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AN ANALYSIS OF AN AGE STRUCTURED PREDATOR — PREY MODEL

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This paper is a critique and an elaboration on the question of stability of solutions of an age structured predator-prey model examined by Hastings and Wollkind (1982). In studying their paper, we found some questionable places in their derivations. We will present our analysis on the subjects and give alternative derivations.

1. Introduction

A predator-prey model is a description of an ecological situation involving two species, one of which preys on the other while the other lives on a different source of food. An example is foxes and rabbits in a closed forest; the foxes prey on the rabbits and the rabbits live on the vegetation in the forest. The problem can be generalised to any number of species of predators and preys, thus creating a food chain.

Most predator-prey models, from the classical Lotka (1924) – Volterra (1926) models to the more contemporary Kolmogorov (1948) models, are phrased in terms of autonomous systems of ordinary differential equations (o.d.e.) of the form:

$$\frac{dN_i}{dt} = F_i(N_1, \ldots, N_m)$$

with initial conditions

$$N_i(0) = N_{i,0} \geq 0$$

$$i = 1, \ldots, m,$$

where $N_i = N_i(t)$ is the size of the i^{th} population at time t and each $F_i(\cdot)$ describes the dynamics of the i^{th} population. Appropriate assumptions on each $F_i(\cdot)$ characterizes the type of interactions between the species. Each $F_i(\cdot)$ is then assumed to satisfy a Lipshitz condition w.r.t. $N = (N_1, \ldots, N_m)$ on some open region D in $\mathbb{R}^m$ That is, for the vector $F = (F_1, \ldots, F_m)$ and every $N' \in D$, there exists a constant $k > 0$ such that

$$|F(N) - F(N')| < k|N - N'|$$

whenever $|N - N'|$ is sufficiently small. This assures the uniqueness of solutions.

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By their inherent structure, autonomous o.d.e. systems are unable to incorporate age structure effects. As a result, the extensive use of autonomous o.d.e. systems in the literature means that usually the effects of age structure in population dynamics have been ignored. In recent years, models involving age structure have been studied more.

Von Foerster (1959) first studied the effects of age structure in ecological models using systems of partial differential equations (p.d.e.) of the form:

$$\frac{\partial n_i}{\partial a} + \frac{\partial n_i}{\partial t} = -u_i(t) \cdot n_i(a, t)$$

with the boundary conditions

$$n_i(0, t) = \int_0^\infty n_i(a, t) \cdot b_i(a, t)\, da$$

$$i = 1, \ldots, m,$$

where $n_i(a, t)$, $b_i(a, t)$ and $u_i(t)$ describe the population density at age a, birth rate at age a and death rate, at time t, respectively, for the i^{th} population.

Models using difference equations were introduced in Hassell (1978). Other variations of age structure models are given in Oester and Takahashi (1974), Oester (1978), Logan et al. (1978) and Hastings and Wollkind (1982).

Except for a few different assumptions, the model developed in this paper is essentially the same one as in Hastings and Wollkind. We found the assumptions used here to be necessary for the validity of ensuing derivations. The model is a general two species predator-prey model described by a pair of equations; a differential equation for the prey and an integral equation for the predator. We will present a linear stability analysis on an equilibrium where both species exist. The main result in this paper is the derivation of sufficient conditions for local asymptotic stability of such an equilibrium.

This paper is divided into five major sections. The development of the general model will be presented in Section 2. Section 3 is devoted to stability analysis. A specific case is worked out in Section 4 to illustrate the use of results obtained in the previous section. Discussions and concluding remarks follow in Section 5.

2. The Model

The model developed in this section is essentially the same as that in Hastings and Wollkind (1982). It is a two species predator-prey model. Age structure will be incorporated only in the dynamics of the predator population. The prey population is independent of age by the assumptions made below. The model will be described by a pair of equations; a differential equation for the prey population and an integral equation for the predator population. We present a derivation in detail below.

2.1 Assumptions

The following assumptions were made in constructing the model:

H-1: Death of each member of the prey population occurs only through predation and is age independent. Thus age structure in the prey population is ignored here.

H-2: There is an abundant food supply for the prey population.

H-3: The rate of reproduction for the predator population will increase with an increase in the prey population, and vice-versa. This affects the rate of increase in the predator population through its birth rate.

H-4: Each predator has a fixed life span. Death for each predator occurs only as a result of attaining that age.

Assumption (H-4) makes the model age dependent. An implicit assumption in any two species predator-prey model is that both species exist in a closed habitat. That is, the predators depend solely on the prey population for food and they do not compete for the prey population with any other species.

2.2 Derivation

Let H (t) denote the population of prey and C (t) the population of predator, at time t. We need the following definition before going any further.

Definition: The predator functional response $\phi(H)$, is defined as the number of prey killed, per unit time, by the average predator. It can be construed as the attack rate of each predator. Thus, $C \cdot \phi(H)$ gives the total number of prey killed by predators at time t.

It follows that

$$\frac{dH}{dt} = H \cdot r(H) - C \cdot \phi(H) \equiv F(H, C)$$

describes the growth rate of the prey population with respect to time. Here, r (H) is the intrinsic per capita growth rate of the prey population when predator is absent.

Note: In order to retain some generality in the model, we do not make specific assumptions on r (H) and ϕ (H) other than that they are continuously differentiable in their arguments. However, it is reasonable for r (H) to have the following properties:

$$r(0) > 0 \text{ and } \lim_{H \to \infty} r(H) < 0$$

These imply that the prey population can grow while the prey are rare but that the prey population is bounded above. If the habitat has a carrying capacity, then for some $K > 0$, $r(K) = 0$.

To derive the dynamics of the predator population, we assume that the lifespan of each predator is D units of time (by H-4). Let $\overline{C}(A, t)$ be the population density, per unit of age, of predators of age A at time t $(A < D)$. Then for small $h > 0$ and $\Delta A > 0$, we have the approximate relation

$$\overline{C}(A, t-h) \cdot \Delta A \cong \overline{C}(A+h, t) \cdot \Delta A.$$

Note that the left side represents the number of predators at time t-h between the ages A and A+ΔA. The same predators at time t have ages between A+h and A+h+ΔA, as represented on the right side. This approximate relation is given in exact form by

$$\int_{A}^{A+\Delta A} \overline{C}(\alpha, t-h)\, d\alpha = \int_{A+h}^{A+h+\Delta A} \overline{C}(\beta, t)\, d\beta$$

Letting $\beta = \alpha + h$ in the right integral we have

$$\int_{A}^{A+\Delta A} \overline{C}(\alpha, t-h)\, d\alpha = \int_{A}^{A+\Delta A} \overline{C}(\alpha+h, t)\, d\alpha$$

Therefore, for all $A > 0, h \geq 0$ and $A + h + \Delta A \leq D$,

$$\int_{A}^{A+\Delta A} [\overline{C}(\alpha, t-h) - \overline{C}(\alpha+h, t)]\, d\alpha = 0.$$

We assume $\overline{C}(A, t)$ is continuous in A. Hence, dividing the above expression by ΔA and taking the limit as $\Delta A \to 0$, we get by the Integral Mean Value Theorem,

$$\overline{C}(A, t-h) = \overline{C}(A+h, t)$$

for $h \geq 0$, $A \geq 0$ and $A + h \leq D$. If we set $A + h = B$ and $t - h = r$, then this implies

$$\overline{C}(B-h, r) = \overline{C}(B, r+h).$$

Thus, the original relation holds for negative h as well if $A + h > 0$. From this we get

$$\frac{\overline{C}(A, t-h) - \overline{C}(A, t)}{h} = \frac{\overline{C}(A+h, t) - \overline{C}(A, t)}{h}$$

Letting $h \to 0$, we get

$$\frac{\partial \overline{C}}{\partial A} + \frac{\partial \overline{C}}{\partial t} = 0 \tag{2}$$

The birth rate of the predator population is

$$\lim_{\Delta A \to 0} \frac{1}{\Delta A} \int_{0}^{\Delta A} \overline{C}(\alpha, t)\, d\alpha = \overline{C}(0, t).$$

Thus to describe the birth rate of the predator population, the boundary condition

$$\overline{C}(0, t) = [\varepsilon \cdot h(\phi(H)) \cdot C](t) \tag{3}$$

is imposed. Here the per capita birth rate of the predator is made up of $h(\phi)$ to account for the conversion of prey into predator (as food supply) multiplied by the intrinsic efficiency factor $\in$

Note that this indicates that the birth rate of the predator is assumed to be independent of age. The parameter $\in$ is included to allow for an examination of parametric variation at equilibrium. The function $h(\phi)$ is assumed to be continuously differentiable and that

$$\frac{dh}{d\phi} > 0 \text{ for all } \phi > 0.$$

By the assumption of death at age D, the predator population, at time t, is given by

$$C(t) = \int_0^D \overline{C}(A, t)\, dA \tag{4}$$

The system of equations (2) – (4) is a p.d.e. boundary value problem which can be solved by the method of characteristics (Abbott, 1966).

The system of o.d.e. associated with the p.d.e. (2) is

$$\frac{dt}{1} = \frac{dA}{1} = \frac{d\overline{C}}{0}$$

Two independent general solutions of this system are

$$t - A = c_1, \overline{C} = c_2; c_1, c_2 \text{ constants.}$$

Thus, the solutions of (2) are given by

$$\overline{C}(A, t) = r(t - A) \tag{5}$$

where r is an arbitrary C^1 function of a single variable. By (3) we have

$$r(t) = \overline{C}(0, t) = [\in h(\phi(H))\, C](t)$$

This together with (4) and (5) imply

$$C(t) = \int_0^D [\in h(\phi(H)) \cdot C](t - A)\, dA$$

Now by appropriate time scaling we may assume D = 1. Then substituting s = t – A in the above we get

$$\begin{aligned} C(t) &= \int_{t-1}^{t} [\in h(\phi(H))\, C](s)\, ds \\ &= \int_{t-1}^{t} [G(H, C)](s)\, ds \end{aligned} \tag{6}$$

where

$$G(H, C) = \in h(\phi(H))\, C \tag{7}$$

Note that (6) implies

$$\frac{dC}{dt} = G(H, C)(t) - G(H, C)(t - 1).$$

It now becomes evident that it is necessary to stipulate H and C for the initial time interval $t \in [-1, 0]$ rather that just at the initial time $t = 0$. Thus, we prescribe the initial conditions

$$H(t) = H_0(t)$$
$$C(t) = C_0(t) \quad (8\text{-a})$$
$$\text{for } t \in [-1, 0]. \quad (8\text{-b})$$

The model described by (1) and (6) for $t \geq 0$ together with the initial conditions (8a, b) becomes a well-posed initial boundary value problem.

We consider next an analysis of the stability of a possible equilibrium for the above model.

3. Stability Analysis

The analytical techniques presented below are similar to those given by Hastings (1977) and May (1974). For their model, Hastings and Wollkind (1982) considered three classes of equilibrium points (H^*, C^*):

(i) $H^* = 0,\ C^* = 0$

(ii) $H^* > 0,\ C^* = 0$

(iii) $H^* > 0,\ C^* > 0$

We note that the equilibrium points (H^*, C^*) satisfy $F(H^*, C^*) = 0$ and $G(H^*, C^*) = C^*$ by (1) and (6). The type (iii) equilibria, where both species are present is of main interest, so we will describe in detail the stability analysis only for this case.

We perform a linear stability analysis of the equilibrium populations (H^*, C^*) following the general outline described below.

Given the constant solutions or equilibrium populations (H^*, C^*), we are interested in the asymptotic behavior of solutions which are slightly perturbed. That is, we write

$$H(t) = H^* + u(t)$$
$$C(t) = C^* + v(t)$$

where $u(t)$ and $v(t)$ represent the perturbations from (H^*, C^*). We assume $u(0)$ and $v(0)$ are small. We are then interested in what happens to (u, v) as $t \to \infty$ To answer this question, we proceed as follows:

(i) Derive the system of differential equations satisfied by (u, v).

(ii) Assuming u and v are very near zero, we linearize the system of differential equations in (i) for (u, v).

(iii) Examine the behavior of (u, v) for the linearized system as $t \to \infty$.

