INTERACTED MULTIPLE ANT COLONIES OPTIMIZATION APPROACH FOR THE SINGLE MACHINE TOTAL WEIGHTED TARDINESS PROBLEM

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ABSTRACT. Single Machine Total Weighted Tardiness Problem (SMTWTP) is an important combinatorial optimization problem that considers the job scheduling for sequential processing on a single machine and the target is to minimize the total tardiness of all jobs. This is a crucial task in manufacturing and production planning. The framework of Interacted Multiple Ant Colonies Optimization (IMACO) is a recent proposition. It divides the ants' population into several colonies and employs certain techniques to organize the work of these colonies. This paper considers the tackling of SMTWTP using IMACO. It also proposes the idea of different ant colonies use different types of problem dependent heuristics. The performance of IMACO was demonstrated and compared with the best performing ant algorithms the Ant Colony System (ACS). The Computational results show the dominance of IMACO.

Keywords: Multiple Ant Colonies Optimization, Ant colony, optimization

INTRODUCTION

Machine scheduling problems, traveling salesman problem, quadratic assignment problem, vehicle routing problem, and network routing problem are some well known examples of Combinatorial Optimization Problems that have great importance in research and development. These problems have a discrete set of feasible solutions and the goal is to find the optimal solution (the best solution from the feasible solutions). These problems are theoretically proven as *NP* - hard problems. The only way to tackle these problems is to use approximate (heuristic) algorithms such as tabu search, evolutionary computation, simulated annealing, genetic algorithms and Ant Colony Optimization (ACO).

The recently proposed IMACO framework tries to improve the performance of ACO algorithms by utilizing several ant colonies with certain techniques to organize the work of these colonies. The proposed framework composes necessary techniques that encourage the controlled exploration of the search space in couple with a good exploitation of previously obtained good solutions. As exploration is the means of ants to search for new solution, this should be done under certain control to avoid exploring a very wide area from search space that might be far from the optimal solution. On the other hand a good exploitation of the search history is necessary to search the solution space in the neighborhood of previously good solution. However, very strong exploitation is not required because it increases the convergence speed of ants towards the same solution obtained in previous iterations (Aljanaby et al., 2010a, 2010b).

In this paper, the SMTWTP is first described. The explanation of the framework of IMACO and its incorporated techniques is then given. An experimental study of using IMACO with different problem dependent heuristics to solve all 375 available instances of SMTWTP is conducted and the results have been shown and compared with the results of other ACO algorithms.

THE SINGLE MACHINLE PROBLEM

SMTWTP can be stated as follows. Each of n jobs is to be processed without pre-emption on a single machine that can handle no more than one job at a time. The processing and set-up requirement of any job are independent of its position in the sequence. The release time of all jobs is zero. Thus, jobs j (j=1, ..., n) becomes available at time zero, requires uninterrupted positive processing time p_j, which includes set-up and knock-down times on the machine, has a positive weight w_j, and has a due time d_j by which it should ideally be finished. For a given processing order of the jobs, the completion time c_j and the tardiness $T_j=\max\{0, c_j-d_j\}$ of job j can be computed. The problem is to find a processing order of the jobs with minimum total weighted tardiness $\sum_{j=1}^{n} w_j T_j$ (Besten, Stützle, & Dorigo, 2000, Baggio, Wainer, & Ellis, 2004).

The SMTWTP is an NP-hard scheduling problem for which instances with more than 50 jobs often can not be solved to optimality with state of the art branch and bound algorithms (Congram, Potts, & van de Velde, 2002). The total number of available instances is 125 for values of n=40, n=50 and n=100. Optimal values of solutions are available for 124 and 115 of 40 and 50 job problem instances respectively. The values for unsolved problems are the best known solution to Crauwells et al. (1998). These solutions appear to be optimal since they have not been enhanced for a long time. The best known solutions to date of the 100-job instances are available and most of them are according to Crauwells et al. (1998) and Congram et al. (2002).

Three types of problem specific heuristic are examined in this work. These problem specific heuristic are easily calculated and have been studied in the literature (Besten, Stuzle, & Dorigo, 2000) and are as follows.

