Advances and Applications in Statistics
© 2021 Pushpa Publishing House, Prayagraj, India
http://www.pphmj.com
http://dx.doi.org/10.17654/AS066020165
Volume 66, Number 2, 2021, Pages 165-174 **P-ISSN: 0972-3617**

ON EFFECTS OF STOCHASTIC VOLATILITY AND LONG MEMORY TOWARDS MORTGAGE INSURANCE MODELS: AN EMPIRICAL STUDY

Mohammed Alhagyan[1,*], Masnita Misiran[2] and Zurni Omar[2]

[1]Department of Mathematics
Community College in Al-Aflaj
Prince Sattam Bin Abdulaziz University
Kingdom of Saudi Arabia
e-mail: m.alhagyan@psau.edu.sa

[2]Department of Mathematics and Statistics
School of Quantitative Sciences
Universiti Utara Malaysia
Kedah, Malaysia
e-mail: masnita@uum.edu.my
zurni@uum.edu.my

Abstract

The change in collateral price is one of the challenges in modeling mortgage insurance. Current work mostly considers collateral price similar to addressing risky asset modelling, in which geometric Brownian motion is being used to model its underlying processes. This assumption has been heavily criticized due to its lack of fundamental dependencies in its distribution. This work provides

Received: November 16, 2020; Accepted: December 20, 2020
2020 Mathematics Subject Classification: 62P05, 97M30.
Keywords and phrases: geometric fractional Brownian motion, stochastic volatility, long memory, mortgage insurance, risk mitigation.
*Corresponding author

empirical investigation towards valuing of loss in mortgage insurance while taking into account the dependency, i.e., memory in its underlying model. Findings suggest that the model with memory and stochastic volatility significantly affects the calculation of loss in mortgage insurance.

1. Introduction

Mortgage insurance aims to protect lenders in the event when the borrower defaults on payments, dies, or are unable to meet the contractual obligation of the mortgage. It mitigates exposure of risk among lenders as this risk is transferred from lenders to insurers. A good mortgage insurance model will contribute to the growth in house financing. Among challenges faced by mortgage insurance models include heavy dependency toward inflation, other risk faced by insurer and unexpected change in collateral prices.

Change in collateral (risky asset) price plays a critical role in the pricing of mortgage insurance contracts since the amount that the insurer has to pay lender significantly depends on the price of collateral. Current literatures assume the changes in collateral prices to follow a geometric Brownian motion (GBM) model, such as in Bardhan et al. [5] and Chen et al. [6], as GBM is known to mimic movement that reflects as likely as possible the dynamics of these prices. However, GBM essentially stems from the random movement of stationary increments in its distribution, thus, fails to acknowledge the presence of any dependencies and correlation (memory) in its dataset. As such, this assumption is appropriate only when the changes in the price of collaterals follow normal distribution with stationary increments.

Scholars such as Mandelbrot and Van Ness [13], Ross [14], Bakshi et al. [4], Chronopoulou and Viens [7, 8], Wang and Zhang [18] and Wang et al. [19] had showed that the use of fractional Brownian motion (FBM) is more suitable to model financial environments. This is because the correlation between the increments that varies consistently with Hurst parameter H that represents memory helps to capture the correlation dynamics (memory) of data and therefore produce better forecasting results. Thus, it is natural to extend the assumption of GBM to GFBM in order to adapt theoretical advantages of such a property.

In this article, we consider GFBM model as underlying process for collateral prices in mortgage insurance model. We also make use of long memory stochastic volatility (LMSV) in GFBM's parameter, which is an additional improvement to constant volatility that is normally used in the literature for easy calculation for GBM/GFBM model. Note that LMSV is considered since GBM model with constant volatility has been under heavy criticism by many empirical studies such as in Bakshi et al. [4], Aït-Sahalia and Lo [1] and Stein [17], as they argue that financial entities considered under this model failed to reflect financial environment.

