DECISION MAKING USING MODIFIED S-CURVE MEMBERSHIP FUNCTION IN FUZZY LINEAR PROGRAMMING PROBLEM

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ABSTRACT

In order to develop approaches to solve a fuzzy linear programming problem, it is necessary to study first the formulation of membership functions and then the methodology for applying the solution to real life problems. A S-curve membership function is proposed in this paper. It is important to note that the S-curve membership function has to be flexible to describe the fuzziness in the problem. Fuzziness may occur in several levels of an industrial production management such as manpower requirements, resource availability such as software and the demand to be met. In order to show that the S-curve membership function works well for fuzzy problems, a numerical example is demonstrated. A thorough study on how the non linear membership function used in dealing with fuzzy parameters and fuzzy constraints is also presented. Only one case where all three coefficients (such as objective coefficients, technical coefficients and resource variables) that normally occur in production planning problem, are considered and fuzzified. However, there are several other cases. The result obtained from this paper is to provide confidence in using the proposed S-curve membership function in a real life production planning industrial problem.

Key words : S-Curve Function, Vagueness, Fuzzy Parameters and Degree of Satisfaction

1.0 INTRODUCTION

The methodology for this fuzzy linear programming (FLP) has references to various works by Zimmermann (1985), Werner (1987), Chanas (1983), Verdegay (1982), Carlsson and Korhonen (1986), Rommelfanger (1996), Chen and Chou (1996), Wang (1997), Parra et.al (1999), Guu and Wu (1999), Maleki et.al (2000), Buckley and Feuring (2000), Inuiguchi and Ramik (2000), Nishizaki and Sakawa (2000) and Sengupta et.al (2001). In a decision process using FLP model, source resource variables may be fuzzified, instead of precisely given numbers as in a crisp linear programming (CLP) model. For example, machine hours, labour force and materials needed in a manufacturing centre, are always imprecise, because of incomplete information and uncertainty in various potential suppliers and environments. Therefore, they should be considered as fuzzy resources, and the FLP problem should be solved using fuzzy set theory.

The general methodology to solve FLP is given below :
A general model of crisp linear programming is formulated as :

$$Max \quad z = cx \qquad \text{Non fuzzy formulation}$$

$$\text{Subject to} \quad \begin{aligned} Ax &\le b \\ x &\ge 0 \end{aligned} \tag{1}$$

where c and x are n dimensional vectors, b is an m dimensional vector, and A is m x n matrix.

Since we are living in an uncertain environment, the coefficients of objective function (c), the technical coefficients of matrix (A) and the resource variables (b) are fuzzified. Therefore the above variables and coefficients can be represented by fuzzy numbers, and hence the problem can be solved by FLP approach.

The fuzzy linear programming problem is formulated as

$$Max \quad z = \tilde{c}x \qquad \text{Fuzzy formulation}$$

$$\text{Subject to} \quad \begin{aligned} \tilde{A}x &\tilde{<} \tilde{b} \\ x &\ge 0 \end{aligned} \tag{2}$$

where x is the vector of decision variables ; $\tilde{A}$, $\tilde{b}$ *and* $\tilde{c}$ are fuzzy quantities; the operations of addition and multiplication by a real number of fuzzy quantities are defined by Zades's extension principle (Zadeh, 1975) and (Delgado et al., 1989); the inequality relation $\tilde{<}$ is given by a certain fuzzy relation and the objective function, z, is to be maximized in the sense of a given crisp LP problem. The Carlsson and Korhonen (1986) and Buckley and Feuring (2000) approaches are seen as having solved the FLP problem (2) which is fully trade-off, meaning that the solution will be with a certain degree of satisfaction.