- Earliest Due Date (EDD): this heuristic puts the jobs in non-decreasing order of the due dates d_j and given by:

$$H_{ij} = \frac{1}{d_j} \tag{1}$$

- Modified Due Date (MDD): this heuristic puts the jobs in non-decreasing order of the modified due dates mdd_j which given by $mdd_j=max\{C+p_j, d_j\}$, where C is the sum of the processing times of the already scheduled jobs. This heuristic is given by:

$$H_{ij} = \frac{1}{mdd_j} \tag{2}$$

- Apparent Urgency (AU): this heuristic puts the jobs in non-decreasing order of the apparent urgency which given by

$$au_j = \frac{w_j}{p_j} \cdot \exp\left(-\frac{\max\{d_j - C_j, 0\}}{k\bar{P}}\right) \tag{3}$$

Where $\bar{P}$ is the average processing time of the remaining jobs, k is a scaling parameter which set to 2 (Besten, Stuzle, & Dorigo, 2000). The heuristic is given by:

$$H_{ij} = \frac{1}{au_j} \tag{4}$$

INTERACTED MULTIPLE ANT COLONIES OPTIMIZATION

IMACO framework is recently proposed in pervious work of the author (Aljanaby et al., 2010a, 2010b, 2010c, 2010d). In this framework there are two levels of interaction the first one is the colony level and the second one is the population level. The colony level interaction can be achieved through the pheromone depositing process within the same colony; the pheromone updating mechanism is responsible for the implementation of this kind of interaction. The population level interaction is achieved by evaluating the pheromones of different colonies using some evaluation function; the responsibility here is of the pheromone evaluating mechanism.

The work activities of a single colony in the proposed IMACO algorithm are based on ACS. Each colony has its own pheromone that is used as an interaction between the ants of the same colony. The interaction between ant colonies using pheromone can be organized in different terms. The IMACO algorithm is described as follows. M colonies of m ants each are working together to solve some combinatorial problem. The probabilistic decision of the ant k belongs to the colony v to move from node i to node j is defined as:

$$j = \begin{cases} \arg \max_{l \in N_i^k} \{ f(P_{il}) H_{il}^{\beta} \} & if \quad q \le q_0 \\ S & \text{otherwise} \end{cases} \qquad (5)$$

The random variable S is selected according to the following probabilistic rule:

$$S \begin{cases} \dfrac{f(P_{ij}) H_{ij}^{\beta}}{\sum_{l \in N_i^k} f(P_{il}) H_{il}^{\beta}} & \text{if } j \in N_i^{kv} \\ 0 & \text{otherwise} \end{cases}$$

(6)

Where N_i^{kv} is the set of remaining nodes to be visited by the k^{th} ant of colony v located at node i and P_{ij}^{v} is the pheromone of colony v on the edge (i,j). $f(P_{ij})$ is the evaluation function of the pheromone on the edge (i, j) and will be discussed in next subsection.

Global and local pheromone updating are used in IMACO. Global pheromone updating includes that best ant of each colony deposits an amount of pheromone on its own path. The best ant refers to the ant that got the so far best (global) solution since the starting of the algorithm execution or the ant that got the best solution in the current iteration of the algorithm execution. In this work a combination of so far best and iteration best ants are allowed to update the pheromone.

After all ants of all colonies complete their tours (i.e., one algorithm iteration), the ant that finds the so far best solution in its colony is allowed to deposit an amount of the colony's pheromone on the edges of its tour according to the following global pheromone update:

$$P_{ij}^{v} = (1 - \sigma) P_{ij}^{v} + \sigma \Delta p_{ij}^{v,bs} \qquad (7)$$

Where σ is a pheromone evaporation parameter its value is in the range [0, 1] and $\Delta P_{ij}^{v,bs}$ is the pheromone quantity added to the connection (i, j) belonging to the best solution of the v^{th} colony $L^{v,bs}$ and is given by:

$$\Delta p_{ij}^{v,bs} = \begin{cases} 1/L^{v,bs} & if \ (i,j) \text{ belongs to the best tour of colony } v \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

To create a search diversification IMACO uses iteration best solution once in the pheromone updating after each 50 times of using the global best solution. Local pheromone updating includes that each ants reduces the amount of pheromone on paths it uses in order to give a more chance to other paths to be chosen by the future generations. Local pheromone update is applied by each ant on the visited edges. It is very important rule as it is performed during the solution construction this helps to yield different pheromone evaluation values for the same edge in the same iteration at different solution construction steps and it is given by:

$$P_{ij}^{c} = (1 - \gamma) P_{ij}^{c} + \gamma p_0 \quad (9)$$

where P_0 is the initial pheromone value and γ is another pheromone evaporation parameter with a value in the range [0, 1].