In response to this drawback, researchers such as Scott [15], Hull and White [12], Stein and Stein [16], Heston [11], Hagan et al. [10], Comte and Renault [9], Chronopoulou and Viens [7, 8] and Wang and Zhang [18] have been cooperating SV in GBM model, while recently, Alhagyan et al. [2, 3] have successfully developed GFBM model with stochastic volatility.

To date, no work is available that investigates collateral prices that follows GFBM model with stochastic volatility, and LMSV. In the following, we provide comparative study among models that are able to produce different results for the calculation of collateral prices.

2. Mortgage Insurance Modeling

In standard mortgage insurance contract, insurer has to pay lender certain amount (*Loss*) if standard default occurs at time t by following the model:

$$Loss(t) = \max\{0, \min(B(t-1) - V(t), L_R B(t-1))\}, \tag{1}$$

where $B(t) = \frac{y}{c}\left(1 - \frac{1}{(1+c)^{T-t}}\right)$ is a loan balance with installment y and mortgage rate c during period T. L_R represents loss ratio while $V(t)$ is a collateral (risky asset) price. Equation (1) implies that if the collateral value is greater than the remaining loan balance, then the insurer will not pay to the lender and then the *loss* is zero, while if the value of the collateral is less than the loan balance, then the maximum *loss* is equal to $L_R B(t-1)$.

In this work, we compute collateral values by using four models including of GBM with constant volatility assumption (GBM-Con), GBM with stochastic volatility assumption (GBM-STO), GFBM with constant volatility assumption (GFBM-Con), and GFBM with stochastic volatility assumption (GFBM-STO). Also, we assume that the stochastic process obeys fractional Ornstein-Uhlenbeck (FOU) process (see Table 1). Thus, we make use of long memory stochastic volatility properties to be included in this empirical study and investigate further the expected loss to lenders. The comparison for insurer's loss is further investigated.

Table 1. The models under consideration

Model	Formula
GBM-Con	$dV_t = \mu V_t dt + \sigma V_t dB(t)$
GBM-STO	$dV_t = \mu V_t dt + \sigma(Y_t) V_t dB(t)$ $dY_t = \alpha(m - Y_t) dt + \beta dB_{H_2}(t)$
GFBM-Con	$dV_t = \mu V_t dt + \sigma V_t dB_{H_1}(t)$
GFBM-STO	$dV_t = \mu V_t dt + \sigma(Y_t) V_t dB_{H_1}(t)$ $dY_t = \alpha(m - Y_t) dt + \beta dB_{H_2}(t)$

In Table 1, $\{V_t;\, t \in [0, T]\}$ represents collateral price process, μ is mean of return, σ is constant volatility, $\sigma(Y_t)$ represents stochastic volatility of stochastic process Y_t that obeys fractional Ornstein-Uhlenbeck process (one of LMSV models). α, β and m represent mean reverting of volatility, volatility of volatility, and mean of volatility, respectively. $B(t)$ is a

Brownian motion, $B_{H_1}(t)$ and $B_{H_2}(t)$ are two independent FBM processes where H_1 and $H_2 > 0.5$.

a. Data

For investigation purposes, we use available data online at http:// www.nationwide.co.uk. This data represents total house price index in UK. The quarterly house price indices from fourth quarter of 1973 (4Q73) to first quarter of 2017 (1Q17) are considered with total observation of 174 quarters. These data reveal long time dependency with Hurst parameter of $H = 0.85$. The return series are calculated in logarithm to avoid high volatility in the data. Figure 1 and Figure 2 show the house price index and its return series, while Table 2 represents the parameters' values considered in this empirical investigation.

