Fuzzy Multi Criteria Evaluation for Performance of Bus Companies

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Abstract

A multi criteria decision making in ranking the bus companies using fuzzy rule is proposed. The proposed method uses the application of fuzzy sets and approximate reasoning in deciding the ranking of the performance of several bus companies. The proposed method introduces data normalization using similarity function which dampens extreme values that exist in the data. The use of the model is suitable in evaluating situation that involves subjectivity, vagueness and imprecise information. Experimental results are comparable to several previous methods.

Keywords: Fuzzy sets, Multicriteria decision making, Approximate reasoning

1. Introduction

A highly reliable and effective performance evaluation rule is an essential process in decision making environments. In real problems, evaluation techniques engage handling cases like subjectivity, fuzziness and imprecise information. Application of fuzzy sets theory in evaluation systems can improve the evaluation (Zhou et al., 2002). Several researchers have tried to solve this problem through analytical hierarchy process (AHP), for example in personnel selection (Saaty, 1995; Taylor et al., 1998) and shipping performance evaluation (Chou & Liang, 2001), where evaluation was done by aggregating all the fuzzy sets. However, the presence of imprecision, vagueness and subjectivity at each level further accumulates greatly the undesired elements in aggregating the marks.

In the literature, various concepts have been proposed focusing on the combination of fuzzy logic model with multi objective decision that can assist in reducing errors in making a judgment (Yamashita, 1997; Turban et al., 2000). These researches provide approaches of judgment procedure on personnel selection through the development of AHP fuzzy multi criteria. It is cited as being able to minimize the subjectivity. Several researches in fuzzy evaluation methods have been discussed in Chen & Lee (1999), Law (1996), Biswas (1995), Chu (1990) and Kuo & Chen (2002). The authors have proposed algorithms based either on fuzzy similarity function or fuzzy synthetic decision and ranking procedure through satisfaction function. Fuzzy sets membership enables the interpretations of linguistic variables in a very natural and plausible way to formulate and solve various problems. The expression of the linguistic variable by singleton fuzzy sets such as in Capaldo & Zollo (2001) and Weon & Kim (2001) could loss much important information and would additionally complicate the course of action.

Although many evaluation methods for selecting or ranking have been suggested in the literature, there is yet a method which can always give a satisfaction solution to every situation. For this reason, a fuzzy evaluation method is proposed by combining the concepts introduced by Chu (1990) and Biswas (1995) and integrating it with a fuzzy rule that is derived automatically from an input data. In evaluating student answer scripts, Biswas (1995) introduced fuzzy set mark and standard fuzzy set for grading. However, evaluations are not consistent because they are given by evaluators. In Chu (1990), teaching quality is evaluated by obtaining fuzzy synthetic decision matrix through operation of vector dot product between normalized original data and the weight. The decision matrix is then compute using decision criteria set and fuzzy approximate reasoning which uses fuzzy rule that is automatically generated from input data. Lastly, the ranking is determined by calculating the satisfaction function. The author, however, did not consider data in the form of fuzzy sets.

The paper proceeds as follows. The case background is presented in section 2 while the proposed model is described in section 3. The algorithm of the proposed model is highlighted in section 4. Section 5 presents the numerical results and concluding remarks are given in section 6.

2. Case Background

The data used was adopted from Yeh et al. (2000) which comprises of the performance evaluation study on 10 bus companies (labeled as A_1, A_2, A_3, A_4, A_5, A_6 A_7, A_8, $A_9$and A_{10}) in Taipei, Taiwan as listed in Table 1. The criteria which include the service attributes and description, used to rank the performance of bus companies are listed in Table 2. The attributes include safety, comfort, convenience, operation and social duty.