Evaluation Technique

The pheromone of different colonies has been evaluated using two mechanisms. The first mechanism evaluates the pheromone as an average of the pheromone values of all colonies on some edge. This means that an ant will make its decision to choose some edge based on the average of the available experiences of ants of all colonies that visited this edge in the past. This variant of IMACO is referred hereafter as IMACO-AVG.

Given that for each edge there are M pheromone values each belongs to a single colony. Average pheromone evaluation function evaluates the pheromone on any edge as an average of the available M values. The average pheromone evaluation function $f(P_{ij})$on the edge (i, j) for IMACO-AVG will be defined as:

$$f(P_{ij}) = \frac{\sum_{c=1}^{M} P_{ij}^{c}}{M} \quad (10)$$

The second mechanism evaluates the pheromone as the maximum value of the pheromone values of all colonies on some edge. This mechanism, referred as IMACO-MAX, chooses the max value among the available M values. The pheromone evaluation function for IMACO-MAX is defined as:

$$f_2(P_{ij}) = \underset{c=1}{\overset{M}{Max}} \; P_{ij}^{c} \quad (11)$$

The above rule lets an ant's decision to choose some new edge be based on the best available experience of ants of all colonies that previously visited this edge. This kind of cooperation using max pheromone evaluation is trying to make an early exploitation of the history of the search by choosing the max (best) available pheromone value. The result of this max pheromone evaluation function lets an ant to follow the best available information about the goodness of particular edge. However, since best pheromone comes from different colonies, this will provide necessary diversification that helps ants' to avoid the attraction to a one good solution.

The above two mechanism are pure average and max evaluation that depends 100% on the average evaluation function. The following rule is a more general which evaluates the pheromone as a composition between the pheromone values of the ant own colony and the value of the pheromone evaluation function based on some pheromone evaluation rate. Consider that the composition rate is 0.5; an ant will build 50% of its decision based on its own colony's experience and the other 50% based on the experiences of other colonies. This new variant will be called IMACO-AVG E λ and IMACO-MAX E λ where λ is the pheromone evaluation rate; its value is in the range [0, 1]. The pheromone evaluation function is then defined as:

$$f'(P_{ij}) = \lambda P_{ij}^{s} + (1-\lambda)\ f(P_{ij}) \tag{12}$$

Where P_{ij}^{s} is the pheromone belongs to colony s on edge (i, j). Note that IMACO-AVG E0 and IMACO-MAX E0 represent the pure pheromone evaluation and IMACO-AVG and E1 IMACO-AVG represent no interaction between utilized ant colonies.

Exploration technique

Each ant makes a probabilistic decision when it needs to move to a new node. The probabilistic decision is based on heuristic information (cost) and pheromone information. Pheromone represents information about previous experiences of the ant's own colony and of the other colonies. While heuristic represent a priori information about the goodness of a solution. Exploration and exploitation is controlled by the parameter q_0 whose value is in [0, 1]. It is usually used in ant's probabilistic decision as trade-off between exploitation (choosing the edge with the higher value of the multiplication of pheromone and heuristic values) and exploration (choosing the edge randomly according to some probability distribution). Setting q_0 to zero means that the algorithm uses a pure exploration while pure exploitation is reached by setting q_0 to one. However, the value used for q_0 in many research papers usually between 0.5 and 0.9 (Dorigo & Stützle, 2002, 2004, Dorigo & Blumb, 2005). Most of the work done using ACS in solving different problems was with q_0 =0.9 which gives the algorithm a high chance of exploitation without loosing the chance of exploration.