Table 2. Involved parameters' values

Parameter	μ	α	m	β	H_1	H_2	σ	$\sigma(Y_t)$
Value	0.01782	0.6936	0.00066	0.00115	0.8538	0.8541	0.02586	0.000659

σ: constant volatility $\sigma(Y_t)$: stochastic volatility

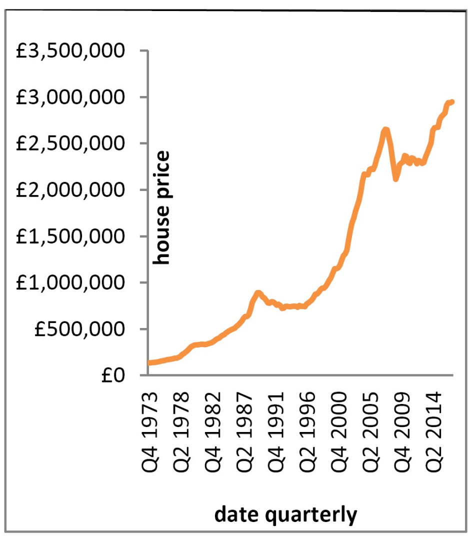

Figure 1. Quarterly house price index in UK from 4Q73 to 1Q17.

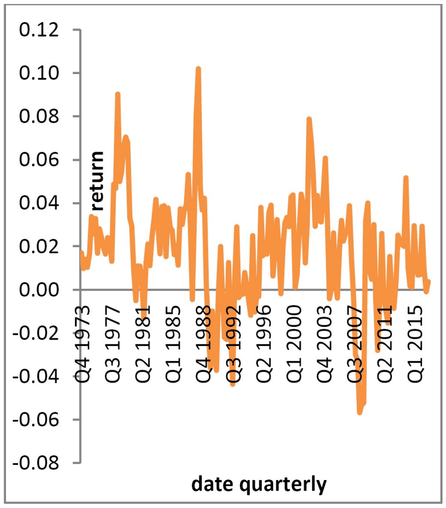

Figure 2. Quarterly return of house price index in UK from 4Q73 to 1Q17.

The annual insurer's potential loss is computed by following parameters: insured property, $V_0 = £100000$; annual installment, $y = £15000$; mortgage rate, $c = 0.042$; loss ratio, $L_R = 0.75$; and time period, $T = 15$ years.

b. Valuing insurer's potential loss

Table 3 shows the computed values of loan balance, collaterals via different models in Table 1 and their corresponding insurer's potential loss in equation (1). Figure 3 illustrates the level of computed potential losses for the first six years. Potential losses after six years happen to be zero.

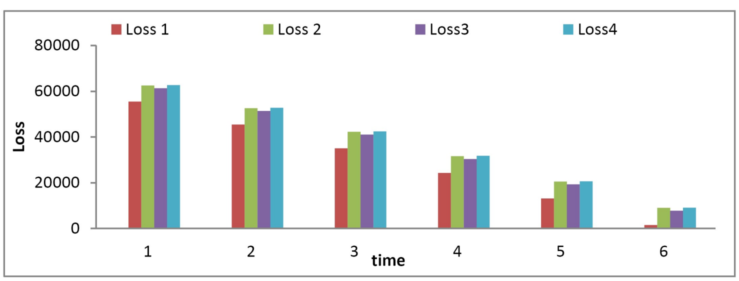

Figure 3. Comparison between the levels of potential losses in the first six years.