Definition: An equilibrium point (x_0, y_0) is said to be asymptotically stable if it is stable and each solution $x = x(t)$, $y = y(t)$ satisfies

$$\lim_{t\to\infty} x(t) = x_0, \quad \lim_{t\to\infty} y(t) = y_0$$

provided $x(0) = x_0$ and $y(0) = y_0$ are small.

As pointed out later, if all (u, v) in the linearized system satisfy $(u, v) \to (0, 0)$ as $t \to \infty$, then (H^*, C^*) is asymptotically stable. Otherwise, (H^*, C^*) is said to be unstable. We will establish sufficient conditions for (H^*, C^*) to be asymptotically stable.

3.1 Linearization

The analysis of small disturbances about the equilibrium populations (H^*, C^*) begins by considering perturbed populations of the form

$$H(t) = H^* + Me^{\lambda t} \tag{9-a}$$

$$C(t) = C^* + Ne^{\lambda t} \tag{9-b}$$

where M, N are constants not both of which are zero and λ is real or complex. Here the $e^{\lambda t}$ terms measure the perturbations from the equilibrium populations, and are initially small by assumption.

The linearized system of differential equations for a perturbed solution near (H^*, C^*) is obtained by a Taylor's expansion up to linear terms of G(H, C) and F(H, C) in (7) and (1) around the equilibrium points (H^*, C^*). Thus we replace G and H by

$$G_0(H, C) = G^* + \left(\frac{\partial G}{\partial H}\right)^* (H - H^*) + \left(\frac{\partial G}{\partial C}\right)^* (C - C^*)$$

$$F_0(H, C) = F^* + \left(\frac{\partial F}{\partial H}\right)^* (H - H^*) + \left(\frac{\partial F}{\partial C}\right)^* (C - C^*)$$

where '*' means that the function is evaluated at (H^*, C^*). Now the linearized form of (1) is

$$\frac{dH}{dt} = F_0(H, C).$$

Substituting (9-a) and (9-b) in this, we have

$$\frac{d}{dt}(H^* + Me^{\lambda t}) = F^* + \left(\frac{\partial F}{\partial H}\right)^* Me^{\lambda t} + \left(\frac{\partial F}{\partial C}\right)^* Ne^{\lambda t}$$

Simplifying the above equation using the fact that

$F^* = F(H^*, C^*) = 0$, we get

$$0 = M\left[\left(\frac{\partial F}{\partial H}\right)^* - \lambda\right] + N\left(\frac{\partial F}{\partial C}\right)^* \qquad (10$$

Substituting (9-a) and (9-b) in the linearized version of (6),

$$C(t) = \int_{t-1}^{t} [G_0(H, C)]\,(s)\,ds$$

we get

$$C^* + Ne^{\lambda t} = \int_{t-1}^{t} \left[G^* + \left(\frac{\partial G}{\partial H}\right)^* Me^{\lambda s} + \left(\frac{\partial G}{\partial C}\right)^* Ne^{\lambda s}\right] ds \tag{11}$$

Recall that

$$G(H, C) = \in h(\phi(H)) \cdot C$$

so that

$$\left(\frac{\partial G}{\partial C}\right)^* = \in h(\phi(H^*)) .$$

But by (6)

$$C^* = \int_{t-1}^{t} \in h(\phi^*) C^* ds = \in h(\phi^*) C^* = G^*$$

so we must have

$$\in h(\phi^*) = 1$$

and hence

$$\left(\frac{\partial G}{\partial C}\right)^* = 1.$$

Now (11) becomes

$$Ne^{\lambda t} = [M\left(\frac{\partial G}{\partial H}\right)^* + N] \int_{t-1}^{t} e^{\lambda s} ds \tag{11'}$$

But

$$\int_{t-1}^{t} e^{\lambda s} ds = \begin{cases} e^{\lambda t} & \neq 0 \\ \frac{1}{\lambda}(1 - e\lambda) e^{\lambda t}; & \lambda \neq 0 \end{cases}$$

Thus for all λ, (11') is equivalent to

$$0 = MK(\lambda)\left(\frac{\partial G}{\partial H}\right)^* + N(K(\lambda) - 1) \tag{12}$$

where

$$K(\lambda) = \begin{cases} 1 & \lambda = 0 \\ \frac{1}{\lambda}(1 - e\lambda), & \lambda \neq 0 \end{cases} \tag{13}$$

Note that $K(\lambda)$ is continuous at $\lambda = 0$.

Equations (10) and (12) can be written in matrix form

$$J\& = 0 \tag{14}$$

where &' = (M, N) is the transpose of & and

$$J = \begin{bmatrix} \left(\frac{\partial F}{\partial H}\right)^* - \lambda & \left(\frac{\partial F}{\partial C}\right)^* \\ K(\quad)\left(\frac{\partial G}{\partial H}\right)^* & K(\lambda) - 1 \end{bmatrix}$$

The matrix J is known as the community matrix which is of biological interest and significance. Since M and N are not both zero, we want & $\neq 0$ in (14). For a non-trivial solution of (14), we need $|J| = 0$ where $|J|$ is the determinant of J. This may be written us

$$\lambda - \lambda K(\lambda) + \left(\frac{\partial F}{\partial H}\right)^* - \left(\frac{\partial F}{\partial C}\right)^* \left(\frac{\partial G}{\partial H}\right)^* \lambda K(\lambda) - \left(\frac{\partial F}{\partial H}\right)^* = 0$$

or

$$\lambda - \lambda K(\lambda) + \alpha K(\lambda) + \beta = 0 \tag{15}$$

where

$$\alpha = \left(\frac{\partial F}{\partial H}\right)^* - \left(\frac{\partial F}{\partial C}\right)^* \left(\frac{\partial G}{\partial H}\right)^* \tag{16-a}$$

$$\beta = -\left(\frac{\partial F}{\partial H}\right)^* \tag{16-b}$$

Equation (15) is the characteristic equation for the linear system given by (14). Generally, there exists an infinite number of solutions to characteristic equations such as (15). The criterion for asymptotic stability of (H*, C*) for the non-linear model is that Re $(\lambda) < 0$ for all real or complex numbers λ such that (9-a) and (9-b) give a nontrivial solution to the linearized problem at (H*, C*). This follows as a result of Theorem I, p. 74 of El'sgol'ts (1966). That is, if solutions to (15) have Re $(\lambda) < 0$, then the equilibrium is asymptotically stable; if there exist solutions with Re $(\lambda) > 0$, it is unstable.

3.2 Region of Stability

We will now proceed to establish the region of stability in the parameter space α and β, where Re $(\lambda) < 0$. To accomplish this, we will employ the method of D-partition suggested by El'sgol'ts and Norkin (1973).

3.2.1 The Method of D-partition.

Given any suitable characteristic equation $\Psi(\lambda) = 0$ with non-zero coefficients, the zeroes of Ψ are generally continuous functions of its coefficients. We can divide the space of coefficients into regions by hypersurfaces corresponding to the zeroes $\lambda = u + i\theta$ for the cases $\lambda = 0$ and $\lambda = i\theta$. Such a decompositon is called a D-partition.

The points in each region of such a D-partition correspond to λ with the same number of zeroes with positive real parts. This follows from a theorem in Krall (1967, p. 106).

Thus to every region R_k of the D-partition, we can assign a number k which is the number of zeroes with Re $(\lambda) > 0$ of Ψ defined by the points in the region. We can also find the regions R_0 corresponding to Ψ which do not even have a single root with Re $(\lambda) > 0$. These regions are the regions of asymptotic stability.

For the case of two parameters, α and β, the characteristic equation

$$\Psi(\lambda, \alpha, \beta) = 0 \tag{17'}$$

defines λ as a function of α and β. That is, there is a function $\lambda = g(\alpha, \beta)$ for a solution to (17'). Taking the total differential of (17') when $\lambda = g(\alpha, \beta)$, we have

$$\frac{\partial\Psi}{\partial\lambda} d \quad + \frac{\partial\Psi}{\partial\alpha} d\alpha + \frac{\partial\Psi}{\partial\beta} d\beta = 0$$

or

$$\frac{\partial\Psi}{\partial\lambda} d\lambda = -\left(\frac{\partial\Psi}{\partial\alpha}\right) d\alpha + \frac{\partial\Psi}{\partial\beta} d\beta).$$

Assuming $\dfrac{\partial\Psi}{\partial\lambda} \neq 0$ for $\lambda = g(\alpha, \beta)$, we get

$$\begin{aligned} d\lambda &= -\left(\frac{\partial\Psi}{\partial\alpha} d\alpha + \frac{\partial\Psi}{\partial\beta} d\beta\right) / \frac{\partial\Psi}{\partial\lambda} \\ &= -Q(\alpha, \beta, \lambda, d\alpha, d\beta). \end{aligned} \tag{17}$$

Letting $\lambda = u + i\theta$ for u and θ real, we have

$$\begin{aligned} d(u + i\theta) &= du + id\theta \\ &= -\mathrm{Re}(Q) - i\mathrm{Im}(Q) \end{aligned}$$

where Re (Q) and Im (Q) are the real and imaginary parts of Q. Thus

$$du = -\mathrm{Re}(Q) \tag{18}$$

We can then use (18) to compute du on some boundary of the D-partition for a change in only one of the parameters, $d\alpha$ or $d\beta$. This will enable us to clarify how the number of roots with $u > 0$ changes as some boundary of the D-partition is crossed. The increase or decrease in the number of roots with $u > 0$ is determined by the algebraic sign of du w.r.t. $d\alpha$ or $d\beta$. We proceed next to apply the above method to (15).

3.2.2 Computations

Denote the left side of (15) a function of λ and the parameters α and β

$$\psi(\lambda, \alpha, \beta) = \lambda - \lambda K(\lambda) + \alpha K(\lambda) + \beta \tag{19}$$

We first find the curve in α, β space corresponding to $\lambda = 0$. Since $K(0) = 1$ by (13), then

$$\psi(0, \alpha, \beta) = \alpha + \beta$$

and (15) implies that $\alpha + \beta = 0$. We thus have the line

$$\alpha + \beta = 0 \tag{20}$$

as one of the boundaries of the D-partition.

Now, suppose $\psi(\cdot)$ has the pure imaginary root $\lambda = i\theta$. Equation (15) takes the form (recall $\lambda K(\lambda) = (1 - e^{-\lambda})$)

$$
\begin{aligned}
0 &= i\theta - i\theta K(i\theta) + \alpha K(i\theta) + \beta \\
&= i\theta - (1 - e^{-i\theta}) + \frac{\alpha}{i\theta}(1 - e^{-i\theta}) + \beta \\
&= (-1 + \cos\theta + \frac{\alpha \sin\theta}{\theta} + \beta) \\
&\quad + i(\theta - \sin\theta - \frac{\alpha}{\theta}(1 - \cos\theta)).
\end{aligned}
$$

Equating real and imaginary parts to zero and solving for α and β, we obtain parametric equations for another of the D-partition boundaries;

$$\alpha(\theta) = \frac{(\sin\theta - \theta)\theta}{\cos\theta - 1} \tag{21-a}$$

$$\beta(\theta) = 1 - \cos\theta - \frac{\sin\theta(\sin\theta - \theta)}{\cos\theta - 1} \tag{21-b}$$

$$\theta \in \mathbb{R}$$

Note that both $\alpha(\theta)$ and $\beta(\theta)$ are even functions of θ, so we consider only the values of (α, β) for $\theta \in \mathbb{R}^+$. Denote the family of curves given by (21-a, b) for $\theta \in \mathbb{R}^+$ by C. Thus

$$C = (C_j \mid j = 1, 2, \ldots) \tag{22}$$

where C_j is the curve defined by (21-a, b) for $2(j-1)\pi < \theta < 2j\pi$. For $j = 1, 0 < \theta < 2\pi$. For $j = 1$, $0 < \theta < 2\pi$, the curve C_1 can be shown to lie in the first quadrant. For each $j > 1$, C_j forms a parabola-like curve with C_j lying above C_k for $j > k$ and $j, k > 1$. A computer generated plot confirms this assertion (see also Appendix I). These curves together with the line (20) form the D-partition shown in Figure 1.