Linguistic terms are found to be easy to express the subjective and imprecise assessments (Yeh et al., 2000). Linguistic terms with the corresponding membership function as defined in Table 3 are used to facilitate the subjective assessment in evaluating the bus company performance. The membership functions are characterized by the triangular fuzzy number defined as triplet (a_1, a_2, a_3). The triangular fuzzy numbers are the average performance rating values in range of a_1 and a_3 The membership function is defined as

$$\mu(x_i) = \begin{cases} 0 & , \quad x_i \leq a_1 \\ \dfrac{x_i - a_1}{T - a_1} & , \quad a_1 \leq x_i \leq T \\ \dfrac{a_3 - x_i}{a_3 - T} & , \quad T \leq x_i \leq a_3 \end{cases} \qquad (1)$$

where, x_i is fuzzy evaluation of alternative in term of triangular fuzzy number, T is the vertex of the triangular fuzzy number and a_1 and a_3 are the two endpoints. Figure 1 below displays the triangular fuzzy number of the memberships of these linguistic terms defined as in Table 1.

The weightage of each sub-criteria is given in Table 4 which represents the importance of each criterion used in the evaluation. Grades A, B, C, D and E were used to represent the performance score and the evaluation set as shown in Table 5. The linguistic data are obtained by asking the passengers directly using structured questionnaire. Table 6, 7, 8, 9 and 10 show the quantitative assessment data and corresponding linguistic assessment results.

3. The Proposed Fuzzy Model

The model starts with the calculation of the membership set of score. The membership set score is computed by using the membership function defined as in Equation (1). The generated fuzzy set characterizes the membership values $\mu_A(x) \in [0, 1]$. Table 11 depicts part of the membership set score of the accident rate C_{11}. The model uses the standard fuzzy sets grade. The standard fuzzy sets grade is needed to feed the proposed fuzzy model with knowledge in expressing the grade linguistically. The notion of mid-point is introduced to obtain the range for grades A, B, C, D, and E (Turksen & Wilson, 1994) as displayed in Table 12.

The third step is to construct the fuzzy set membership for each criterion. The fuzzy set membership f_{ij} can be translated as the degree score given by the respondent to the alternatives given by

$$f_{ij} = \{ \mu_{f_{ij}}(x) / x, x \in X \} \qquad (2)$$

where f_{ij} is the fuzzy set membership of subjective evaluation mark (i = 1, 2, ...n, alternatives and j = 1, 2, ...m, the criteria environment), $\mu_{f_{ij}}(x)$ is the fuzzy set score of average fuzzy performance rating of alternatives according to criteria. These fuzzy set memberships are used to illustrate the degree achievement score obtained in grades A, B, C, D and E for all the factors. The symbol "/" is used to indicate the relationship between percentage score and the grade. Fuzzy set membership for first criterion is then constructed as shown in Table 13. The fuzzy sets grade is then define as shown in Table 14 (Biswas, 1995).

The grade for each criterion of ten alternatives is accorded by solving the fuzzy similarity function discussed in Chu (1990). The calculations for similarity values use the following equation:

$$S(F,M) = \frac{\hat{F}.\hat{M}}{\max(\hat{F}.\hat{F}, \hat{M}.\hat{M})}, \qquad (3)$$

where $\hat{F}$ = ($\mu_F(x_1)$, ($\mu_F(x_2)$, ...), $\hat{M}$ = ($\mu_M(x_1)$, ($\mu_M(x_2)$, ...) are the vectors and $\hat{M}$ denotes the transpose vectors A^T, B^T. C^T, D^T and E^T. F represents transpose vector of fuzzy set f_{ij} where i = 1, 2, 3, 4, 5, ..., 10 and j = 1,2 3, 4... Set X = (x_1, x_2, ...,x_n) represents the set of universe of discourse and '•' is the dot product. Similarity values for bus company A_1, are presented in Table 15.

The maximum similarity value is determined by identifying the maximum of the similarity values in Table 15. Next, the grade is mapped to the appropriate mid-interval mark. In this step the similarity value and the similarity curve are used to map the mark. The following guideline is used in allocating the midpoint mark.