Table 3. Collateral values and their corresponding potential loss

Time t	$B(t-1)$	$V_1(t)$	$V_2(t)$	$V_3(t)$	$V_4(t)$	$Loss_1(t)$	$Loss_2(t)$	$Loss_3(t)$	$Loss_4(t)$
1	164467	108997	101976	103242	101835	55470.3	62491.3	61225.1	62632.1
2	156375	110920	103810	105063	103666	45455.3	52565.4	51311.6	52708.8
3	147943	112876	105676	106917	105530	35066.5	42266.7	41026.1	42412.7
4	139156	114868	107576	108803	107428	24289.0	31580.3	30353.8	31728.9
5	130001	116894	109510	110722	109359	13107.4	20490.7	19279.1	20642
6	120461	118956	111479	112675	111325	1505.47	8981.84	7786.13	9135.82
7	110520	121054	113484	114662	113327	0	0	0	0
8	100162	123189	115524	116685	115364	0	0	0	0
9	89369.1	125362	117601	118743	117439	0	0	0	0
10	78122.7	127574	119715	120838	119550	0	0	0	0
11	66403.8	129824	121868	122969	121699	0	0	0	0
12	54192.8	132114	124059	125139	123888	0	0	0	0
13	41468.9	134444	126290	127346	126115	0	0	0	0
14	28210.6	136816	128560	129592	128383	0	0	0	0
15	14395.4	139229	130872	131878	130691	0	0	0	0

$V_1(t)$: collateral computed by GBM-Con $\quad$ $Loss_1(t)$: potential loss corresponding to $V_1(t)$

$V_2(t)$: collateral computed by GBM-STO $\quad$ $Loss_2(t)$: potential loss corresponding to $V_2(t)$

$V_3(t)$: collateral computed by GFBM-Con $\quad$ $Loss_3(t)$: potential loss corresponding to $V_3(t)$

$V_4(t)$: collateral computed by GFBM-STO $\quad$ $Loss_4(t)$: potential loss corresponding to $V_4(t)$

$B(t-1)$: loan balance at time $t-1$

Table 3 reveals an inverse relationship between collateral values and their potential losses, i.e., as collateral value increases, the loss decreases. In the first six years, the value of loan balance $B(t-1)$ is greater than the computed values of collateral computed by $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_4(t)$. Thus, the insurers have to pay a certain amount equal to $Loss_1(t)$, $Loss_2(t)$, $Loss_3(t)$ and $Loss_4(t)$ corresponding to collateral values $V_1(t), V_2(t), V_3(t)$ and $V_4(t)$, respectively. In the seventh year onwards, collateral values are greater than loan balances. Thus, insurer's loss equals to zero.

The proposed model of GFBM-STO provides the greatest value of insurer's loss $(Loss_4(t))$, while GBM-Con provides the smallest value of insurer's loss $(Loss_1(t))$. These findings imply two perspectives, from the insurer and the loaner. From the insurer's perspective, the loss computed by GBM-Con is the best, while the loss computed by GFBM-STO is the worst, while from the loaner's perspective, the loss computed by GFBM-STO is the best, while the loss computed by GBM-Con is the worst.

Table 3 also showed a significant difference between the potential loss computed via GBM-Con, and the other three potential computed losses (GBM-STO, GFBM-Con and GFBM-STO). These results reflect the level of affection of memory and stochastic volatility assumption on potential loss of insurer. Therefore, we strongly recommended taking memory and stochastic volatility into account in mortgage insurance contracts.

3. Discussion

We computed the collateral values using four models listed in Table 1. After, we computed the corresponding potential loss of insurer, the findings have been summarized into Table 3 and Figure 3.

The findings showed that the insurer's loss computed by using GFBM-STO model provides the largest value while GBM-Con provides the least. These results can be read from two viewpoints - insurer's viewpoint and loaner's viewpoint. From the insurer's viewpoint, the loss calculated by GBM-Con is the best, and the loss calculated by GFBM-STO is the worst, while from the loaner's viewpoint, the loss calculated by GFBM-STO is the best, and the loss calculated by GBM-Con is the worst.

The findings indicate a significant difference between the potential loss calculated depending on GBM-Con and the other three potential losses calculated depending on GBM-STO, GFBM-Con and GFBM-STO. These results reveal the level of affection of memory and stochastic volatility assumption on potential loss of insurer. Consequently, we strongly recommend taking memory and stochastic volatility into account in mortgage insurance contracts.

Acknowledgement

This publication was supported by the Deanship of Scientific Research at Prince Sattam Bin Abdulaziz University.

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