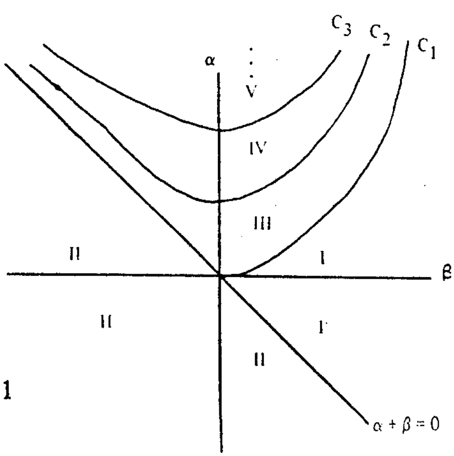


Figure 1

We are now ready to use (18) to study the behavior of Re (λ) = u as each boundary of the D-partition is crossed. Recall that each region of the D-partition has the same number of roots with positive real parts.

On the line $\alpha + \beta = 0$, where $\lambda = 0$, we find from (19) and (13) that

$$\frac{\partial \psi}{\partial \lambda} = 1 - K(\lambda) + (\alpha - \lambda) K'(\lambda) \mid \lambda = 0$$

$$= \alpha K'(\lambda) \mid \lambda = 0$$

$$= \alpha \lim_{\lambda \to 0} \left(\frac{K(\lambda) - 1}{\lambda} \right)$$

$$= \alpha \lim_{\lambda \to 0} \left(\frac{1 - e^{-\lambda} - \lambda}{\lambda^2} \right)$$

$$= - \frac{\alpha}{2.}$$

Also, we get

$$\frac{\partial \psi}{\partial \alpha} = K(\lambda) \mid \; = 0 = 1$$

Thus for $\alpha \neq 0$ and β fixed ($d\beta = 0$), equation (18) becomes

$$du = - \mathrm{Re} \left(\frac{\partial \psi}{\partial \alpha} \Big/ \frac{\partial \psi}{\partial \lambda} \right) d\alpha$$

$$= - \mathrm{Re} \left(\frac{1}{-\alpha/2} \right) d\alpha$$

$$= \frac{2}{\alpha} d\alpha.$$

Therefore, on the line $\alpha + \beta = 0$, we conclude that

$$\frac{du}{d\alpha} \begin{cases} > 0, & \alpha > 0 \\ < 0, & \alpha < 0 \end{cases} \tag{23}$$

More generally, on the curves C, we have for constant β,

$$du = - \mathrm{Re} \left[\frac{K(\lambda)}{1 - K(\lambda) + (\alpha - \lambda) K'(\lambda)} \right] d\alpha$$

Recall that for $\lambda \neq 0$, we have

$$K(\lambda) = \frac{1}{\lambda} (1 - e^{-\lambda}).$$

Hence

$$K'(\lambda) = \frac{1}{\lambda} (e^{-\lambda} - K(\lambda)$$

so we get

$$du = -\mathrm{Re}\left[\frac{K(\lambda)}{\lambda K(\lambda) - \frac{\alpha}{\lambda}K(\lambda) + \frac{\alpha}{\lambda}e^{-\lambda}}\right] d\alpha$$

$$= -\mathrm{Re}\left[\frac{\lambda(e^{\lambda} - 1)}{(e^{\lambda} - 1)(\lambda^2 - \alpha) + \alpha\lambda}\right] d\alpha$$

Recalling that $\lambda = i\theta$ on C, we find

$$du = -\mathrm{Re}\left[\frac{-\theta \sin\theta + i\theta(\cos\theta - 1)}{(\alpha + \theta^2)(1 - \cos\theta) + i(\alpha\theta - \sin\theta(\alpha + \theta^2))}\right] d\alpha$$

$$= \alpha\left[\frac{\theta^2(1 - \cos\theta)}{(\alpha + \theta^2)^2(1 - \cos\theta)^2 + (\alpha\theta - \sin\theta(\alpha + \theta^2)^2}\right] d\alpha$$

On each C_i defined by (22), the expression in braces is always positive and α is positive as we see in Figure 1. Thus, on each C_j, $j = 1, 2, \ldots.$

$$\frac{du}{d\alpha} > 0 \tag{24}$$

For $(\alpha, \beta) = (0, 1)$ which is in region I (see Figure 1), (15) reduces to

$$\lambda + e^{-\lambda} = 0$$

which has solutions with only negative u (see Appendix II). Thus (15) has no roots with positive real parts in Region I. Upon passing from region I to II across the line $\alpha + \beta = 0$, a root with positive real part appears. This follows from (23), since for $\alpha < 0$, $du > 0$ for $d\alpha < 0$. Therefore, for decreasing α and constant β, the real part of the root equal to zero on this line receives a positive increment. On the boundary curve C_1 of the D-partition (that is on the curve defined by (22), $0 < \theta < 2\pi$), we have from (24), $du > 0$ for $d\alpha > 0$. Thus upon crossing the curve C_1 from region I to region III, a pair of conjugate complex roots gain positive real parts. The analysis on the other boundaries of the D-partition is completely analogous.

In summary, region I has no roots with positive real parts, region II has one with positive real part, region III has two roots with positive real parts, region IV with four roots with positive real parts and so on. We conclude that region I is the region of stability. Figure 2 summarizes the results in this analysis. The shaded region indicates the region of stability and p indicates the number of roots with positive real parts in each region.

Further analysis shows that $\alpha + \beta < 0$ is unattainable at equilibrium. Hastings and Wollkind (1982) made the same claim for their model. The assertion is true if the predator functional response is taken to be $\phi = \phi(H)$ as we do here. Hastings and Wollkind considered $\phi = \phi(H, C)$ for which $\alpha + \beta > 0$ does not necessarily hold at equilibrium.

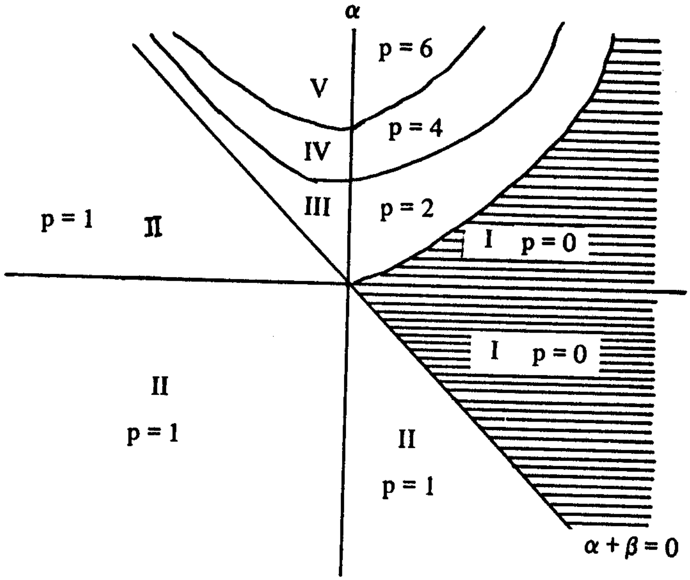


Figure 2

We observe that (16 - a, b) gives

$$\alpha + \beta = -\left(\frac{\partial G}{\partial H}\right)^{*}\left(\frac{\partial F}{\partial C}\right)^{*} \tag{25}$$

Now if $\phi = \phi\,(H, C)$, equations (1) and (7) become

$$F\,(H, C) = Hr\,(H) - \phi\,(H, C) \cdot C \tag{26}$$

$$G\,(H, C) = \varepsilon h\,(\phi\,(H, C)) \cdot C \tag{27}$$

so that

$$\left(\frac{\partial G}{\partial H}\right)^{*} = \varepsilon eh'\,(\phi\,(H^{*}, C^{*}))\left(\frac{\partial \phi}{\partial H}\right)^{*} \cdot C^{*} \tag{28}$$

$$\left(\frac{\partial F}{\partial C}\right)^{*} = -\left(\phi\,(H^{*}, C^{*}) + C^{*} \cdot \left(\frac{\partial \phi}{\partial C}\right)^{*}\right) \tag{29}$$

Recall that $\phi(H, C)$, the predator functional response, is defined as the number of prey killed, per unit time, by the average predator. It is reasonable then to assume that it is positive at equilibrium, and that it decreases with increasing predator population and increases with increasing prey population. That is,

$$\phi^{*} = \phi\,(H^{*}, C^{*}) > 0$$

$$\left(\frac{\partial \phi}{\partial C}\right)^{*} < 0$$

and

$$\left(\frac{\partial \phi}{\partial H}\right)^* < 0$$

Substituting (28) and (29) into (25), we have

$$\alpha + \beta = \left(\phi^* + C^* \left(\frac{\partial \phi}{\partial C}\right)^*\right)\left(\mathcal{E}h'(\phi^*)\left(\frac{\partial \phi}{\partial H}\right)^* \right) C^*) > 0$$

if and only if

$$\phi^* + C^* \left(\frac{\partial \phi}{\partial C}\right)^* > 0 \qquad (30)$$

since h' (ϕ) > 0 and $\mathcal{E} > 0$. But (30) would not necessarily hold. Note that (30) will always hold when $\phi = \phi$ (H) since then

$$\left(\frac{\partial \phi}{\partial C}\right) \equiv 0.$$

Further, taking $\phi = \phi$ (H, C) would violate the assumption that $\left(\frac{\partial G}{\partial C}\right)^* = 1$. This would change the appearance of (15) and consequently, the validity of the stability analysis. If $\phi = \phi$ (H, C), we have from (27)

$$\frac{\partial G}{\partial C} = \mathcal{E}h(\phi(H, C)) + \mathcal{E}h'(\phi(H, C)) \cdot C\frac{\partial \phi}{\partial C}.$$

Hence

$$\left(\frac{\partial G}{\partial C}\right)^* = \mathcal{E}h(\phi^*) + \mathcal{E}h'(\phi^*) \cdot C^* \left(\frac{\partial \phi}{\partial C}\right)^*.$$

But by (6),

$$C^* = \int_{t-1}^{t} \mathcal{E}h(\phi^*) \cdot C^* ds = \mathcal{E}h(\phi^*) \cdot C^*$$

or

$$\mathcal{E}h(\phi^*) = 1.$$

Thus $\left(\frac{\partial G}{\partial C}\right)^* = 1$ only if

$$h'(\phi^*) = 0 \text{ or } \left(\frac{\partial \phi}{\partial C}\right)^* = 0.$$

Neither condition was assumed to hold in Hastings and Wollkind (1982) but the second does when ϕ depends only on H as we assume here.

We conclude this section by giving sufficient conditions for asymptoti. stability. It can be shown that C1 (Figure 2) always lies above the line $\alpha = 2\beta$ (see Appendix I). We can therefore conclude that sufficient conditions for asymptotic stability are

$$\alpha + \beta > 0 \text{ and } \alpha < 2\beta. \tag{31}$$

From Figure 2, we can also conclude that a sufficient condition for instability is

$$\beta < 0 \tag{32}$$

The general conditions derived in this section, in particular inequalities (31) and (32), will be used in the next section to examine the stability of a specific model.

4. A Specific Example

Equation (1), describing the dynamics of the prey population, will be taken to have the form

$$\frac{dH}{dt} = \eta\left(1 - \frac{H}{K}\right) \cdot H - \rho\left(1 - e^{-\gamma H}\right) \cdot C \equiv F(H, C) \tag{33}$$

That is, the intrinsic growth rate of the prey population, r(H) is given by the Vershulst logistic term

$$r(H) = \eta\left(1 - \frac{H}{K}\right); \quad \eta > 0, k > 0$$

where K is the carrying capacity of the habitat. The predator functional response is given by

$$\phi(H) = \rho\left(1 - e^{-\gamma H}\right); \rho, \gamma > 0$$

as suggested by Ivler (1961) This form of the predator functional response suggests that the predation rate is proportional to H* for small prey populations, but saturates to a constant ρ per average predator for large H (Figure 3).