If the SIM(f_{ij}, Grade) ≥ 3.0

then take the midpoint mark

Else

If skew of similarity curve to the left then ¾ of midpoint

Else

If skew of similarity curve to the right then ¼ of midpoint.

Else

If the similarity curve distributed evenly to two grades then takes

the midpoint mark enclose by the two grade.

The results of allocating an appropriate mid-point and mid-interval mark to each criterion for the first bus alternative are shown in Table 16.The normalized synthetic score is then built as shown in Table 17. Each element in the table is the summation of the product between maximum similarity value and the factor weightage as listed in the Appendix.

The decision criteria C_i, i = 1, 2, 3, ..., 5, is the intersection or combination of factor rules which is the antecedent of the rule (refer Table 18). The combination multi-criteria rules are described in Table 18 can be generalized as follows:

$$\text{If } (C_i = \bigcap_{j=1}^{5} \bigcup F_j) \text{ then } A_k$$

where DC_i is the decision criteria, C_j is the factor rules, A_k is the linguistic variables and k represents the grade. For example, the decision criteria C_1 rule can be written as

$$\text{If } DC_1 = C_1 \cap C_4 \text{ then } A_1 \text{ satisfactory } A_1(v) = v,$$

The appraisal set, v, is defined as $A = \{A_k\}$, where $v \in V$, $V = \{v_l\}$ = {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1} and l = 1, 2, ..., 11. V is unit appraisal space in [0,1] and l is the number of appraisal set in V. The factor rule value, $\tilde{c}(u_m)$ of Table 19 is obtained by processing the elements of Table 7 with the fuzzy rules given in Table 8. For example, for decision criteria C_1 and alternative U_1, the factor rule value is identified by choosing the minimum between 0.7375, 0.6775 and 0.7250.

Then the appraisal fuzzy value, $(d_j(m,l))$, of Table 20 is computed as follows (Chu, 1990):

$$d_j(m,l) = 1 \wedge (1 - \tilde{c}(u_m) + A_k(v_l))$$

where j = 1, 2, 3, ...,7, m = 1, 2, 3, ..., 5, l = 1, 2, ..., 11 and $\tilde{c}(u_m)$ is the factor rule value.

Each entry of Table 21 is the product of the appraisal fuzzy value for all the decision criteria. Achievement score can be ranked using the satisfaction value, $SV(m)$, as proposed by Lee et al. (1994) and this is calculated as follows:

$$SV(m) = \frac{1}{\alpha_{\max}} \sum_{l=1}^{11} H_l(E_{m\alpha}) \Delta\alpha_l$$

where α = degree of appraisal product value, $\Delta\alpha_l = \alpha_l - \alpha_{l-1}$, $\alpha_0 = 0$, $H_l(E_{m\alpha})$ = mid-point V_l (l = 1,2,3...,11) and $\alpha_{\max}$ = maximum degree of appraisal product value. The calculated values of the range of α, $\Delta\alpha_l$, and $H_l(E_{m\alpha})$ is tabulated in Table 22. The bus companies are ranked according to the satisfaction value where the bigger the value indicates a higher rank as indicated in Table 23.

4. Fuzzy Evaluation Algorithm

The fuzzy evaluation algorithm consist 9 steps as listed below:

Step 1 : Calculate membership set of score.

Step 2 : Determine grade range and midpoints.

Step 3 : Construct fuzzy set membership for each criterion.

Step 4 : Define student fuzzy sets for the grades

Step 5 : Calculate maximum similarity value and determine grade

Step 6 : Calculate the normalized synthetic score value

Step 7 : Determine multicriteria rules combination and calculate factor rule value

Step 8 : Calculate appraisal fuzzy value and the appraisal product value

Step 9 : Compute satisfaction value and ranking.