First of all, formulate the membership functions for the fuzzy parameters of $\tilde{c}, \tilde{A}$ *and* $\tilde{b}$. Here a non-linear membership function such as the S-curve function is employed. The membership functions are represented by $\mu_{a_{ij}}, \mu_{b_i}$ *and* μ_{c_j} , where a_{ij} are the technical coefficients of matrix A for $i=1,\ldots,m$ and $j=1,\ldots,n$, b_i are the resource variables for $i=1,\ldots,m$ and c_j are the coefficients of objective function z for $j=1,\ldots,n$.

Next, through the appropriate transformation with the assumption of trade-off between fuzzy numbers of $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ $\forall$ i and j , an expression for $\tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{c}_j$ will be obtained. After trade-off between $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ the solution will always exist at (Carlsson and Korhonen, 1986) :

$$\mu = \mu_{c_j} = \mu_{a_{ij}} = \mu_{b_i} \quad \textit{for all } i = 1,\ldots,m \textit{ and } j = 1,\ldots,n \tag{3}$$

Therefore, we can obtain:

$$c = g_c(\mu) \quad , \quad A = g_A(\mu) \quad \text{and} \quad b = g_b(\mu) \tag{4}$$

where $\mu \in [0,1]$ and g_c, g_A and g_b are inverse functions (Carlsson and Korhonen,1986) of μ_c , μ_A and μ_b respectively. Equation (2) becomes

$$\begin{aligned} &\text{Max } z = [g_c(\mu)]\,\text{x} && \text{Fuzzy inverse formulation} \\ &\text{Subject to } \quad [g_A(\mu)]\,\text{x} \le g_b(\mu) && \\ &\qquad\qquad \text{x} \ge 0 && \end{aligned} \tag{5}$$

Using the above methodology, one can find an optimal compromise 'in between' as a function of the grades of imprecision in the parameters. Furthermore the optimal solutions (z_k^*: k = 1, 2, 3, 4,...) as function of the membership functions in 2 dimensional and 3 dimensional graphical modes can be plotted. The graphics offer a decision maker a clear holistic perception of how the objective function behaves for varying grades of precision, and enable him to arrive at appropriate conclusions. This compromised fuzzy solution will be given to the implementer for further discussion and for implementation. In order to solve the FLP, it is assumed that the non-linear membership functions and operators are consistent with the judgements of the decision maker and the implementer and also, with the rationality in the fuzzy decision making processes.

2.0 MEMBERSHIP FUNCTIONS FOR OBJECTIVE COEFFICIENTS, TECHNICAL COEFFICIENTS AND RESOURCE VARIABLES

In the following section the formulation of membership function for Objective coefficients, technical coefficients and resource variables are given.

2.1 Membership Function of the Coefficient of the Objective Function $\tilde{c}_j$

The membership function of $\tilde{c}_j$ is given as :

$$\mu_{c_j} = \begin{cases} 1.000 & c_j < c_j^a \\ 0.999 & c_j = c_j^a \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)}} & c_j^a < c_j < c_j^b \\ 0.001 & c_j = c_j^b \\ 0.000 & c_j > c_j^b \end{cases} \qquad (6)$$

where μ_{c_j} is the membership function of c_j and c_j^a *and* c_j^b are the lower and the upper boundaries of the fuzzy coefficient of $\tilde{c}_j$, respectively. The number $\mu_{c_j} = 1.000$ and $\mu_{c_j} = 0.000$ correspond to the 'crisp' values .

$c_j^a < \tilde{c}_j < c_j^b$ in the fuzzy region, and the value of $B = 1$, $C = 0.001001001$ and $\alpha = 13.81350956$ (Pandian, 2002).
The following section shows the derivation of fuzzy coefficient of objective function $\tilde{c}_j$

2.1.1 Fuzzy Coefficient for Objective Function $\tilde{c}_j$

The membership function for $\tilde{c}_j$ is given as :

$$\mu_{c_j} = \frac{B}{1 + Ce^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)}}$$

Rearranging exponential term $$e^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)} = \frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right)$$

Taking $\log_e$ both sides $$\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right) = \ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right)$$

Hence $$c_j = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right) \tag{7}$$

Since c_j is a fuzzy coefficient for the objective function as in equation (7), it is denoted as $\tilde{c}_j$.