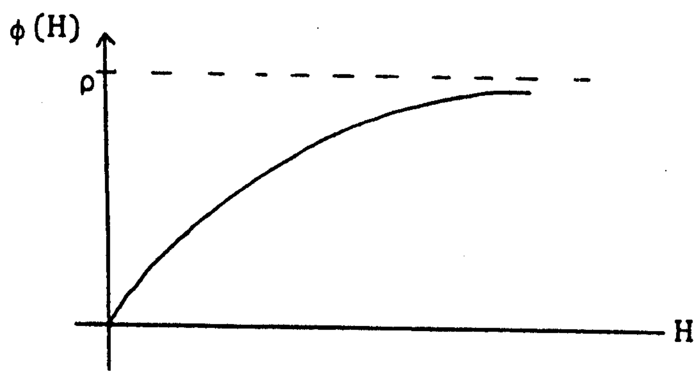


Figure 3

The description of the dynamics of the predator population begins by taking

$$h(\phi) = \delta\phi = \delta\rho\left(1 - e^{-\gamma H}\right); \quad \delta > 0$$

where $h(\phi)$ is the conversion rate of prey into predator as food. Equation (6) becomes

$$C(t) = \int_{t-1}^{t} [\varepsilon\delta\rho(1 - e^{-\gamma H}) \cdot C](s)\, ds \qquad (34)$$

$$= \int_{t-1}^{t} [G(H, C)](s)\, ds$$

where

$$G(H, C) = \varepsilon\delta\rho(1 - e^{-\gamma H}) \cdot C \qquad (35)$$

At equilibrium $G(H^*, C^*) = C^*$ so (35) implies

$$\varepsilon\delta\rho(1 - e^{-\gamma H^*}) = 1 \qquad (36)$$

Then

$$H^* = \frac{1}{\gamma} \ln \left(\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1} \right) \qquad (37)$$

We assume $\varepsilon\delta\rho > 1$ so that H^* exists and satisfies $H^* > 0$ Now, $F(H^*, C^*) = 0$ implies

$$C^* = \frac{\eta H^* (1 - \frac{H^*}{K})}{\rho(1 - e^{-\gamma H^*})} = \varepsilon\delta\rho\, H^* (1 - \frac{H^*}{K}) \qquad (38)$$

We assume $H^* < K$ since K is the carrying capacity of the habitat, and then $C^* > 0$.

We can now apply the stability results in section 3 to the equilibrium populations (37) and (38). From (33) and (35), we compute the following derivatives:

$$\frac{\partial F}{\partial C} = -\rho(1 - e^{-\gamma H}) \qquad (39\text{-a})$$

$$\frac{\partial F}{\partial H} = \eta\left(1 - \frac{2H}{K}\right) - \rho\gamma C e^{-\gamma H} \qquad (39\text{-b})$$

$$\frac{\partial G}{\partial C} = \varepsilon\delta\rho(1 - e^{-\gamma H}) \qquad (39\text{-c})$$

$$\frac{\partial G}{\partial H} = \varepsilon\delta\rho\,\gamma C e^{-\gamma H} \qquad (39\text{-d})$$

From (16-a, b) and (36) we then find

$$\alpha + \beta = -\left(\frac{\partial F}{\partial C}\right)^* \left(\frac{\partial G}{\partial H}\right)^*$$

$$= \rho(1 - e^{-\gamma H^*})\rho\delta\varepsilon\gamma C^* e^{-\gamma H^*}$$

$$= \rho\gamma C^* e^{-\gamma H^*} \qquad (40)$$

By (36) and (38), we get

$$\alpha + \beta = \gamma\eta H^* (L - \frac{H^*}{K})(\rho\delta\varepsilon - 1). \tag{41}$$

Now,

$$\beta = -(\frac{\partial F}{\partial H})^*$$

$$= \eta(\frac{2H^*}{K} - 1) + \rho\gamma C^* e^{-\gamma H^*} \tag{42}$$

This together with (40) implies

$$\alpha = -\eta(\frac{2H^*}{K} - 1) \tag{43}$$

Let $\xi = H^*/K$. Then by (37), we have

$$\xi = \frac{1}{\gamma K} \ln(\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1}) \tag{44}$$

We want $0 < \xi < 1$ so that $0 < H^* < K$ and $C^* > 0$. Hence we want $\varepsilon\delta\rho > 1$ and $\ln(\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1}) < \gamma K$, or

$$\varepsilon\delta\rho > \frac{e^{\gamma K}}{e^{\gamma K} - 1} > 1, \quad \gamma K > 0 \tag{45}$$

We then have from (41)

$$\alpha + \beta = \gamma\eta K\xi(1 - \xi)(\rho\delta\varepsilon - 1) > 0.$$

So, we need only to check for the condition $\alpha < 2\beta$ for stability. From (42) and (43), we find

$$2\beta - \alpha = -3\alpha + 2\rho\gamma C^* e^{-\gamma H^*}$$

$$= 3\eta(2\xi - 1) + 2\rho\gamma C^* e^{-\gamma H^*}$$

By (38) and writing $H^* = K\xi$, we get

$$2\beta - \alpha = 3\eta(2\xi - 1) + 2\varepsilon\delta\rho\gamma\eta K\xi(1 - \xi)e^{-\gamma K\xi}.$$

But from (44)

$$\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1} = e^{\gamma K\xi}$$

so that

$$\varepsilon\delta\rho = \frac{e^{\gamma K\xi}}{e^{\gamma K\xi} - 1} \tag{46}$$

Hence

$$\begin{aligned} 2\beta - \alpha &= 3\eta\,(2\xi - 1) + \frac{2\eta\gamma K\xi(1-\xi)}{e^{\gamma K\xi} - 1} \\ &= \eta\,[3\,(2\xi - 1) + \frac{2\gamma K\xi\,(1-\xi)}{e^{\gamma K\xi} - 1}] \\ &= \eta \cdot g(\xi) \end{aligned} \tag{47}$$

Since $\eta > 0$, $2\beta - \alpha$ will have the same sign as $g(\xi)$. Now from (44) we see that depends on the two parameters, γK and $\varepsilon\delta\rho$. Let us introduce

$$\tau = \gamma K\xi = \ln\left(\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1}\right) \tag{48}$$

We can choose any $\gamma K > 0$ and find $0 < \tau < \gamma K$ to give any values we want for $0 < \xi < 1$. Alternately, we can interpret τ and ξ as independent parameters on ranges $\tau > 0$ and $0 < \xi < 1$. Then for $\tau > 0$, we define

$$\begin{aligned} G(\tau,\xi) :&= g(\xi)\,\frac{e^{\gamma K\xi} - 1}{\gamma K\xi} \\ &= 3(2\xi - 1)\,\frac{e^{\tau} - 1}{\tau} + 2(1 - \xi) \end{aligned} \tag{49}$$

Thus we want to find in the (τ, ξ) plane where $G(\tau, \xi)$ is positive to get $\alpha < 2\beta$ for stability. We begin by finding the curve where $G(\tau, \xi) = 0$. This is zero if and only if

$$\frac{e^{\tau} - 1}{\tau} = \frac{2(\xi - 1)}{3(2\xi - 1)}$$

(see figures 4 and 5).

For $\tau > 0$,

$$\frac{e^{\tau} - 1}{\tau} > 1$$

and

$$\frac{2(\xi - 1)}{3(2\xi - 1)} \geq 1$$

if and only if $\xi < \frac{1}{2}$ and $\xi \geq \frac{1}{4}$. Hence, for $\frac{1}{4} < \xi < \frac{1}{2}$, we get $\tau > 0$ where $G(\tau, \xi) = 0$ and τ increases with ξ. There $(2\xi - 1) < 0$, so increasing τ makes $G(\tau, \xi) < 0$. For $\tau > 0$ and $\frac{1}{2} < \xi < 1$, $(2\xi - 1) > 0$, so $G(\tau, \xi) > 0$ by (49)

$$\varepsilon h(\phi(H)) = \varepsilon \delta \rho (1 - e^{-\gamma H}).$$

For large H, this saturates to $\varepsilon \xi \rho$. Hence, H^*, $C^* > 0$ exist only if the per capita predator birth rate is larger than one when a plentiful supply of prey is available.

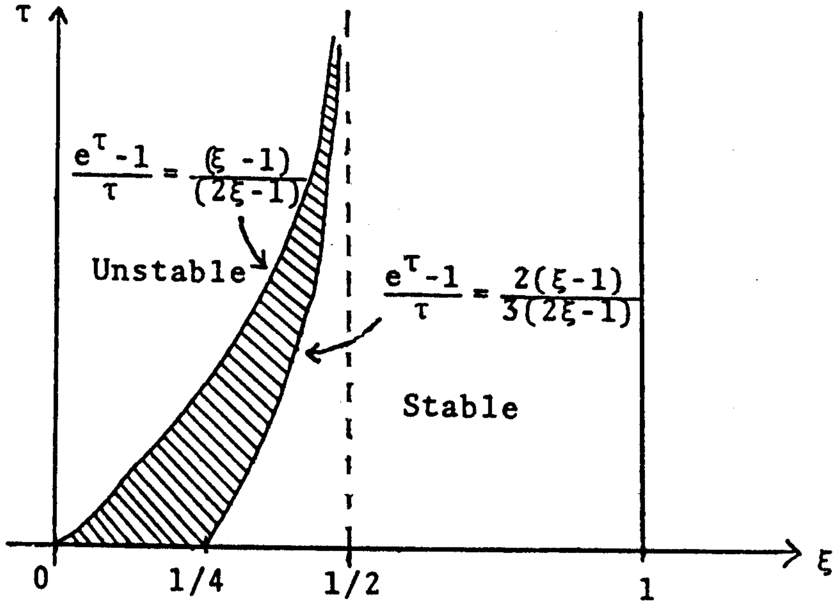


Figure 8

5. Conclusions

In concluding this paper, we discuss some of the features of the model derived in section 2, its values and limitations, and some applications of the stability analysis presented. We will do this by discussing the example worked out in section 4.

A particular feature of the model in this paper is that the prey equilibrium population can be altered only by changing the birth rate of the predators. This is due to the fact that the predator population has a vertical isocline. Keeping in mind equations (1) and (6), the predator isocline is defined as the locus of points (H, C) satisfying the condition

$$G(H, C) = C \tag{50}$$

and the prey isocline satisfies

$$F(H, C) = 0 \tag{51}$$

The point of intersection of these two curves represents the equilibrium point. The prey isocline (51) implicitly define C as a function of H. That is, we have from (1),

$$r(H) \cdot H - \phi(H) \cdot C = 0$$

(see figures 4 and 5).