The proposed method uses the data that are represented in terms of linguistic terms. This presentation of data is simpler and easier to gather compared to Biswas (1995) approach that used fuzzy set data constructed by the evaluator. The method takes advantage of Chu's approach in representing data. The proposed method differs from Chu (1990) where the frequency data are transformed into fuzzy membership set. The main advantage of the proposed method is that the fuzzy membership set are not predetermined by the expert. This is important to ensure the consistency of the decision. The transformation enables the method to gather as much information as possible. The model used similarity function to normalize the data in order to dampen the fluctuation among data.

5. Numerical Result

Comparison of results between Yeh et al. (2000) and the proposed method are depicted in Table 24. The accuracy of ranking the performance of bus companies between the proposed method are computed based on the results given by Yeh et al. (2000). High satisfaction value implies that the decision maker (DM) is satisfied with the subjective assessment of bus company performance. From the results the selection of bus company performance based on satisfaction values can be ordered as A_6, A_4, A_2, A_8, A_3, A_{10}, A_5, A_9, A_1 and A_7 respectively. The experimental results show that the proposed method is comparable to Yeh et al. (2000), even with the use of small number of rule.

The proposed model with the concepts of combining Chu (1990) and Biswas (1995) shown advantages in generalizing the evaluation of the performance achievement where the evaluation process can be conducted consistently with the use of the set degree of membership. Furthermore, the proposed method based upon fuzzy sets has initiated the idea of membership set score valued evaluation of each criterion alternative enables to include requirements which are incomplete and imprecise. Therefore, the problem of the traditional evaluation method which is too loose and too wide measurement scalar used, the subjectivity element increases exponentially resulting from the aggregating all the marks given to the criteria could be solved. Furthermore, the method could also lessen the presence of imprecision, vagueness and subjectivity. The approximate reasoning of the method allows decision maker to make the best choice in accordance of human thinking and reasoning processes. The method is suitable for dealing with evaluations in situations that involve subjectivity, vagueness and imprecise information, such as the grading system of evaluation which involves many hedges like "*good*", "*bad*" and "*satisfactory*". The proposed method introduces data normalization process which reduces the irregular data and produces highly reliable data. The reliability of the data indicates the stability and consistency with which the proposed method generates fuzzy rules and evaluating performance quality or the alternatives. Hence, the suggested method is able to produce good and precise ranking results in fuzzy environments.

6. Conclusion

A new fuzzy model has been proposed for the evaluation of bus company performance. Experimental results produced are comparable to results obtained by Yeh et al. (2000). The model has been implemented using C++ programming language and is suitable for various fuzzy environments. The model could be used as an alternative approach in solving the problems that involve uncertainty. The main contribution of the research model was the usage of fuzzy expert system consisting of set of rules in the form of IF (antecedent) THEN (Conclusion). The evaluation output comes nearer to precision if the combination factors were accurately defined. Further research will be in obtaining a universal view on an appropriate combination factors and the classification of midpoint, which could improve the performance of the proposed model.

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Table 1. Alternative of Bus Companies

Alternative	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Bus Company	City bus	Hsin-hsin	Ta-nam	Ta-yu	Kuang-hua	Chung-shing	Ch-nam	Taipei	San-c hung	Shou -tu

Table 2. Service Attributes

Service Attribute	**Performance Measure**
Safety (C_1)	Average vehicle age, average vehicle breakdown rate, and traffic offence
Comfort (C_2)	Cleanliness, seat comfort, driver's driving skills, driver's appearance and driver's friendliness
Convenience (C_3)	Punctuality of the bus service, route transferability, terminal space and service reliability
Operation (C_4)	Cost efficiency, cost effective and service efficiency
Social duty (C_5)	Vehicle air pollution and vehicle noise level

Table 3. Linguistic Terms

Linguistic term	Very poor (VP)	Poor (P)	Fair (F)	Good (G)	Very Good (VG)
Membership Function	(1, 1, 3)	(1, 3, 5)	(3, 5, 7)	(5, 7, 9)	(7, 9, 9)