Therefore $$\tilde{c}_j = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right) \tag{8}$$

The membership function for μ_{c_j} and the fuzzy interval , c_j^a *to* c_j^b , for $\tilde{c}_j$ is as given in Figure 1.

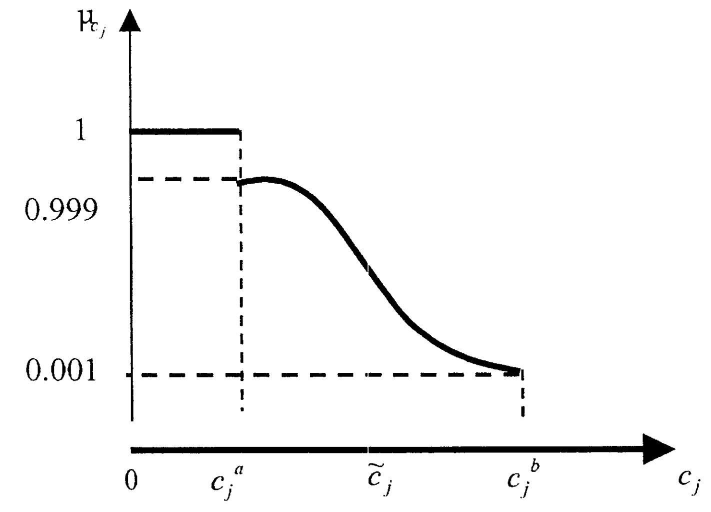

Fig. 1: Membership Function μ_{c_j} and Fuzzy Interval for $\tilde{c}_j$

In a similar way the membership function for fuzzy technical coefficients and fuzzy resource variables and its derivations (Pandian, 2002) can be formulated. According to Watada (1997), a triangular or trapezoidal membership function shows a necessity level and a sufficiency level at their grades 1 and 0 respectively. On the other hand, considering a non-linear membership function as a flexible S-curve, a necessity level or a sufficiency level may be approximated at the points with grade $\mu_{c_j} = 0.999$ *when* $c_j = c_j^a$ and

$$\mu_{c_j} = 0.001 \text{ when } c_j = c_j^b \tag{9}$$

3.0 S-CURVE MEMBERSHIP FUNCTION IN FLP

A numerical example on the parametric case is considered in order to illustrate FLP approach using S-curve membership function. This problem was discussed by Carlsson and Korhonen (1986) using exponential membership function. A general S-curve membership function is classified as a flexible membership function (Bells, 1999). The usefulness of the proposed form of the S-curve membership function is illustrated through a numerical example.

In the following formulation the parameters of the model are defined on a fuzzy interval denoted by $[k_1, k_2)$ where $k_1 < k_2$. The first number k_1 represents a ' crisp ' value and the second number k_2 represents a ' fuzzy ' value. The FLP formulation is given as :

$$
\begin{aligned}
&\text{Max } [1, 1.5)x_1 + [1, 3)x_2 + [2, 2.2)x_3 \\
&\text{Subject to } [2, 3)x_1 + [0, 2)x_2 + [1.5, 3)x_3 \le [18, 22) \\
&\qquad [0.5, 1)x_1 + [1, 2)x_2 + [0, 1)x_3 \le [10, 40) \\
&\qquad [6, 9)x_1 + [18, 20)x_2 + [3, 7)x_3 \le [96, 110) \\
&\qquad [6.5, 7)x_1 + [15, 20)x_2 + [8, 9)x_3 \le [96, 110)
\end{aligned}
\tag{10}
$$

The fuzzy interval for $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are defined as follows :

$[c^a_j, c^b_j)$ is a fuzzy interval of the coefficient of the objective function whereby c^a_j is the lower boundary and c^b_j is the upper boundary .

$[a^a_{ij}, a^b_{ij})$ is a fuzzy interval of the technical coefficient matrix whereby a^a_{ij} is the lower boundary and a^b_{ij} is the upper boundary .