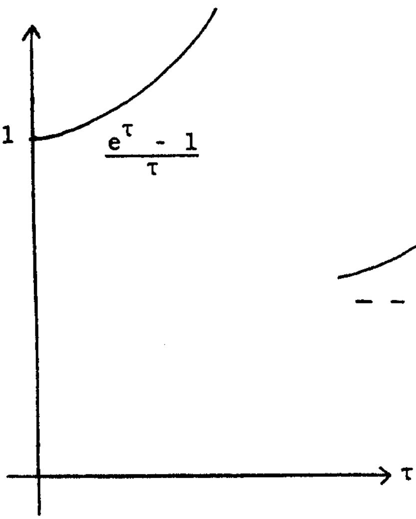


Figure 4

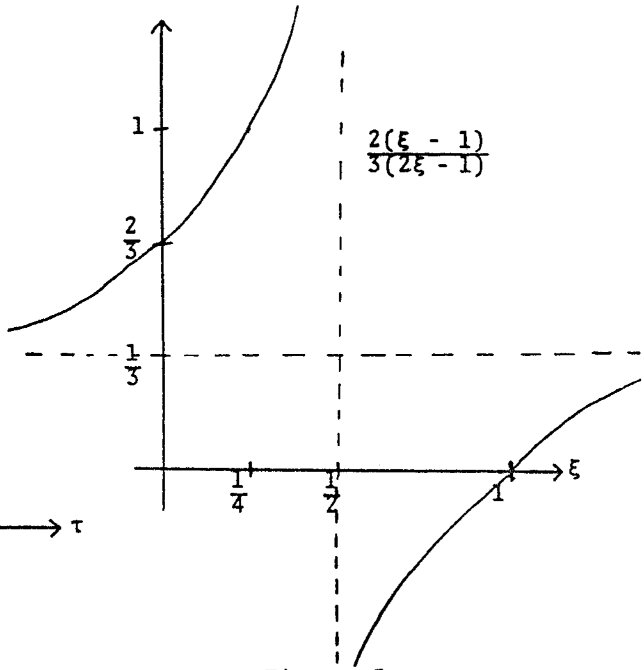


Figure 5

since then we have

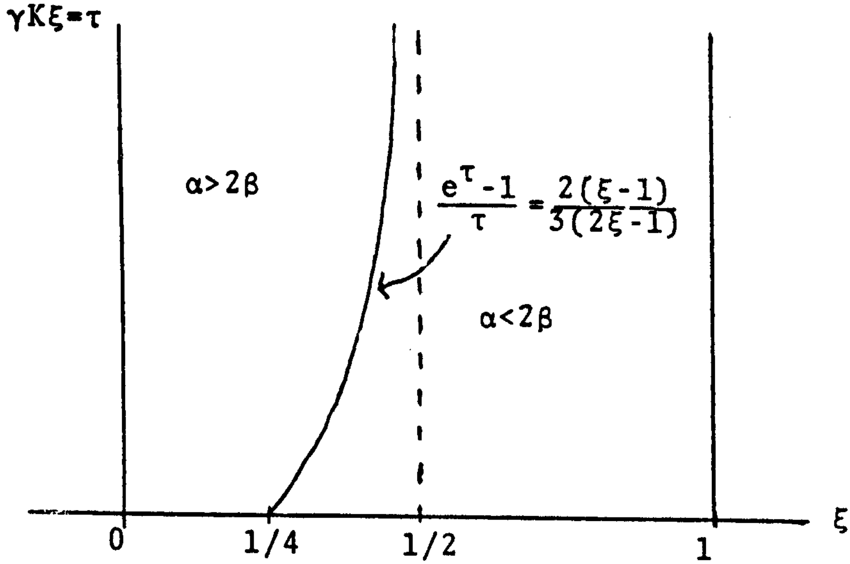
$$\frac{e^\tau - 1}{\tau} > \frac{2(\xi - 1)}{3(2\xi - 1)}$$

Hence, we have divided the (τ, ξ) plane for $\tau > 0$ and $0 < \xi < 1$ corresponding to the cases $\alpha < 2\beta$ and $\alpha > 2\beta$ (Figure 6). Since $\alpha < 2\beta$ is only a sufficient condition for stability, we can only conclude that the equilibrium populations (37) and (38) are stable if

(i) $\frac{1}{2} \leq \xi < 1$

or (ii) if $\frac{1}{4} < \xi < \frac{1}{2}$ and Υ and K satisfy

$$\frac{e^\tau - 1}{\tau} < \frac{2(\xi - 1)}{3(2\xi - 1)}, \quad \tau = \Upsilon K \xi$$



A stability plot in the $\tau - \xi$ plane. Shaded region corresponds to stability. The unshaded region is ambiguous.

The unshaded region is ambiguous since we cannot determine in a closed form the curve C_1 (see Figure 2) in the (τ , ξ) plane. We can however use (32) to find region in the (τ , ξ) plane pertaining to instability. From (42) and (44), we have

$$\beta = \eta(2\xi - 1) + \eta \frac{\gamma K\xi\,(1 - \xi)}{e^{\gamma K\xi} - 1}$$

$$= \eta\left[(2\xi - 1) + \frac{\gamma K\xi\,(1 - \xi)}{e^{\gamma K\xi} - 1}\right]$$

$$= \eta \cdot \bar{g}(\xi); \quad \eta > 0$$

Note that $\bar{g}(\xi)$ is similar to $g(\xi)$ in (47). Analogously, for $\tau = \gamma K\xi > 0$, we define

$$B(\tau,\xi) = \frac{e^{\gamma K\xi} - 1}{\gamma K\xi} \cdot \bar{g}(\xi)$$

$$= (2\xi - 1)\,\frac{e^{\gamma K\xi} - 1}{\gamma K\xi} + (1 - \xi).$$

We then find in the (τ , ξ) plane where $B(\tau,\xi)$ is negative to get $\beta < 0$ for instability. We first find the curve where $B(\tau,\xi) = 0$. This holds if and only if

$$\frac{e^{\tau} - 1}{\tau} = \frac{(\xi - 1)}{(2\xi - 1)}$$

By arguments similar to those on page 37 we have for $0 < \xi < \frac{1}{2}$, $\tau > 0$ where $B(\tau, \xi) = 0$. The region where $B(\tau, \xi) < 0$ is given by (see Figure 7)

$$\frac{e^{\tau} - 1}{\tau} > \frac{(\xi - 1)}{(2\xi - 1)}, \quad 0 < \xi < \frac{1}{2}$$

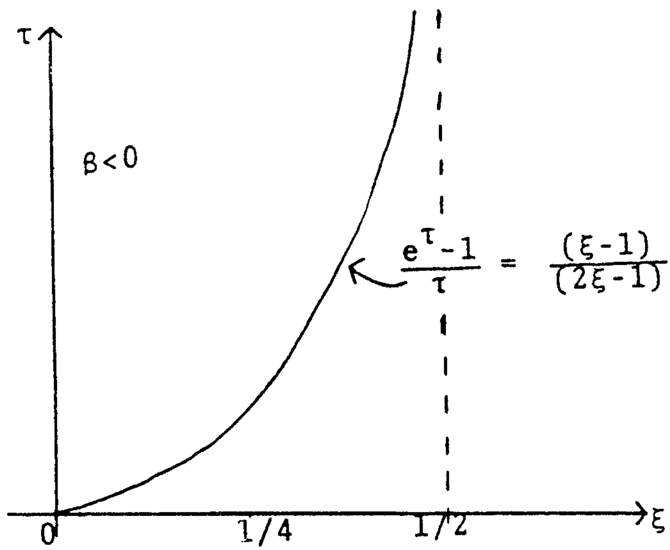


Figure 7

Except for the ambiguous case described by

$$\frac{2(\xi - 1)}{3(2\xi - 1)} < \frac{e^{\tau} - 1}{\tau} < \frac{(\xi - 1)}{(2\xi - 1)}$$

we have completely determined regions of stability and instability in the τ - ξ plane. Figure 8 summarizes the stability analysis for this model.

A stability diagram in the τ - ξ plane. The cross-hatching designates the ambiguous region while the shaded region corresponds to stability and the unshaded one to instability.

An important assumption throughout the above analysis is $\rho\delta\epsilon > 1$. This was necessary for the existence of H*, C* > 0. Note from (3) that the predator's per capita birth rate is given by and solving for C gives

$$C = \frac{r(H) \cdot H}{\phi(H)} = \theta(H).$$

When $r(H) = \eta(1 - H/K)$ and $\phi(H) = \rho(1 - e^{-\gamma H})$, as in our example, we have

$$C = \theta(H) = \frac{\eta H(1 - H/K)}{\rho(1 - e^{-\gamma H})} \tag{52}$$

as the prey isocline. Now we consider the predator isocline

$$G(H, C) = C.$$

Writing $h(\phi(H))$ in (6) as $\tilde{h}(H)$, we get

$$\varepsilon \tilde{h}(H)\, C = C$$

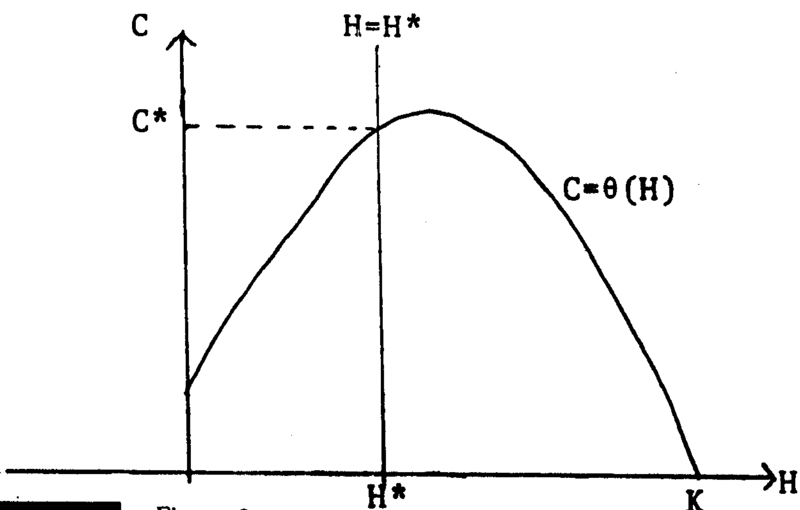
Assuming $\tilde{h}(H)$ is one-to-one on the domain $H \geq 0$ and range $\tilde{h}(H) \geq 0$, we have

$$H = \tilde{h}^{-1}(\varepsilon^{-1}) = H^* \tag{53}$$

where $\tilde{h}^{-1}(\cdot)$ is the inverse function of $\tilde{h}(\cdot)$. For our example, we have

$$H = H^* = \frac{1}{\gamma} \ln\left(\frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1}\right)$$

as the predator isocline. Hence, the predator isocline is the vertical line $H = H^*$. This is a consequence of the non-existence of intraspecific interaction among the predators, and by the independence of the conversion function, $h(\phi(H)) = \tilde{h}(H)$, of C. Figure 9 shows the isoclines (52) and (53).



[redacted] Figure 9

To illustrate how the prey equilibrium population, H*, can be altered by changing the per capita birth rate of the predators, we recall from our example,

$$H^* = \frac{1}{\gamma} \ln \frac{\varepsilon\delta\rho}{\varepsilon\delta\rho - 1}$$

From (6), we find the per capita birth rate of the predators is given by

$$\varepsilon h(H) = \varepsilon\delta\rho(1 - e^{-\gamma H}).$$

It is evident then that H* can be altered by changing γ or $\varepsilon\delta\rho$ the limiting per capita birth rate.

We can use the previous fact to "stablilize" the equilibrium populations, if they are unstable, by artificially changing the parameters concerned. This provides a means of biological control. Given that we know the stability region in the α - β plane (section 4), one can plot the values of (α, β) corresponding to different values of the parameters ε, γ, δ, ρ and η, and thereby see which combinations would lie in the stable region. We can also do the same for τ and ξ in Figure 8.

It would also be interesting to study the effects various types of predator functional response, $\phi(H)$, might have on stability. Hastings and Wollkind (1982) worked out an example involving the Holling's type functional response

$$\phi(H) = \frac{aH}{b + H} \; ; \; a, b > 0$$

and found it to have a stabilizing effect on the equilibrium (H*, C*). In our example, where we used the Ivler functional response,

$$\phi(H) = \rho(1 - e^{-\gamma H}); \; \rho, \gamma > 0$$

stability is not guaranteed for all H*, C* > 0. Hastings **et al.** (1982) and Oaten and Murdoch (1975) gave an in-depth analysis of these subjects. Interested readers should consult them.

Though simple and arguably unrealistic, the model in this paper provides a good example to illustrate the analytical study of the stability of an equilibrium. Various assumptions could be added to the model to make it more general but at the expense of complicating the mathematics. Prey age structure and environmental fluctuations, for example, could be considered. It would appear that the more realistic the model the more difficult it would become to describe in a precise way the interactions of various effects which serve to separate the stable and unstable domains.