Table 4. Weightage

Criteria	**Weight**						
$\mathbf{W_1}$	(7, 9, 9)		(1, 3, 5)				
$\mathbf{W_2}$	(7, 9, 9)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(1, 1, 3)	(1, 1, 3)
$\mathbf{W_3}$	(7, 9, 9)	(5, 7, 9)	(1, 3, 5)	(7, 9, 9)			
$\mathbf{W_4}$	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)				
$\mathbf{W_5}$	(5, 7, 9)	(3, 5, 7)					

Table 5. Evaluation Set

Grade	**Linguistic Variable**	**Fuzzy Set**
A	Very Good	{0, 0, 0, 0, 0, 0, 0, 0.3, 0.7}
B	Good	{0, 0, 0, 0, 0, 0.5, 1, 0.5, 0}
C	Fair	{0, 0, 0, 0, 0.5, 1, 0.5, 0, 0}
D	Not good	{0, 0.5, 1, 0.5, 0, 0, 0, 0, 0}
E	bad	{0.3, 0.7, 0, 0, 0, 0, 0, 0, 0}

Table 6. Quantitative Assessment Data and Corresponding Linguistic Assessment Results for Safety

Safety (C_1)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Accident rate (C_{11})	F	F	P	P	P	VG	F	F	VP	F
Average Vehicle age (C_{12})	VP	G	G	G	F	G	G	G	VP	P

Table 7. Level of Comfort

Comfort (C_2)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Air-conditioned vehicle (C_{21})	P	P	P	G	G	Vg	P	G	F	VG
On-board information (C_{22})	P	G	G	VG	P	P	P	P	F	VG
Vehicle cleanliness (C_{23})	F	P	F	F	F	P	P	G	F	F
Seat comfort (C_{24})	F	F	P	G	P	P	P	G	F	F
Driver's skill (C_{25})	G	G	F	P	P	G	F	G	G	F
Driver's appearance (C_{26})	F	P	P	P	P	P	P	F	P	G
driver's friendliness (C_{27})	P	G	P	P	P	P	P	G	P	P

Table 8. Level of Convenience

Convenience (C_3)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Punctuality (C_{31})	F	G	G	P	G	G	P	G	P	VG
Route transferability (C_{32})	VG	F	F	F	F	G	VP	VG	VG	P
Terminal space (C_{33})	G	G	P	P	P	P	F	G	VG	VG
Service reliability (C_{34})	G	G	G	F	P	G	G	F	F	F

Table 9. Operation Performance for Private Bus

Operation (C_4)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Cost efficiency (C_{41})	VP	P	G	VG	F	G	F	P	VG	F
Cost effective (C_{42})	VP	P	F	VG	F	P	P	P	G	P
Service efficiency (C_{43})	G	G	F	G	G	P	P	VG	F	VP

Table 10. Social Duty

Social duty (C_5)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Vehicle air pollution level (C_{51})	G	F	G	G	F	G	VP	P	F	G
Vehicle noise level (C_{52})	P	P	VP	P	P	P	P	P	P	P

Table 11. Membership Set Score

Alternative	**C_{11}**								
	1	2	3	4	5	6	7	8	9
A_1	0	0	0	0.5	1	0.5	0	0	0
A_2	0	0	0	0.5	1	0.5	0	0	0
A_3	0	0.5	1	0.5	0	0	0	0	0
A_4	0	0.5	1	0.5	0	0	0	0	0
A_5	0	0.5	1	0.5	0	0	0	0	0
A_6	0	0	0	0	0	0	0	0.5	1
A_7	0	0	0	0.5	1	0.5	0	0	0
A_8	0	0	0	0.5	1	0.5	0	0	0
A_9	1	0.5	0	0	0	0	0	0	0
A_{10}	0	0	0	0.5	1	0.5	0	0	0

Table 12. Range, Grade Mid-Point and Mid-Interval Mark

Grade	**Range**	**Mid-Point**	**Mid-Interval**
A	90.0 – 100.0	95.0	92.5, 97.5
B	70.0 – 90.0	80.0	75.0, 85.0
C	50.0 – 70.0	60.0	55.0, 65.0
D	30.0 – 50.0	40.0	35.0, 45.0
E	00.0 – 30.0	15.0	7.5, 22.5