$[b^a_i, b^b_i)$ is a fuzzy interval of the resource variable whereby b^a_i is the lower boundary and b^b_i is the upper boundary.

4.0 FORMULATION OF FUZZY LINEAR PROGRAMMING PROBLEM (FLPP) .

The FLP problem, formulated in equation (10) can be written as :

$$
\begin{aligned}
&Max \sum_{j=1}^{3} \tilde{c}_j x_j \\
&subject\ to \sum_{i=1}^{4} \tilde{a}_{ij} x_j \le \tilde{b}_i \\
&where\ x_j \ge 0 , j = 1,2,3.
\end{aligned}
\tag{11}
$$

Using equations (8) and (10), the formulation (11) is made equivalent to :

$$Max \quad \sum_{j=1}^{3}\left(c_j^a + \left[\frac{c_j^b - c_j^a}{\alpha}\right] \ln\frac{1}{C}\left[\frac{B}{\mu_{c_j}} - 1\right]\right) x_j$$

subject to $$\sum_{i=1}^{4}\left(a_{ij}^a + \left[\frac{a_{ij}^b - a_{ij}^a}{\alpha}\right] \ln\frac{1}{C}\left[\frac{B}{\mu_{a_{ij}}} - 1\right]\right) x_j \le b_i^a + \left[\frac{b_i^b - b_i^a}{\alpha}\right] \ln\frac{1}{C}\left[\frac{B}{\mu_{b_i}} - 1\right]$$

where $x_j \ge 0, \quad j = 1,2,3 \;, \quad 0 < \mu_{c_j}, \mu_{a_{ij}}, \mu_{b_i} < 1 \;, \quad 0 < \alpha < \infty .$

(12)

The complete set of resultant equations is given below as :

$$Max\left(1+\frac{0.5}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{c_1}}-1\right]\right)x_1 + \left(1+\frac{2}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{c_2}}-1\right]\right)x_2 + \left(2+\frac{0.2}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{c_3}}-1\right]\right)x_3$$

Subject to

$$\left(2+\frac{1}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a11}}-1\right]\right)x_1 + \left(\frac{2}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{12}}}-1\right]\right)x_2 + \left(1.5+\frac{1.5}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{13}}}\right]\right)x_3 \le$$

$$18+\frac{4}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{b_1}}-1\right]$$

$$\left(0.5+\frac{1}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{21}}}-1\right]\right)x_1 + \left(1+\frac{1}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{22}}}--1\right]\right)x_2 + \left(\frac{1}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{23}}}-1\right]\right)x_3 \le$$

$$10-\frac{30}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{b_2}}-1\right]$$

$$\left(6+\frac{3}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{31}}}-1\right]\right)x_1+\left(18+\frac{2}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{32}}}-1\right]\right)x_2+\left(3+\frac{4}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{33}}}-1\right]\right)x_3 \leq$$

$$96+\frac{14}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{b_3}}-1\right]$$

$$\left(6.5+\frac{0.5}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{41}}}-1\right]\right)x_1+\left(15+\frac{5}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{42}}}-1\right]\right)x_2+$$

$$\left(8+\frac{1}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{43}}}-1\right]\right)x_3 \leq 96+\frac{14}{\alpha}\ln\frac{1}{C}\left[\frac{B}{\mu_{b_4}}-1\right] \quad (13)$$

where α=13.81350956, C=0.001001001, B=1 *and* $0<\mu<1$ *as in equations* (12) *and* (13)

In equation (13), the best value for the objective function at the fixed level of μ is reached (Carlsson and Korhonen, 1986) when

$$\mu=\mu_{c_j}=\mu_{a_{ij}}=\mu_{b_i} \quad for \quad i=1,2,3,4 \quad ; j=1,2,3 \quad (14)$$

Such a fixed level of μ representing same measure of precision is considered here to simplify the complexity of the problem.