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Appendix I

For $\lambda = i\theta$, the curve C_1 is given parametrically by

$$\alpha(\theta) = \frac{\theta(\theta - \sin\theta)}{1 - \cos\theta} \qquad \text{A-1}$$

$$\beta(\theta) = 1 - \cos\theta - \frac{\sin\theta(\theta - \sin\theta)}{1 - \cos\theta}$$

$$= \frac{2 - 2\cos\theta - \theta\sin\theta}{1 - \cos\theta} \qquad \text{A-2}$$

$$0 < \theta < 2\pi.$$

To show that C_1 corresponds to an increasing function in the first quadrant in the (α, β) plane for $\theta \in (0, 2\pi)$, it is sufficient to show

(i) $\lim\limits_{\theta \to 0} \alpha(\theta) = \lim\limits_{\theta \to 0} \beta(\theta) = 0$

(ii) (ii) $\alpha(\theta) > 0, \beta(\theta) > 0$ for all $\theta \in (0, 2\pi)$

(iii) $\alpha(\theta) > 2\beta(\theta)$

(i) Can be easily verified by L' Hospital's rule. Since $\sin\theta < \theta$ and $\cos\theta < 1$ for $\theta \in (0, 2\pi)$, we have $\alpha(\theta) > 0$. To show $\beta(\theta) > 0$ in (A -2), we only need to show

$$\beta(\theta) = 2 - 2\cos\theta - \theta\sin\theta > 0,$$
$$\theta \in (0, 2\pi).$$

But

$$\beta'(\theta) = \sin\theta - \theta\cos\theta$$

$$\tilde{\beta}''(\theta) = \theta\sin\theta$$

Since $\tilde{\beta}(0) = \tilde{\beta}'(0) = \tilde{\beta}''(0) = 0$ and $\tilde{\beta}''(\theta) > 0$ for $\theta \in (0, \pi)$, it follows that

$$\tilde{\beta}'(\theta), \tilde{\beta}(\theta) > 0 \text{ for } \theta \in (0, \pi)$$

For $\theta \in [\pi, 2\pi)$, we have $\sin\theta \leq 0$ and $1 - \cos\theta > 0$, so $\tilde{\beta}(\theta) \geq 2$. Thus $\tilde{\beta}(\theta) > 0$ for $\theta \in (0, 2\pi)$. To show (iii), construct the function

$$Q(\theta) = \alpha(\theta) - 2\beta(\theta)$$

$$= \frac{\theta(\theta - \sin\theta)}{1 - \cos\theta} - 2 + 2\cos\theta + \frac{2\sin\theta(\theta - \sin\theta)}{1 - \cos\theta}$$

$$= \frac{4\cos\theta - 4 + \theta\sin\theta + \theta^2}{1 - \cos\theta}$$

Now we can show that the curve C_1 always lies above the line $\alpha = 2\beta$ by showing $Q(\theta) > 0$ for $\theta \in (0, 2\pi)$. We need only to show

$$Q(\theta) = 4\cos\theta - 4 + \theta \sin 0 + \theta^2 > 0,$$
$$0 \in (0, 2\pi)$$

To see this, we calculate the following derivatives:

$$Q'(\theta) = -3\sin\theta + 2\theta + \theta\cos\theta$$
$$Q''(\theta) = 2 - 2\cos\theta - 0\sin\theta = \beta(\theta)$$

We have shown $\beta(\theta) > 0$ for $\theta \in (0, 2\pi)$. Since

$$Q(0) = Q'(0) = 0$$

it follows that

$$Q(\theta) > 0 \text{ for } 0 \in (0, 2\pi).$$

Hence

$$Q(\theta) > 0 \text{ for } 0 \in (0, 2\pi).$$

and (iii) follows.

Appendix II

We will show that all the roots of

$$\lambda + e^{-\lambda} = 0 \qquad \text{A-1}$$

have $\mathrm{Re}(\lambda) < 0$. Let $\lambda = u + i\theta$, $u, \theta \in R$ in (A-1).
Setting the real and imaginary parts to zero, we get

$$u + e^{-u}\cos\theta = 0 \qquad \text{A-2a}$$

$$\theta - e^{-u}\sin\theta = 0 \qquad \text{A-2b}$$

From (A - 2b)

$$e^{-u} = \frac{\theta}{\sin\theta}$$

or

$$u = \ell n\left(\frac{\sin\theta}{\theta}\right) \qquad \text{A-3}$$

Substituting (A-3) into (A-2a), we get

$$\ell n\left(\frac{\sin\theta}{\theta}\right) + \frac{\theta\cos\theta}{\sin\theta} = 0$$

or

$$\frac{\sin \theta}{\theta} = e^{-\theta \cot \theta} \qquad \text{A-4}$$

In the figure the graphs of $\frac{\sin \theta}{\theta}$ and $e^{-\theta \cot \theta}$ are shown for $\theta \in (-\pi, \pi)$. It is evident that the solutions to (A - 4) occur at $\theta = \pm\theta_1$ where $0 < \theta_1 < \frac{\pi}{2}$ and

$$\frac{\sin (\theta_1)}{\theta_1} = \frac{\sin (-\theta_1)}{-\theta_1} < 1.$$

The real part for the corresponding complex conjugate roots is thus

$$u_1 = \log \left[\frac{\sin \theta_1}{\theta_1} \right] < 0$$

From (A - 2a, b), we get

$$\sin \theta = e^u \theta, \ \cos \theta = - e^u u$$

so

$$1 = e^{2u} (\theta^2 + u^2)$$

or

$$e^{-2u} = \theta^2 + u^2.$$

If there exist any roots with $\theta^2 > (\frac{\pi}{2})^2$, we have then

$$e^{-2u} > (\frac{\pi}{2})^2 + u^2 \geq (\frac{\pi}{2})^2 > 1$$

So, u would still have to be negative.

PENDENGARAN TANPA MENDENGAR: SATU ANALISIS TERHADAP PERANAN MENDENGAR DALAM BERKOMUNIKASI

Safri Samit
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Keberkesanan kita sebagai seorang Pentadbir, Pengarah, Penyelia atau apa jua kerjaya kita, bergantung kepada keefisyenan kita berkomunikasi. Dalam lima panca indera (penglihatan, pendengaran, percakapan, penghiduan dan perisaan) yang kita gunakan sebagai 'alat' untuk berkomunikasi, 'PENDENGARAN' paling minima diberi pendedahan sebagai punca penghalang komunikasi harmonis. Tergendalanya sesuatu proses komunikasi mungkin kurangnya perhatian sipenerima berperanan sebagai PENDENGAR (listens). Seseorang yang MENDENGAR dengan teliti bukan sahaja akan mengetahui setiap fakta, tetapi juga akan berupaya mengesan perasaan (feelings) yang tersembunyi di sebalik sesuatu yang dilafazkan. Mustahak bagi kita memberi respon kepada 'perasaan' yang kadangkala tidak dinyatakan ini, di samping memberi respon kepada perkataan lisan. Pembabitan kita dengan masalah peribadi, perasaan sendiri, tugas harian dan tanggungjawab merupakan 'penghalang' untuk kita betul-betul MENDENGAR dengan ertikata sebenar. Fakta-fakta di sebalik kelemahan ketidak upayaan kita ini termasuk saingan dari punca-punca lain untuk mendapatkan perhatian, kelalaian, emosi, kekurangan usaha menilai dan kadangkala tabii malas kita sendiri. Artikel ini cuba menghuraikan fenomena ini dan dimuatkan juga di bahagian akhir, ujian nilai diri anda untuk mengetahui profail anda sebagai seorang PENDENGAR.

Komunikasi secara amnya boleh dibahagikan kepada dua aspek: Lisan dan tanpa lisan. Ekoran dari bahagian lisan pula akan membawa kita kepada penulisan, pembaca-an, percakapan dan juga pendengaran. Kebanyakan kita mungkin telah diperagakan dengan secara tidak seimbang kepada keempat-empat jurusan yang disebutkan ini.

Bermula dari sekolah rendah lagi telah banyak kita tumpukan masa untuk belajar membaca, menulis dan bertutur secara efektif. Akan tetapi jarang sekali kita mengikuti pembelajaran formal tentang bagaimana kita boleh mendengar dengan lebih berkesan. Kajian terkini menunjukkan hanya 5 atau 6 peratus* sahaja yang betul-betul berupaya menjadi pendengar yang baik sedangkan kita menghabiskan sehingga hampir 60% masa kita berkomunikasi bukan dengan membaca, menulis atau bertutur tetapi adalah dengan MENDENGAR.

MENDENGAR merupakan komponen penting dalam proses komunikasi Berkomunikasi bukan beerti hanya bertutur secara lisan atau secara bertulis dengan orang lain, tetapi ia melibatkan pengaliran sesuatu mesej untuk membangkit atau menghasil-

* "Your Personal Listening Profile", by Dr. Lyman K. Stail, *Sperry Booklet.* Scott: Foresman and Company, 1981

kan sesuatu respon di pihak penerima. Mendengar merupakan tabii yang utuh dalam proses komunikasi dan tidak kurang mustahaknya seperti juga pertuturan.

Sebahagian besar manusia tidak menyedari mereka mempunyai sifat penghalang yang sedia ada didiri apabila berperanan sebagai seorang pendengar. Tabiat begini merupakan pedang dua mata kerana dengan sikap mereka yang tidak begitu mahu mendengar, orang juga tidak akan mahu mendengar kepada apa yang mereka perkatakan. Dalam erti kata lain, dengan sendirinya akan menghalang keujudan komunikasi yang harmonis. Seorang ahli perniagaan barat yang terkenal menulis tentang topik ini demikian:

> "Kebanyakan kita menyangkakan diri kita sebagai seorang pendengar yang baik. Saya tahu kerana masa memulakan kehidupan dahulu, juga mempunyai perasaan demikian. Tetapi semakin lama mengecapi kehidupan ini, saya menyedari bahawa MENDENGAR bukannya satu tabiat yang ujud semulajadi dalam diri seseorang. Ianya merupakan seni yang boleh dipupuk.
>
> Kebanyakan kita, mendengar – samada semasa perbualan atau sedang bermesyuarat – hanya merupakan DIAM sejenak yang dirasakan perlu diberi kepada seseorang yang bercakap sehingga tiba giliran kita untuk mengeluarkan pendapat sendiri. Ini bukannya MENDENGAR yang dimaksudkan dalam komunikasi.
>
> Mendengar bukannya merupakan satu aktiviti yang pasif di mana kita membiarkan pemikiran kita menceroboh atau mengganggui keatas apa yang sedang dilafazkan oleh orang lain.
>
> Untuk mendengar secara aktif terhadap apa yang sedang diperkatakan, memerlukan kekuatan daya fikir, konsentrasi dan daya usaha mental yang berat.
>
> Ganjaran yang akan diperolehi darinya begitu tinggi sekali apabila kita mendengar secara begini. Hanya ketika itu baru kita mempelajari sesuatu tentang orang lain, perasaan mereka, cita-cita mereka, aspirasi, siapa mereka sebernarnya, apakah rungutan mereka dan apakah kehendak mereka.
>
> Anda akan terperanjat betapa banyak lagi akan anda pelajari dari orang lain dengan mendengar cara begini, samada dari rakan seperjuangan atau pun orang yang lebih tinggi kedudukan dari kita. Akan berkuranganlah perselisihan faham yang mungkin berpunca dari tekanan emosi daripada berlandaskan kepada fakta.

Anda akan lebih bersedia untuk menghargai serta menilai buah fikiran dan kehendak orang lain – orang bawahan, ketua atau rakan sejawat – dari segi pandangan mereka dan tidak dari kacamata anda sendiri!"

(Petikan dari 'What Every Excutive Should Know About Himself – oleh J.C. Penney)

Kita perlu mendengar dengan teliti dan tidak berpura-pura sahaja. Kita harus berkebolehan untuk mengetahui apa yang dirasai dan difikirkan oelh orang lain. Tanpa kita mendengar dengan ertikata sebenar, pengetahuan atau sifat untuk 'merasai' itu tidak akan diperolehi. Kita mendengar untuk mengetahui sikap orang lain. Ini merangkumi lebih dari sekadar mendengar kepada hanya apa yang tersurat tetapi juga perlu kita menganalisis keatas segala yang mungkin tersirat dari pihak punca.

Komponen Mesej Lisan

Jika diteliti tentang percakapan atau perbualan yang kita hadapi setiap hari, tak dapat dinafikan sebahagian besar mengandungi 'fakta' dan juga 'perasaan'. Fakta-fakta yang terkandung mungkin berupa maklumat-maklumat seperti tarikh sesuatu perjumpaan, spesifikasi untuk sesuatu alat, ajenda yang terkandung di dalam mesyuarat atau informasi lain yang berkaitan.