Table 13. Fuzzy Set Membership

Alternative	**C_{11}**								
A_1	0.00/1	0.00/2	0.00/3	0.50/4	1.00/5	0.50/6	0.00/7	0.00/8	0.00/9
A_2	0.00/1	0.00/2	0.00/3	0.50/4	1.00/5	0.50/6	0.00/7	0.00/8	0.00/9
A_3	0.00/1	0.50/2	1.00/3	0.50/4	0.00/5	0.00/6	0.00/7	0.00/8	0.00/9
A_4	0.00/1	0.50/2	1.00/3	0.50/4	0.00/5	0.00/6	0.00/7	0.00/8	0.00/9
A_5	0.00/1	0.50/2	1.00/3	0.50/4	0.00/5	0.00/6	0.00/7	0.00/8	0.00/9
A_6	0.00/1	0.00/2	0.00/3	0.00/4	0.00/5	0.00/6	0.00/7	0.50/8	1.00/9
A_7	0.00/1	0.00/2	0.00/3	0.50/4	1.00/5	0.50/6	0.00/7	0.00/8	0.00/9
A_8	0.00/1	0.00/2	0.00/3	0.50/4	1.00/5	0.50/6	0.00/7	0.00/8	0.00/9
A_9	1.00/1	0.50/2	0.00/3	0.00/4	0.00/5	0.00/6	0.00/7	0.00/8	0.00/9
A_{10}	0.00/1	0.00/2	0.00/3	0.50/4	1.00/5	0.50/6	0.00/7	0.00/8	0.00/9

Table 14. Grade Fuzzy Set

Grade	Linguistic Variable	Fuzzy Set
A	Excellent	{1.0, 1.0, 0.9, 0.8, 0.0}
B	Very Good	{0.8, 0.9, 0.9, 0.8, 0.0}
C	Good	{0.2, 0.4, 0.9, 0.8, 0.1}
D	Satisfactory	{0.0, 0.2, 0.4, 0.9, 0.4}
E	Unsatisfactory	{0.0, 0.0, 0.2, 0.4, 1.0}

Table 15. Similarity Value

		Factor							
		C_1		C_2				C_5	
Alternative	Grade	C_{11}	C_{12}	C_{21}	C_{22}	C_{23}	....	C_{51}	C_{52}
A_1	A	0.0	0.0	0.0	0.0	0.0		0.1	0.0
	B	0.2	0.0	0.0	0.0	0.2		1.0	0.0
	C	0.7	0.0	0.0	0.0	0.7		0.7	0.0
	D	0.2	0.2	1.0	0.8	0.2		0.0	1.0
	E	0.0	0.7	0.1	0.1	0.0		0.0	0.1

Table 16. Maximum Similarity Value

Alternative	Attribute		Max Similarity Value	Grade	Fuzzy mark
	C_1	C_{11}	0.7	C	60
		C_{12}	0.7	E	15
A_1	C_2	C_{21}	1	D	40
		C_{22}	0.8	D	40
		C_{23}	0.7	C	60
		C_{24}	0.7	C	60
		C_{25}	1	B	80
		C_{26}	0.7	C	60
		C_{27}	1	D	40
	C_3	C_{31}	0.7	C	60
		C_{32}	0.7	A	95
		C_{33}	1	B	80
		C_{34}	1	B	80
	C_4	C_{41}	0.4	E	15
		C_{42}	0.4	E	15
		C_{43}	1.0	B	80
	C_5	C_{51}	1.0	B	80
		C_{52}	1.0	D	40