Using equation (4) with the above values of α, B and C, values of c_j are computed for the range $\mu_{c_j}=0.001$ to $\mu_{c_j}=0.999$. The interval between two adjacent μ_{c_j} values can be arbitrary but has to be as small as possible to reach a level of precision in optimal solution. Here an interval for μ_{c_j} is considered as 0.0499. Kuzmin (1981) has indicated that the membership function μ_{c_j} can be obtained in several ways. One of the ways is using a functional rule for determining μ_{c_j}. This observation is adopted in forming a function for μ_{c_j} as given in equation (6). Carlsson and Korhonen (1986) have also used their own functional rule for μ_{c_j}. The interval for μ_{c_j} for computation of c_j, in their works as 0.1, is considerably large.

5.0 COMPUTATION OF THE OBJECTIVE FUNCTION, Z^*

The FLP problem has been formulated and all the coefficients are parameterized. However, it will not be possible to use the linear parametric formulation to solve the FLP problem since the membership functions are non-linear (Carlsson and Korhonen, 1986). What is needed is to carry out a series of experiments for 21 membership values: $\mu_{aij} = \mu_{bi} = \mu_{cj} = \mu = 0.0010, 0.0509, 0.1008, ..., 0.9990$ with an interval of 0.0499. These experiments are carried out by using the Simplex Method in the Optimization Tool Box of MATLAB.

μ = Membership value (it is also referred to as degree of satisfaction (Zimmermann, 1985) and (Sengupta et.al, 2001))
b_i^* = Resource values actually used , $i = 1, 2, 3, 4$.
x_j^* = Decision variable , $j = 1, 2, 3$.
z^* = Optimal value of objective function z

The following definitions are employed in the process of obtaining the optimal solution of the objective function.

The procedure for obtaining the optimal solution for the FLP problem is described as follows :

Step 1 : Set the interval for fuzzy parameter μ from 0 to 1 with interval steps of 0.0499.

Step 2 : For each μ, generate the fuzzy parameter of c_j, a_{ij} and b_i , $i = 1, 2, 3, 4$ and $j = 1, 2, 3$ by using MATLAB® programming

Step 3 : Input the value of fuzzy parameters in Simplex Method of MATLAB® (Optimization Tool Box : Linear Programming) to obtain optimal solution, z^*.

The optimal solution z^* versus membership value μ is plotted using MATLAB® (2 Dimensional Graphics) as given in Figure 2 (Marchand,1999).

Figure 2 will be presented to the decision maker then to the implementer for further analysis.