Turut serta di dalam penyampaian fakta-fakta ini juga 'perasaan' pihak penyampai.Untuk betul-betul 'mendengar' perasaan yang dimaksudkan ini adalah lebih sukar. Ada kalanya, sesuatu kenyataan itu hanya mengandungi fakta dan perasaan yang objektif, berkecuali atau mungkin mencerminkan ketiadaan perasaan yang tersembunyi. Tetapi walaupun tidak membayangkan apa-apa pun dengan sendirinya ia juga merupakan satu perasaan. Dalam kebanyakan keadaan, perasaan seseorang individu itu amat ketara kepada seseorang pendengar yang sensitif.

Bayangkan satu situasi di mana seorang murid yang sedang dalam keadaan gelisah memegang keputusan peperiksaannya sambil menjawap pertanyaan bapanya dengan berkata:

"Saya tak tahu apa yang akan menggembirakan ayah! Saya berjaya mendapat gred B dalam satu mata pelajaran yang begitu susah dan ayah bertanyakan kenapa saya tidak mendapat gred A." Tentu sekali banyak fakta yang terdapat di dalam keadaan ini di samping perasaan yang turut terkandung bersama.

Untuk hanya mendengar dan memberi respon kepada fakta akan mengakibatkan tergendalanya sesuatu komunikasi.

Kenapa kita Kurang Ke Efisyenan Mendengar

Ada beberapa sebab kenapa kita tidak menghayati apa yang kita dengar. Saingan untuk mendapat perhatian di dalam masyarakat yang kompleks kini merupakan salah satu fakta. Sungguh pun kita ingin untuk mendengar kepada apa yang disampaikan oleh seseorang itu kepada kita, banyak perkara-perkara sampingan yang turut bersaing untuk menarik perhatian kita.

Kurang menumpukan perhatian kepada apa yang diperkatakan juga menjadi satu sebab. Perkara yang utama menjadi penghalang kepada penelitian kita ialah perbezaan kederasan penggunaan kata lisan (lebih kurang 150 perkataan seminit) berbanding dengan kederasan pendengaran kita (lebih kurang 600 perkataan seminit) [2] Perbezaan ini menunjukkan bahawa sipenerima mempunyai ruang untuk memikirkan berbagai-bagai idea selain dari apa yang disampaikan. Dalam keadaan demikian prestasi untuk mendengar pasti akan terjejas. Emosi kita sendiri juga boleh menjadi penghalang keatas keupayaan kita untuk mendengar dengan teliti.

Mendengar Kepada Fakta-Fakta

Kita semua mungkin telah mengalami tidak berupaya hadir di sesuatu mesyuarat disebabkan oleh alasan-alasan peribadi atau sebagainya. Untuk mendapatkan maklumat dari apa yang telah diperbincangkan, kita menghampiri seorang rakan sambil berkata, "Ali, saya tidak hadir di mesyuarat kelmarin. Awak hadir bukan; Apa yang telah berlaku?"

"Oh, mesyuarat itu sungguh menarik. Awak tahu dengan jumlah tenaga pekerja sebanyak lebih 500 orang dalam institusi ini, kita tentu sekali menghadapi berbagai-bagai masalah personelia dan mesyuarat pun membincangkan hal ini dan berupaya menyelesaikan beberapa masalah." tegas Ali.

"Ya, saya pasti akan perkara itu Ali. Tetapi secara spesifik apa sebenarnya dibincangkan?"

"Ialah, sebagai saya katakan, mesyuarat membincangkan masalah personelia dan dapat menyelesaikan masalah-masalah tertentu."

"Itu, saya faham, Ali! Tetapi apa sebenarnya berlaku?"

Dengan nada yang agak bengis, Ali berkata, "Saya dah beritahu awak! Kami membincangkan persoalan personelia. Sekiranya awak minat sangat, kenapa awak tak datang ke mesyuarat itu sendiri?"

Kini dengan tidak mendapat sebarang informasi tetapi lebih ransang, anda mendampingi Hassan.

"Hassan, awak hadirkah di mesyuarat kelmarin?"

"Ya, sungguh berfaedah sekali. Sayang awak tak turut hadir."

"Yalah, saya pun rasa begitu. Apa yang diperbincangkan?"

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[2]Norman B. Signand, David N. Bateman, "Comunicating in Business"Scot Foresman & Company Texas, 1981 pp. 313.

"Awak tahu, kita mempunyai lebih 500 orang kakitangan kini dan betapakah banyak masalah personelia yang timbul dari jumlah ini. Walau bagaimanapun, kami dapat membincangkan empat perkara penting: Pertama, tentang pengambilan tenaga profesional, kita bersetuju tidak menggunakan pengiklanan melalui akhbar mulai 1 Disember ini tetapi sebaliknya menggunakan khidmat Syarikat Pakarunding. Juga kita mengharapkan sokongan dari pekerja-pekerja kita untuk memberitahu sekiranya ada rakan sejawat mereka di institusi lain yang ingin datang berkhidmat di sini. Kedua, kita akan melipat gandakan peruntukan latihan bagi tahun hadapan. Ketiga, kita akan menemuduga kakitangan-kakitangan yang meletak jawatan untuk mengetahui sebab-sebab mereka berbuat demikian, dan keempat, kami bersetuji agar mereka yang telah berkhidmat untuk sekian lama dengan institusi ini diberi pengiktirafan dengan memberi cuti tambahan kepada cuti tahunan mereka. Ya, begitulah keputusan mesyuarat."

Bandingkanlah betapa bezanya 'pendengaran' Ali dan Hassan.

Kita boleh memperbaiki diri untuk mendengar kepada fakta. Yang diperlukan hanyalah sedikit usaha dari kita sendiri dan kita akan berupaya megenang kembali apa yang kita dengar di sesuatu ceramah, mesyuarat, wawancara, persembahan dan lain-lain dengan mengamalkan panduan seperti berikut:

1. Kenalpasti perkataan-perkataan penting yang digunakan.
2. Imbas kembali idea-idea penting di sesuatu seramah atau syarahan.
3. Hadapi sesuatu ceramah atau syarahan dengan sikap terbuka dan persesuaikan diri.
4. Nilai, tetapi jangan abaikan perkara-perkara yang mungkin kita kurang senang mendengarinya.
5. Bersedia mengatasi gangguan sekililing.
6. Bertekun dalam mendengar secara bersungguh.

Keupayaan untuk mendengar kepada fakta-fakta merupakan satu sifat semula jadi kepada segenap lapisan masyarakat. Kepada pelajar ianya boleh menambah kepada pengumpulan dan pengingatan kembali informasi untuk mempertingkatkan pencapaian gred. Kepada pekerja ianya berupaya meyumbang kepada suasana kerja yang lebih efektif di samping menambah kepada kemajuan mereka di dalam menjalankan tugas.

'Mendengar' Kepada Perasaan

Kita hidup dalam masyarakat yang mempraktikan tata susila dan adat sopan timur yang ada kalanya berat untuk kita menyuarakan apa yang terpendam di hati. Ada juga individu yang berterus terang menyuarakan perasaan mereka dengan lebih terbuka. Tetapi secara keseluruhannya kebanyakan dari kita memilih kepada cara pertuturan yang tidak secara langsung (indirect), terutama sekali tentang perkara-perkara yang berupa sensitif dan emosional. Sebagai contoh, seseorang itu akan berkata begini.

> "Tugas yang tuan berikan ini, sungguh mencabar keupayaan saya! Saya berada di sini sejak malam Isnin, Selasa dan menghabiskan sebahagian dari hari Rabu untuk menyiapkan semua tugas sebelum pukul 2.30 petang ini. Mencari data stok untuk lima tahun yang lalu begitu sukar sekali. Tetapi syukurlah semuanya dapat disiapkan dalam jangkamasa yang ditetapkan. Saya akan lebih gembira sekiranya saya tidak diberi tugas seperti ini lagi!"

Sekiranya anda merupakan ketua kepada individu yang menyuarakan ini dan anda terus berkata, "Syabas! awak telah membuktikan kemampuan awak, seterusnya saya meminta awak mengumpulkan pula data jualan untuk jangkamasa yang sama!" Tentu sekali sesuatu akan berlaku kerana tidak menghiraukan 'perasaan' yang diluahkan oleh pererja ini.

Apakah yang sebenarnya disuarakan oleh pekerja melalui 'rayuannya' tentang malam Isnin, Selasa dan sebahagian dari Rabu itu? Ini merupakan 'perasaan' yang tersembungi yang mungkin terdiri dari salah satu sebab berikut:

i) "Berilah saya tahniah"
ii) "Tuan tidak bertimbang rasa dalam memberikan tugas!"
iii) "Tolonglah! Jangan beri saya tugas begini lagi!"
iv) "Tidakah saya diberi bayaran lebih masa?"

Satu fakta yang jelas ialah si pekerja tidak menyuarakan hasrat sebenar secara terus terang. Bagaimanakah kita boleh mengetahui yang mana satu dari empat "perasaan" itu yang sebenarnya?

Andainya sipekerja diberi sanjungan walhal sebenarnya dia mengharapkan ganjaran kerja lebih masa dari majikannya, komunikasi antara anda dengannya juga akan tergendala. Tentu sekali anda tidak akan bertanya, "Apa sebenarnya maksud awak?" Sekiranya boleh, sudah tentu pekerja itu akan memberitahu anda pada mulanya lagi! Tak perlu dia berselindung seperti yang dilakukannya.

Penyelesaian kepada masalah begini terletak kepada silaturrahim anda dengan pekerja sendiri. Sekiranya kita betul-berul teliti dalam 'mendengar', kita boleh memastikan apa dia perasaan yang dicurahkan melalui apa yang tidak dilafazkan.

FAKTA TENTANG "PENDENGARAN"

Terutama sekali, kita perlu memahami apakah yang dimaksudkan tentang "PENDENGARAN".

Ianya lebih dari hanya mendengar. Mendengar begini, hanya merupakan satu bahagian proses pendengaran... satu bahagian fizikal apabila telinga kita menerima gelombong suara. Ada tiga bahagian lain yang tidak kurang pentingnya. Pertama *In-*

terpretasi ke atas apa yang didengar yang membawa kepada kefahaman, atau ketidak fahaman. Kedua *Penilaian* di mana kita menimbang informasi-informasi yang diterima dan membuat keputusan untuk menggunakannya. Ketiga, berdasarkan kepada apa yang kita dengar dan penilaian yang dibuat, kita *mengorak langkah* atau *membuat sesuatu reaksi.* Inilah yang dikatakan PENDENGARAN.

Sebelum kita berupaya menjadi seorang PENDENGAR yang baik, perlu kita mengetahui kenapa manusia berkomunikasi antara satu sama lain. Terdapat EMPAT jenis asas komunikasi lisan:

(i) Jenis komunikasi "pengenalan" atau berhubung untuk menjalin silaturrahim yang dikenal sebagai "phatic communication."

(ii) Cathartic Communication. Iaitu komunikasi yang berupa pencurahan segala perasaan kepada individu yang rela mendengar.

(iii) Komunikasi Maklumat (Information Communication) iaitu jenis komunikasi di mana idea-idea, data dan informasi diperbincangkan.

(iv) Komunikasi galakan (Persuasive Communication) di mana kegunaannya adalah sebagai pendorong untuk mengubah sikap atau bagi menghasilkan sesuatu tindakan.

"Mendengar" merupakan aktiviti utama dalam komunikasi. Kajian menunjukkan kita menghabiskan lebih kurang 80% dari waktu jaga kita untuk berkomunikasi. 45% dari jumlah ini digunakan untuk mendengar. Para pelajar menghabiskan 60% – 70% dari masa mereka dalam darjah untuk "mendengar". Menakala dalam alam perniagaan, "mendengar" dianggap sebagai Kriteria Pengurusan yang paling kritikal dan sangat perlu bagi seseorang Pengurus.

Tabii kita sebagai seorang "pendengar" bukannya hasil dari sebarang fahaman yang diperolehi tetapi sebenarnya adalah kerana hasil dari kurang pembelajaran terhadapnya.

Carta berikut menunjukkan susunan bagaimana EMPAT kemahiran asas komunikasi dipelajari, perenggan ia dipergunakan dan setakat mana ianya diajar. "Pendengaran" merupakan Kemahiran Komunikasi yang dipergunakan secara meluas tetapi merupakan bidang yang tidak dipelajari (rujuk carta yang disertakan).