Table 17. Normalized Synthetic Score Value

	Attribute				
Alternative	C_1	C_2	C_3	C_4	C_5
A_1	0.47	0.54	0.78	0.32	0.63
A_2	0.66	0.59	0.74	0.50	0.51
A_3	0.52	0.54	0.72	0.67	0.52
A_4	0.52	0.70	0.52	0.91	0.630
A_5	0.46	0.53	0.59	0.65	0.51
A_6	0.91	0.60	0.78	0.70	0.63
A_7	0.66	0.43	0.48	0.55	0.26
A_8	0.66	0.71	0.77	0.54	0.40
A_9	0.15	0.62	0.65	0.86	0.51
A_{10}	0.54	0.75	0.68	0.48	0.63

Table 18. Multicriteria Rules Combination

Decision Criteria	Factor Rule	Linguistic Variable	Description	Appraisal Set
DC_1	$C_1 \cap C_3$	A_1	Satisfactory	v
DC_2	$C_2 \cap C_5$	A_1	Satisfactory	v
DC_3	$C_1 \cap C_4 \cap C_5$	A_2	Very satisfactory	v
DC_4	$C_1 \cap C_3$	A_3	Very very satisfactory	v

Table 19. Factor Rule Value

	Attribute				
Alternative	C_1	C_2	C_3	C_4	C_5
A_1	0.47	0.54	0.78	0.32	0.63
A_2	0.66	0.59	0.74	0.50	0.51
A_3	0.52	0.54	0.72	0.67	0.52
A_4	0.52	0.70	0.52	0.91	0.63
A_5	0.46	0.53	0.59	0.65	0.51
A_6	0.91	0.60	0.78	0.70	0.63
A_7	0.66	0.43	0.48	0.55	0.26
A_8	0.66	0.71	0.77	0.54	0.40
A_9	0.15	0.62	0.65	0.80	0.51
A_{10}	0.54	0.75	0.68	0.48	0.63

Table 20. Appraisal Fuzzy Value for Decision Criteria C_l

Alternative	Appraisal Set									
A_1	0.53	0.63	0.73	0.83	0.93	1	1	1	1	1	1
A_2	0.34	0.44	0.54	0.64	0.74	0.84	0.94	1	1	1	1
A_3	0.33	0.43	0.53	0.63	0.73	0.83	0.93	1	1	1	1
A_4	0.09	0.19	0.29	0.39	0.49	0.59	0.69	0.79	0.89	0.99	1
A_5	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1	1	1	1
A_6	0.09	0.19	0.29	0.39	0.49	0.59	0.69	0.79	0.89	0.99	1
A_7	0.34	0.44	0.54	0.64	0.74	0.84	0.94	1	1	1	1
A_8	0.34	0.44	0.54	0.64	0.74	0.84	0.94	1	1	1	1
A_9	0.14	0.24	0.34	0.44	0.54	0.64	0.74	0.84	0.94	1	1
A_{10}	0.46	0.56	0.66	0.76	0.86	0.96	1	1	1	1	1

Table 21. Appraisal Product Value

Alternative	Appraisal Set										
A_1	0.0879	0.1315	0.1977	0.3011	0.4636	0.6964	0.8900	1	1	1	1
A_2	0.0283	0.0463	0.0765	0.1283	0.2173	0.3680	0.5659	0.8217	0.98	1	1
A_3	0.0365	0.0599	0.0975	0.1597	0.2631	0.4335	0.6562	0.9409	1	1	1
A_4	0.0077	0.0214	0.0447	0.0849	0.1545	0.2735	0.4723	0.7433	0.8900	0.9900	1
A_5	0.0500	0.0803	0.1277	0.2038	0.3271	0.5252	0.7695	1	1	1	1
A_6	0.0029	0.0083	0.0185	0.0389	0.0789	0.1547	0.2921	0.4824	0.7654	0.99	1
A_7	0.0968	0.1469	0.2217	0.3240	0.4529	0.6403	0.8272	1	1	1	1
A_8	0.0416	0.0658	0.1051	0.1709	0.2812	0.4213	0.6317	0.8300	0.9800	1	1
A_9	0.0496	0.1047	0.1858	0.3071	0.4806	0.6336	0.7400	0.8400	0.9400	1	1
A_{10}	0.0407	0.0656	0.1053	0.1708	0.2792	0.4566	0.7000	0.9500	1	1	1