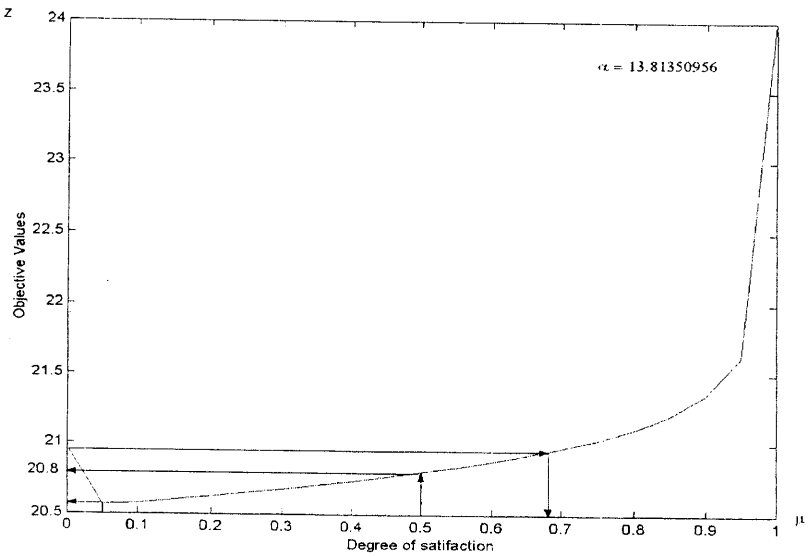

Fig. 2: Degree of Satisfaction and Objective Values for α=13.81350956

5.1 Objective Values for the S-Curve Membership Function

The relationship between the optimal objective values of the fuzzy linear programming problem and their corresponding membership grades (degree of satisfaction) is presented in Figure 2. From Figure 2, we see that the objective function z with an S-curve membership function has a value 24.0000 at $\mu = 0.999$ and 20.5655 at $\mu = 0.001$. It can also be seen that the value of z dropped sharply from 20.9524 ($\mu = 0.0010$) to 20.5655 ($\mu = 0.0509$) and then it increases steadily from 20.5768 ($\mu = 0.1008$) to 21.6275 ($\mu = 0.9491$). There is a very sharp increase for z value from 21.6275 ($\mu = 0.9491$) to 24.0000 ($\mu = 0.9990$). For instance the z value for a degree of satisfaction of 50% is 20.8000. From Figure 3 the decision maker can evaluate how the objective function behaves as a function of μ.

6.0 OBJECTIVE VALUES FOR VARIOUS VAGUENESS VALUES, α

Figure 3, displays the objective values plot for various values of α from 2 to 20. The graph shows the nature of variations of z^* with respect to μ.

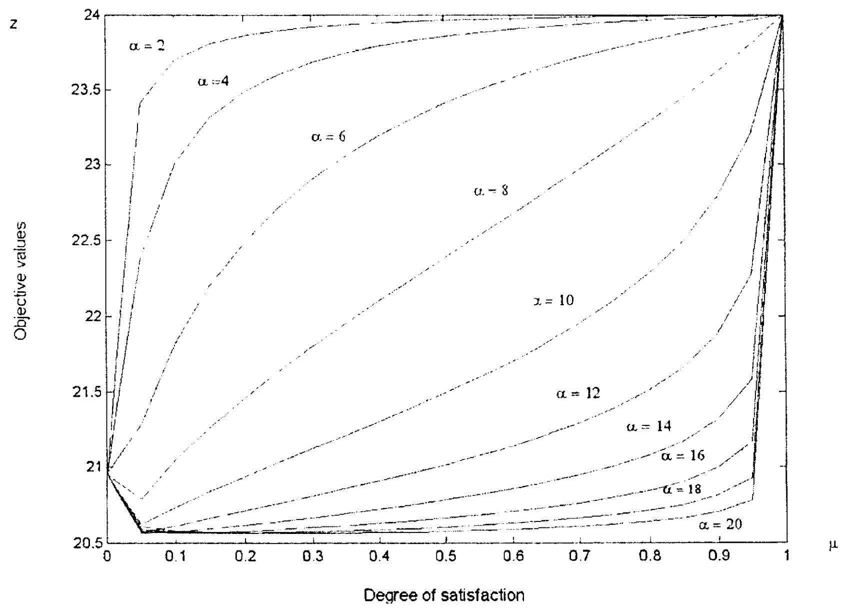

Fig. 3: Degree of Satisfaction and Objective Values for $2 \le \alpha \le 20$

The membership values μ in Figure 3 represents the degree of satisfaction, and z can be the profit function. The result for 50% degree of satisfaction for $2 < \alpha < 20$ and the correspondence value for z is presented in Table 1. The possible realistic solution exists at $\mu = 0.5000$ (Carlsson and Korhonen, 1986) which is considered as compromised satisfactory solution with trade off.

Table 1 : Fuzzy Parameter and Objective Values (50% degree of satisfaction, $\mu = 0.5$)

Fuzzy Parameter (vagueness, α)	Objective Value z^{**}
2	23.9646
4	23.8563
6	23.4056
8	22.3865
10	21.4915
12	21.0177
14	20.7844
16	20.6669
18	20.6074
20	20.5788

The values of the objective function z^* and the vagueness parameter α are obtained μ=0.5000 (50% degree of satisfaction). It can be seen from Table 1 that at μ=0.5000 when α (vagueness) increases, then z^* (objective value) reduces. It means that when the vagueness in the variables increases, z^* decreases for the same degree of satisfaction. This is a realistic solution in fuzzy environment.