Dipelajari	PENDENGARAN	PERTUTURAN	PEMBACAAN	PENULISAN
Kegunaan	*Pertama*	*Kedua*	*Ketiga*	*Keempat*
	Paling Tinggi (45%)	Kedua Tinggi (30%)	Kedua Rendah (16%)	Rendah (9%)
Diajarkan	Rendah	Kedua Rendah	Kedua Tinggi	Tinggi

Kebanyakan individu merupakan PENDENGAR yang kurang efisyen. Banyak ujian[3] telah membuktikan bahawa di akhir ceramah yang disampaikan selama 10 minit, purata pendengar yang dapat mengerti, memahami dan berupaya mengingati tentang apa yang dipersoalkan hanya lebih kurang seperdua dari apa yang disampaikan oleh penceramah. Dan dalam jangka 48 jam kemudian, angka ini menurun 50% kepada 25% tahap yang efektif. Dalam ertikata lain, kita biasanya berupaya *menguasai* dan *mengingati* hanya ¼ dari apa yang telah diperkatakan.

Ketidak efisyenan dan pendengaran yang tidak efektif merupakan kos yang tinggi.

Dengan jumlah tenaga kerja yang melebihi 9.3 juta [4] di negara kita, kesilapan pendengaran yang hanya mungkin berharga $1.00 bagi setiap orang akan mengakibatkan kos sebanyak 9.3 juta lebih. Surat-surat terpaksa diulang taip, temubual terpaksa disusun semula, pengiriman barangan dirombak semula dan banyak perkara yang bersangkutan terpaksa disusun kembali.

Cuba bayangkan pula apabila individu di dalam syarikat-syarikat yang besar tidak "mendengar" antara satu dengan lain, kerugian yang akan dihadapi itu bertambah besar. Idea-idea akan disalah tafsir sehingga lari dari idea-idea asal semasa melalui rangkaian arahan. Pekerja-pekerja akan mengalami jurang yang lebih besar dengan majikan dan akan merasakan seolah-olah terpisah dari pucuk pentadbiran.

Cara MENDENGAR yang baik boleh dipelajari. Di beberapa institusi pengajian di mana program MENDENGAR dijalankan, PENDENGARAN di kalangan pengikut-pengikut program ini telah berlipat ganda dalam jangkamasa beberapa bulan sahaja.

MENDENGAR adalah merupakan *perbuatan aktif* seperti PERCAKAPAN, walaupun kebanyakan orang menganggap bahawa tanggungjawab untuk berkomunikasi adalah terletak di pihak sumber. Cuba kita fikirkan sejenak bagaimana kita akan dapat mengujudkan komunikasi yang lebih berkesan jika kedua-dua pihak penyampai dan penerima bertanggungjawab untuk mengambil 51% dari peranan mereka bagi mengujudkan komunikasi yang harmonis.

PROFAIL ANDA SEBAGAI SEORANG "PENDENGAR"

Berikut ialah 3 soalan untuk menduga kedudukan anda sebagai seorang PENDENGAR. Tidak terdapat sebarang jawapan yang betul ataupun salah kepada soalan-soalan ini. Walau bagaimanapun respon anda akan menunjukkan prestasi diri anda sebagai seorang PENDENGAR dan akan mencerminkan skop atau bidang untuk ditokok tambah bagi kebaikan sendiri dan dengan rakan sejawat amnya.

UJIAN # 1

A. Tandakan kategori yang dirasakan tepat sekali yang mencerminkan diri anda sebagai PENDENGAR.

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[3] Ujian terakhir yang saya jalankan adalah terhadap 24 peserta kursus komunikasi untuk setiausaha yang diadakan pada 29–20 April 1986 di UUM.

[4] Ringkasan Penduduk, Tenaga Buruh dan Guna tenaga, 1980 – 85 – RMK.

☐ Terlalu Baik ☐ Baik ☐ Lebih dari sederhana ☐ Sederhana

☐ Kurang dari ☐ Teruk ☐ Paling teruk

B. Dengan menggunakan skel 0 – 100 (100 = paling tinggi), bagaimanakah anda meletakkan diri anda sebagai seorang PENDENGAR.

____

(0 – 100)

UJIAN # 2

Pada fikiran anda bagaimanakah orang-orang berikut meletakkan prestasi anda sebagai seorang PENDENGAR.

Sahabat karib anda

Boss anda

Individu yang lebih rendah kedudukan dari anda

Suami/isteri

____

(0 – 100)

UJIAN # 3

Sebagai seorang PENDENGAR berapa kerap anda mendapati diri anda terbabit di dalam 10 kategori tabii kurang sihat (dari segi pendengar) ini? Pastikan ruang yang berkaitan terlebih dahulu. Kemudain tulis markah mengikut kategori yang disediakan di bawah dan jumlahkan semua.

Tabii mendengar	Kekerapan				Markah
	Terlalu Kerap	Kerap	Kadang-Kadang	Jarang Sekali	Hampir Tiada Langsung
1. Menganggap topik tidak menarik (tiada minat)	______	______	______	______	______
2. Mengkritik tabii atau cara penyampaian penceramah	______	______	______	______	______
3. Terlalu dipengaruhi oleh sesuatu yang diperkatakan oleh penceramah.	______	______	______	______	______
4. Mendengar hanya kepada fakta-fakta yang diperkatakan	______	______	______	______	______
5. Cuba untuk memberi fahaman kepada semua yang diperkatakan	______	______	______	______	______
6. Berpura-pura memberi perhatian kepada penceramah	______	______	______	______	______
7. Terpengaruh oleh kegiatan-kegiatan sampingan yang menggunggu	______	______	______	______	______
8. Mengelakkan diri daripada bahan yang sudah-susah	______	______	______	______	______
9. Peribadi kerap dipengaruhi oleh perkataan-perkataan yang menyentuh secara emosi	______	______	______	______	______
10. Masa kerap terbuang kerana kerap berkhayal	______	______	______	______	______

JUMLAH ==============

Kunci:

Bagi setiap jawapan 'Terlalu Kerap' markahnya ialah	2
Bagi setiap jawapan 'Kerap' markahnya ialah	4
Bagi setiap jawapan 'Kadang-kadang' markahnya ialah	6
Bagi setiap jawapan 'Jarang Sekali' markahnya ialah	8
Bagi setiap jawapan 'Hampir Tiada Langsung' markahnya ialah	10

ANALISIS PROFAIL

Berikut adalah bagaimana orang lain membuat respon kepada soalan-soalan yang sama.

UJIAN # 1

A. 85% dari kesemua "pendengar" yang disoal menganggap diri mereka sebagai "sederhana" atau "kurang". Kurang dari 5% menganggap diri mereka sebagai "Baik"atau "Terlalu Baik".

B. Berdasarkan kepada skel 0 – 100,
gugusan lazim ialah 35 – 85, dan purata keseluruhan ialah 55.

UJIAN # 2

Apabila membuat perbandingan diri sendiri dengan apa yang dianggap akan tabii orang lain, kebanyakan responden percaya bahawa sahabat karib mereka akan memberi markah tertinggi ke atas prestasi mendengar. Markah ini akan melebihi dari apa yang mereka beri pada diri mereka sendiri dalam Ujian 1 ... di mana jumlah puratanya ialah 55.

Mengapa begini? Kita hanya boleh menganggap bahawa pertalian sahabat merupakan satu kaitan di mana hubungan antara satu dengan lain, tidak akan wujud sekiranya seseorang itu tidak dengar mendengari antara keduanya. Sekiranya tidak, mustahil dua insan berkenaan akan bersahabat baik.

Seterusnya, mereka yang mengambil ujian ini, menganggap Boss mereka akan memberi penilaian yang lebih tinggi dari penilaian yang dibuat untuk diri sendiri. Sebahagian dari ini merupakan kebenaran. Kita akan lebih memberi perhatian kepada ketua kita... samada kerana hormat atau segan atau sebagainya tidak menjadi persoalan.

Gred untuk rakan sejawat dan mereka yang ke bawah dari kita akan sama kedudukannya seperti yang kita beri pada diri sendiri... angka 55 berulang lagi.

Tetapi apabila kita melihat kepada isteri atau suami, terdapat sesuatu yang amat ketara. Skor akan lebih rendah dari tahap sederhana 55 yang diperolehi. Yang pasti ialah angka ini akan menurun. Manakala pasangan yang baru berumahtangga akan menganggap (rate) mereka dan teman hidup sama seperti kawan karib, tetapi setelah perkahwinan mereka berjalan lebih lama, anggapan ini akan terus menurun. Oleh itu di dalam satu kelamin di mana pasangan berkenaan telah berkahwin selama 30 tahun misalnya, akan berganda-gandalah percakapan dan perbualan, tetapi mungkin tidak siapa pun yang betul-betul "mendengar".

UJIAN # 3

Markah purata ialah 62... 7 mata lebih dari 55, iaitu jumlah purata yang diberikan oleh responden yang mengambil Ujian I. Ini bererti bahawa apabila "pendengaran" dibahagikan kepada skop-skop tertentu sebagai pengukuran kebolehan, kita akan menganggap diri sendiri lebih jika dibandingkan apabila kita menganggap "pendengaran" secara keseluruhan sahaja.

Cara yang berkesan untuk mengetahui kedudukan kita sebagai "pendengar" ialah dengan meminta pandangan dari orang yang kerap kita "dengari". Isteri/suami, boss, sahabat karib, dan lain-lain.

	10 kunci mendengar cara efektif	*Pendengar tak efektif*	*Pendengar efektif*
1.	Pilih jurusan yang diminati	Meninggalkan terus jika tidak menarik	Segera bertanya "apa yang akan diperolehi?"
2.	Taksir kandungan dan tidak kaedah penyampaian	Undur diri jika tidak bersesuaian	Teliti kandungan, ketepikan kelemahan penyampai
3.	Dengar dan teliti	Terlibat dalam perbalahan	Tidak memberi keputusan sebelum mendengar keseluruhannya
4.	Dengar kepada idea-idea	Mendengar fakta sahaja	Dengar tema utama
5.	Sesuaikan diri	Ambil nota yang banyak	Ambil nota sedikit sahaja, sekadar merujuk
6.	Ambil berat untuk mendengar	Berpura-pura mendengar	Tekun dan tunjukan respon yang aktif

7.	Hindar dari gangguan	Mudah terpesong dan terganggu	Mengelak dari gangguan – tahu untuk konsentrasi
8.	Riadah pemikiran	Mengelakan bahan-bahan susah-pilih yang mudah, yang berupa rekreasi	Cenderung kepada bahan-bahan yang memerlukan daya fikir
9.	Kewarasan dan pemikiran yang terbuka	Bertindak kepada yang emosi	Buat analisis ke atas perkataan tertentu tetapi tidak membiar tinggal sebarang kesan
10.	Kenal pasti fakta – pemikiran melebihi dari apa yang di perkatakan	Mungkin berkhayal jika penceramah lembab	Cabar, membuat ikhtisar, banding dan dengar apa yang tersirat dan juga nada suara

PENUTUP

Terdapat kata-kata hikmat yang mengatakan kita boleh mendapat reputasi sebagai pemidato asalkan kita sanggup menjadi pendengar yang baik. Tetapi selain dari kebenaran kata-kata ini, kita jaga mendapat ganjaran berikut:

1. Informasi yang membantu kita dalam melaksanakan tugas.
2. Buah fikiran bernas yang akan menolong prestasi kerja samada di pejabat atau hubungan kita dengan orang lain.
3. Kefahaman terhadap kehendak seseorang serta menjadi lebih dekat akan peribadinya.
4. Kerjasama. Apabila seseorang merasa anda betul-betul menghayati serta memahami akan masalah yang dihadapinya, individu itu sesebaliknya akan memberi resiprokasi yang serupa kepada anda. Untuk menjadi seorang pendengar efektif terhadap fakta dan perasaan bukannya semudah yang difikirkan. Ia memerlukan sikap sensitif, pengorbanan dan keinginan mendalam untuk berkomunikasi secara efektif dengan orang di sekeliling kita, samada kita bersetuju atau tidak terhadap pendapat mereka. Ganjaran yang diperolehi berpadanan dengan usaha yang dibuat.

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