Table 22. Calculated range of α , $\Delta\alpha_l$, and $H_l(E_{m\alpha})$

l	Range α	$E_{m\alpha}$	$H_l(E_{m\alpha})$	$\Delta\alpha_l$
1.	0.0000 < α ≤ 0.0879	{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.50	0.0879
2.	0.0879< α ≤ 0.1315	{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.55	0.0436
3.	0.1315< α ≤ 0.1977	{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.60	0.0663
4.	0.1977< α ≤ 0.3011	{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.65	0.1034
5.	0.3011< α ≤ 0.4636	{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.70	0.1624
6.	≤ 0.4636< α ≤ 0.6964	{0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.75	0.2328
7.	0.6964< α ≤ 0.8900	{0.6, 0.7, 0.8, 0.9, 1}	0.80	0.1936
8.	0.8900< α ≤ 1.0000	{0.7, 0.8, 0.9, 1}	0.85	0.1100
9.	1.0000< α ≤ 1.0000	{0.8, 0.9, 1}	0.90	0.0000
10.	1.0000< α ≤ 1.0000	{ 0.8, 1}	0.90	0.0000
11.	1.0000< α ≤ 1.0000	{1}	1	0.0000

Table 23. Satisfaction Value and Ranking

Alternative	Satisfaction value	Ranking
A_1	0.7116	9
A_2	0.7884	3
A_3	0.7676	5
A_4	0.8159	2
A_5	0.7458	7
A_6	0.8584	1
A_7	0.7145	10
A_8	0.7736	4
A_9	0.7359	8
A_{10}	0.7616	6

Table 24. Comparison of Results

Model	Yeh (λ= 0 for Pessimistic DM)		Yeh (λ= 0.5 for Moderate DM)		Yeh (λ= 1 for Optimistic DM)		Proposed Method	
Alternative	Satisfaction	Rank	Satisfaction	Rank	Satisfaction	Rank	Satisfaction	Rank
A_1	**0.539**	9	**0.452**	9	**0.500**	9	0.7116	9
A_2	**0.673**	3	**0.643**	3	**0.690**	3	0.7884	3
A_3	**0.635**	5	**0.571**	5	**0.655**	5	0.7676	5
A_4	**0.731**	2	**0.738**	2	**0.724**	2	0.8159	2
A_5	**0.573**	7	**0.524**	7	**0.569**	7	0.7458	7
A_6	**0.808**	1	**0.762**	1	**0.810**	1	0.8584	1
A_7	**0.365**	10	**0.238**	10	**0.293**	10	0.7145	10
A_8	**0.654**	4	**0.619**	4	**0.672**	4	0.7736	4
A_9	**0.558**	8	**0.476**	8	**0.517**	8	0.7359	8
A_{10}	**0.615**	6	**0.548**	6	**0.586**	6	0.7616	6

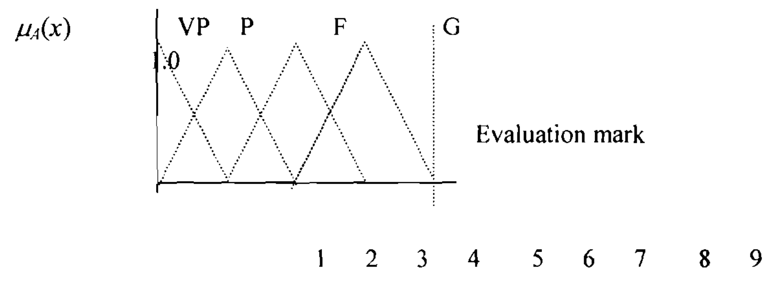

Figure 1. Membership Functions of Linguistic Terms