IMACO considers the case where different ants' colonies have different values for the parameter q_0. The value 0.8 has been assigned to the centre colony whose number equal to *int* (no. of colonies / 2). This value is increased / decreased for the colonies after / before the centre colony by a changing factor called QCF. This technique enables the utilized ant colonies to work with different levels of exploration. Some will prefer high exploration of new areas of search space while other colonies will prefer high exploitation search history.

RESULT AND DISCUSSION

ACS, IMACO-AVG and IMACO-MAX for SMTWTP have been implemented using visual C++. Both versions of IMACO have been applied to all available 375 instances of SMTWTP. Based on past work the number of colonies utilized by IMACO-AVG and IMACO-MAX was 8 colonies, the evaluation rate was λ=0.4 and the exploration / exploitation control parameter was QCF=0.025 (Aljanaby et al., 2010c, 2010d). In addition to use IMACO with EDD, MDD and AU, this section is developing the idea of using IMACO with different combination of the three heuristics. For instance, using EDD-MDD means that half of the utilized ant colonies will use EDD while the other half of these colonies will use MDD.

The global pheromone updating is performed by according to rules **7** and **8**. The value of best solution (global-best or iteration-best) mentioned in rule **8** represents the total weighted tardiness of the jobs sequence of the best solution. Local pheromone updating is performed using rule **9** and P_0 the initial value of pheromone trials that usually assigned a small value computed as $P_0 = \frac{1}{nT_{EDD}}$ where n is the number of jobs and T_{EDD} is the total weighted tardiness of job sequence obtained by EDD.

Table 1 shows the results of experiments done on 125 instances of 40, 50 and 100 job SMTWTP. The results presented in these tables are the number the optimal solution found (out of 125). The results of ACS presented in Table 1 are of the implementation developed with this research work. The reason is that the results of ACS presented in the literature usually with local search while all results presented here are without using local search. As explained in previous section all algorithms ran exactly the same number of computation steps.

Regarding the use of different combination of heuristics, EDD-MDD was the best combination as it always reaches the best results. EDD-MDD-AU heuristic was in the second rank and followed by MDD. This seems normal as previous studies (Besten et al., 2000, Congram et al., 2002) show the ranking of these heuristic according to the goodness of the results obtained was MDD, EDD and AU respectively. The results obtained from IMACO confirmed this getting the best results when using the best two heuristics, i.e., EDD-MDD combination. In fact, the use of a combination of heuristics increases the ability of different colonies to achieve high diversion in the search process and therefore increase the ability to improve the quality of the obtained solutions.

Table 1. Results for 40, 50 and 100 job instances

Algorithm	Heuristic	40-Job	50-job	100-Job
ACS	EDD	39	33	24
	MDD	44	37	27
	AU	36	30	21
IMACO-AVG	EDD	45	38	30
	MDD	53	45	37
	AU	41	34	26
	EDD-MDD	57	50	42
	EDD-AU	43	37	28
	MDD-AU	49	43	34
	EDD-MDD-AU	54	47	38
IMACO-MAX	EDD	43	37	30
	MDD	48	42	34
	AU	38	31	24
	EDD-MDD	53	46	39
	EDD-AU	43	36	30
	MDD-AU	47	40	32
	EDD-MDD-AU	49	43	34

CONCLUSION

It is obvious based on the results of Table 1 is that IMACO-AVG and IMACO-MAX outperform ACS in terms of the number of optimal solution found. IMACO-AVG was the best algorithm that found the best results all the way. It is the ability of IMACO to avoid the stagnation situation and improves its solutions with the time. This comes from the kind of interaction used between ant colonies and the type of information used by ants when making their decision. The proposed interaction plays on two directions which are cooperation and diversification. Pheromone evaluation mechanism plays the main role in cooperation. Pheromone evaluation was the mean to combine the pre-acquired information about the quality of the solutions represented as pheromone values. Average pheromone evaluation was the best technique that puts IMACO-AVG in front of other state-of -the-art ant algorithms. Pheromone evaluation needs a high support from other mechanisms. On the other hand, letting different colonies works with different q or different levels of exploration / exploitation was of great aid in achieving diversification. Some colonies prefer a higher exploration while others prefer a higher exploitation. This provides the whole search process with a wide range of good solution that ants of different colonies choose their best solution from.

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