s \in G_s/\{I_e\}$, satisfied ((Al-Muhja, M., 2015) equation (3)), and Υ_j a sequence of positive linear operators, we have $\Upsilon_j : G_s/\{I_e\} \to G_s/\{I_e\}$, homogeneous group.

Theorem 1.10 (Al-Muhja, M., 2015) Let $A = (a_{jn})$ be nonnegative regular summability matrix, and let Υ_j be a sequence of positive linear operators from $\mathcal{S}_r(z_n)$ into $\mathcal{S}_r(z_n)$. Then for all $s \in \mathcal{S}_r(z_n)$, we have

$$st - lim_j \sum_{n=1}^{\infty} a_{jn} \ \left\|\Upsilon_j(G_s; .) - G_s\right\|_{\mathcal{S}_r(z_n)} = 0,$$

if and only if $s \circ s_\circ = 1$, $G_s(s \circ \tau) = \tau^\lambda s$, $G_s(s \circ \upsilon) = s \circ \gamma_{\tau-1}$, $\ni s_\circ, \tau, \upsilon \in G_{si}$; $i = 0,1,2$; such that $st - lim_j \sum_{n=1}^{\infty} a_{jn} \ \left\|\Upsilon_j(G_{si}; .) - G_{si}\right\|_{\mathcal{S}_r(z_n)} = 0$; $i = 0,1,2$, and G_{si} is a subgroup from G_s ; $i = 0,1,2$.

Theorem 1.11 (Al-Muhja, M., 2015) If $(z_{ni})_{i=0}^{\infty}$ is defined by (1), then there exists a set $\mathfrak{J} = \{z_{n0} < z_{n1} < \cdots < z_{nm} < \cdots\} \subseteq \mathbb{N}$, such that $\delta_\theta(\mathfrak{J}) = 1$ and Υ_j homogeneous group if and only if a sequence $\Upsilon = (\Upsilon_j)$ is lacunary statistically convergence to L.

Now, we present the following result:

Theorem A If $(z_{ni})_{i=0}^{\infty}$ is defined by (1), then there exists a set $\mathfrak{J} = \{z_{n0} < z_{n1} < \cdots < z_{nm} < \cdots\} \subseteq \mathbb{N}$, such that $\delta_\theta(\mathfrak{J}) = 1$ and a sequence $\Upsilon = (\Upsilon_j)$ is bounded statistically lacunary summable to L if and only if it is strongly θ_q-convergence to L.

2. Proof of Theorem A and Concluding

In this section, we want the prove Theorem A, in order to get the installed results in figure 1, it is called Ω striped.

Firstly (Freedman, A. R. & Sember, J. J., 1981), we recall the strong convergence fields of various summability methods. In recent years, using statistical A-summability, Riesz's functional supremum formula via statistical limit and Rouche's sequence is B-statistical A-summability (Edely, O. H., & Mursaleen, M., 2009; Al-Muhja, M., 2014; Al-Muhja, M., Khrajan, M. & Abdul Hussein, H. J., 2015).