The 3 dimensional plot for μ (degree of satisfaction), α (vagueness parameter) and z^* (objective values) is given in Figure 4.

One can observe from Figure 4 that, for $0.000 < \mu < 0.2500$ the objective value, z^*, increases very fast for vagueness, α less then 6. For $0.2500 < \mu < 0.7500$ the z^* values increase steadily as α increases from 6 to 10. A sharp increase in z values is found for $0.7500 < \mu < 1.0000$ as α increases from 12 to 20.

From the above discussion, it is concluded that the optimal decision does not always guarantee the best outcome. Therefore, it is important to note that the optimal decision with the best outcome can be obtained to a certain degree of satisfaction only.

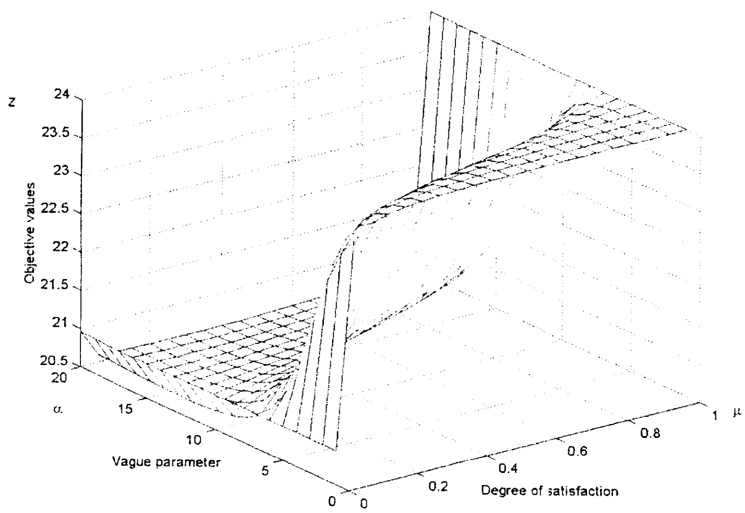

Fig. 4 : Degree of Satisfaction μ, Vagueness α and Objective value z^*

7.0 CONCLUSION

In this paper a numerical example of production planning problem where all constraints are fuzzy is considered. The problem is solved with greater details provided, and the results are encouraging. We accept that this FLP methodology is meaningful.

The main points considered in this paper are as follows :

(i) The number of membership values considered for μ is 21. This is more than those considered by earlier researcher (Carlsson and Korhonen, 1986) when working with exponential membership function with 11 values for μ. More membership values will lead to a more precise solution in the fuzzy environment. The membership values which represent degrees of satisfaction will lead us to obtain more precise solutions in objective function, z^*.

(ii) Since there is no such degree of satisfaction at $\mu = 0$ and $\mu = 1$, the degree of satisfaction at $\mu = 0.001$ and $\mu = 0.999$ are considered. This is because in real life problems it is very hard to obtain either 0% or 100% degree of satisfaction.

(iii) The MATLAB® programming Tool Box is very useful in generating fuzzy parameters (c_j, a_{ij} and b_i) and in giving fuzzy interval for all fuzzy constraints. Here the matching of fuzzy parameters to Simplex Method Tool Box of MATLAB® was done successfully.

(iv) The MATLAB® programming Tool Box saves time in finding the optimal solution for the FLP problem considered in this paper. The study also indicate that MATLAB® programming Tool Box is better than any other linear programming software (conventional method) in solving FLP problems.

The developed methodology of solving FLP problem using MATLAB® has the potential to be applied in real life problems such as in industrial production management problems.

ACKNOWLEDGMENTS

The authors would like to sincerely thank the referees of the paper for giving valuable ideas and comments for the improvement of the paper.

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