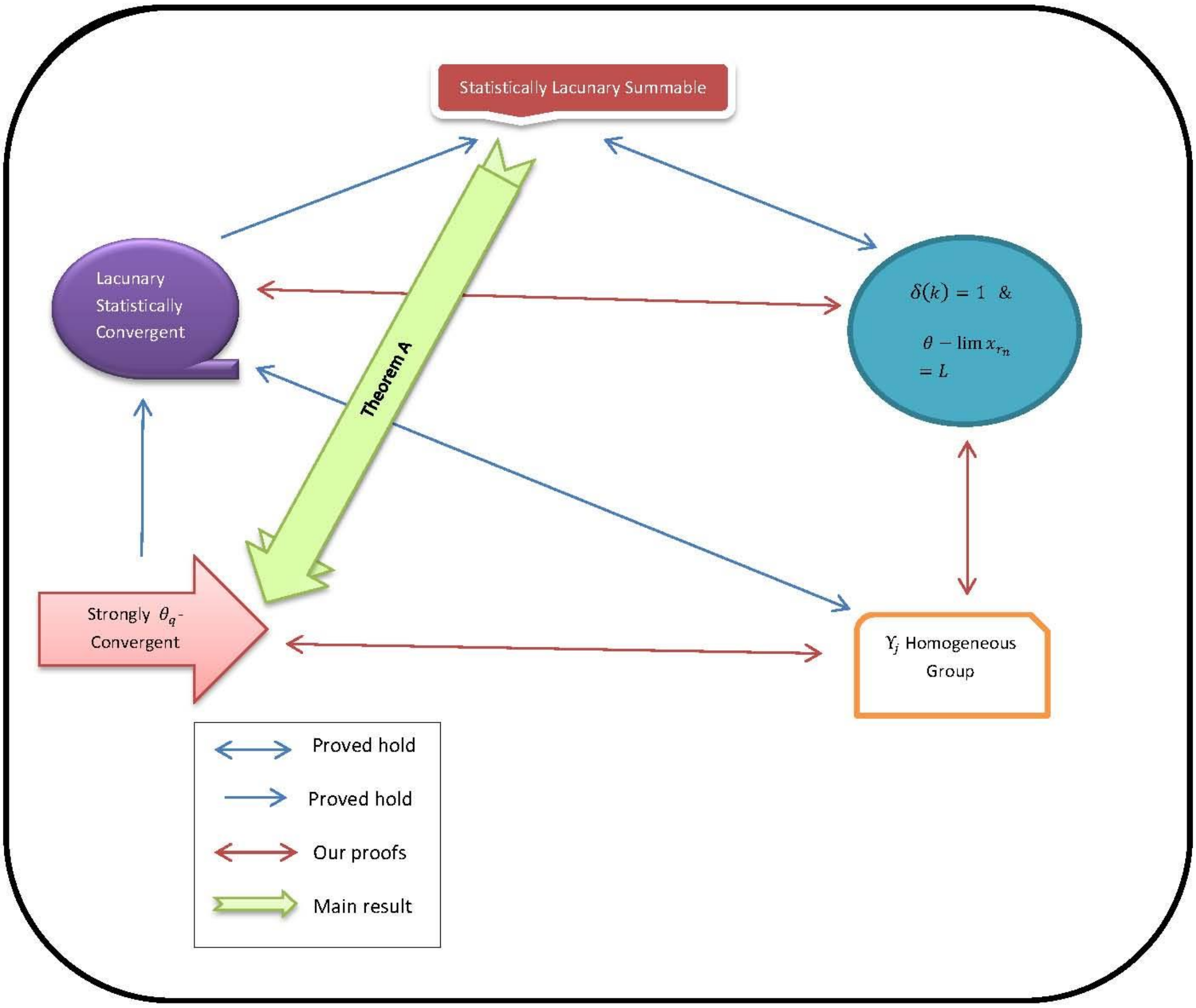

Figure 1. Statistically lacunary summability (SLS) and it is called **Ω** striped. There are new results for Korovkin theorem.

Now, we prove our theorem:

Proof of Theorem A. Assume that $\Upsilon = (\Upsilon_j)$ is bounded strongly θ_q–convergent to limit L, since $\Upsilon = (\Upsilon_j)$ is lacunary statistically convergent to L (Fridy, J. A. & Orhan, C., 1993), implication $\Upsilon = (\Upsilon_j)$ is statistically lacunary summable to L.

Now, assume that $\Upsilon = (\Upsilon_j)$ is statistically lacunary summable to L, then $|t_r(x) - L| \to 0$. Hence,

$\frac{1}{h_r}\sum_{j\in I_r}|x_j - L| \le \frac{1}{h_r}\sum_{j\in I_r}|x_j - L|^q$, ((Al-Muhja, M. & Bhaya, E. S., 2010), theorem 2.4), the inquality become:

$$\le \left|\frac{1}{h_r}\sum_{j\in I_r}(x_j - L)\right|^q = \left|\frac{1}{h_r}\sum_{j\in I_r} x_j - L\right|^q$$

$= |t_r(x) - L|^q \to 0$, as $r \to \infty$, see Definition 1.2, $t_r(x)$ is mean the generalized de la Valle-Poussin.

Hence $x_k \to L[C_\theta]_q$. This is a complete proof. □

Corollary B. A sequence $\Upsilon = (\Upsilon_j)$ is bounded strongly θ_q-convergence to L if and only if there exists a set $\mathfrak{J} = \{z_{n0} < z_{n1} < \cdots < z_{nm} < \cdots\} \subseteq \mathbb{N}$, such that $\delta_\theta(\mathfrak{J}) = 1$ and Υ_j homogeneous group.

Proof. Suppose $\Upsilon = (\Upsilon_j)$ is bounded strongly θ_q-convergence to L. Then from (Mursaleen, M. & Alotaibi, A., 2011; Theorem 2.2) and theorem 1.11, we have Figure 2. (a). Now, assume that Υ_j homogeneous group. Then from theorem 1.11, (Mursaleen, M. & Alotaibi, A., 2011; Theorem 2.1) and Theorem A, we have Figure 2. (b). This is a complete proof. □

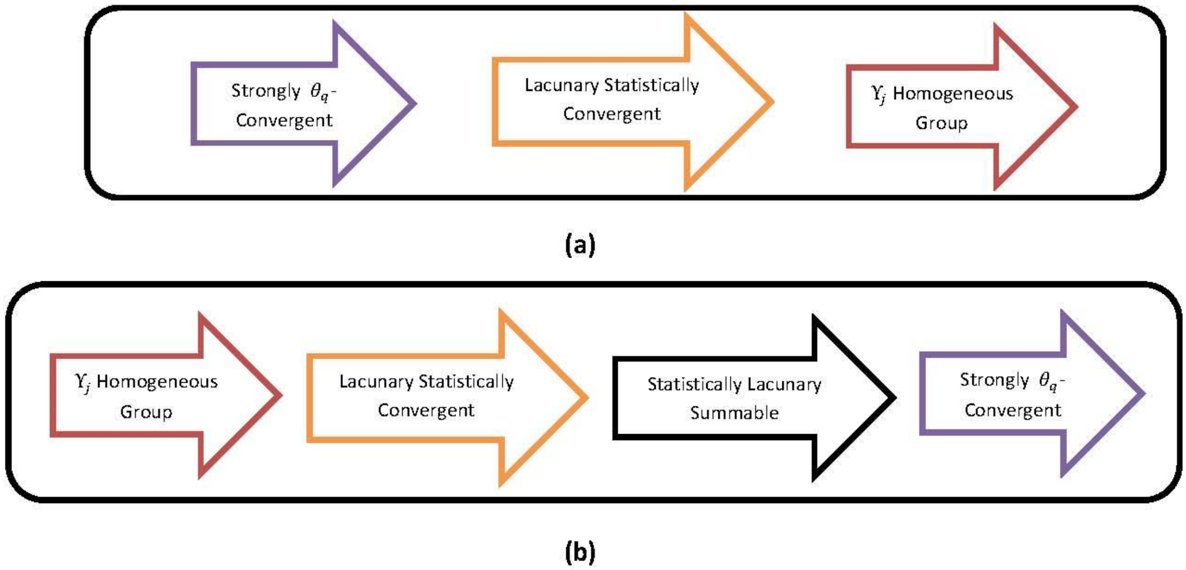

Figure 2. Proof of Corollary B

Corollary C. A sequence $\Upsilon = (\Upsilon_j)$ is statistically lacunary summable to L if and only if Υ_j homogeneous group.

Proof. Suppose $\Upsilon = (\Upsilon_j)$ is statistically lacunary summable to L. Then from Theorem A and Corollary B (first condition), we have Figure 3. (a). Now, assume that Υ_j homogeneous group. Then from theorem 1.11 and (Mursaleen, M. & Alotaibi, A., 2011; Theorem 2.1), we have Figure 3. (b). This is a complete proof. □

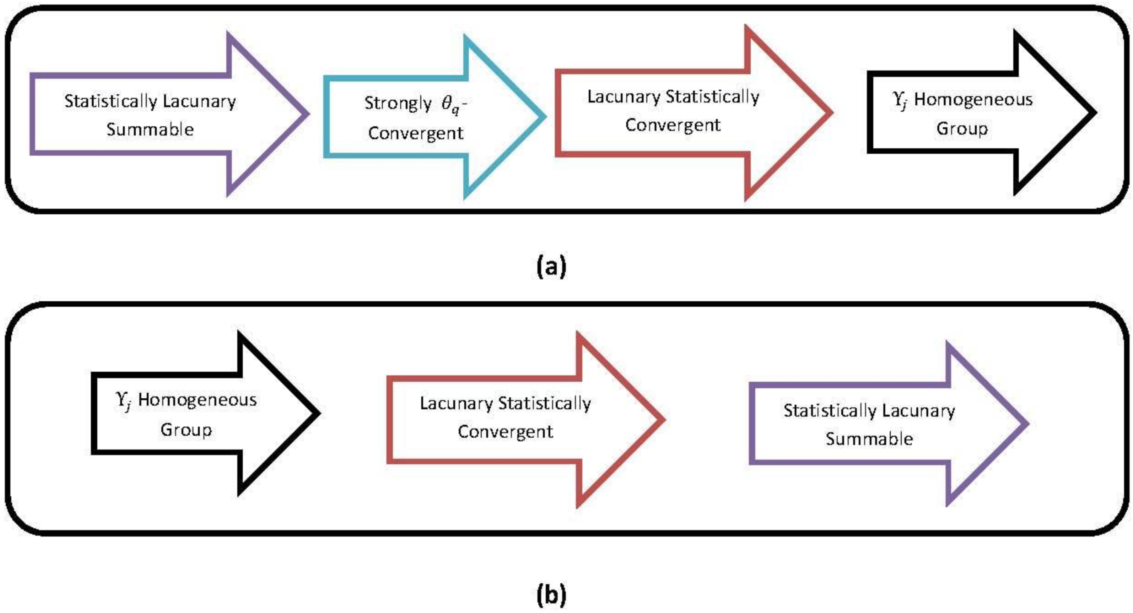

Figure 3. Proof of Corollary C

For an $\Upsilon = (\Upsilon_j)$ sequence is bounded. The following conditions are equivalent:

(SLS 1) Statistically lacunary summable to L.

(SLS 2) A set $\mathfrak{J} = \{z_{n0} < z_{n1} < \cdots < z_{nm} < \cdots\} \subseteq \mathbb{N}$, such that $\delta_\theta(\mathfrak{J}) = 1$ and $\theta - \lim x_{r_n} = L$.

(SLS 3) Υ_j homogeneous group.

(SLS 4) Strongly θ_q-convergence to L.

(SLS 5) Lacunary statistically convergence to L.

Acknowledgements

The authors are grateful for hospitality at the University of Kufa. They thank his fellows for the fruitful discussions while preparing this paper. The authors are thankful to the referees for their careful reading and making some helpful comments which have essentially improved the presentation of this paper.

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