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An Application Of Burrxii-Dal And Weibull-Dal Distributions To Investigates Bank Customers Waiting Time

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ABSTRACT

Background: Data distribution is highly complex and needed a more authentic method to determine by using different distribution models. BURRXII-DAL and WEIBULL-DAL distributions are one if the main models that are used.

Objective: The purpose of the study is to determine application of BURRXII-DAL and WEIBULL-DAL distributions to investigates bank customers waiting time by using different parameters and there comparison with other models.

Methods: BurrXII-DAL and W-DAL distributions each with five parameters are two new lifetime models created from distributions of Burr-G and Weibull-G family. “Burr-XII DAL was compared to six models, which included Kumaraswamy generalised power Weibull (KwGPW, BENH, EGPW, GPW and NH), Beta Exponentiated Nadarajah Haghighi (BENH) and Exponentiated Generalized Power Weibull (EGPW)”. “Weibull -DAL's fit to Weibull Dagum (WDa), Weibull power function (WPF), Weibull Lomax (WLx), generalised power Weibull (GPW), Nadarajah Haghighi (NH), Beta Weibull (BW), and Kumaraswamy Weibull (KwW) distributions”. “One genuine data set is used to test the efficiency of distribution”. “R packages like BFGS (Broyden-Fletcher-Goldfarb-Shanno), SANN (Simulated-Annealing) and NM (Nature-Methods-Methods) are used to process MLEs and the goodness of fit measures containing statistics Criterion such as Akaike Information (AIC), Bayesian Information (BIC), Anderson-Darling (A*), Cramer–von Mises (W*), and K-S statistics (Nelder-Mead)”.

Result: “The results of criteria’s including both Akaike information (AIC) and Bayesian information (BIC) of each of the six models showed that the predicted Burr-XII DAL is as best as the other models comparatively”. MLE using Weibull-DAL are small enough with smaller standard errors in variation among original and fitted values of model in presence of these parameters.

Conclusion: The parameters generated using MLE and goodness of fit techniques have produced Weibull-DAL as the best distribution among all distributions in term of parameters of model estimating.

Keywords: Beta Weibull, Burr-G family distribution, Maximum likelihood estimation, Weibull distribution, Weibull-G family distribution.

INTRODUCTION

Modeling data is highly beneficial in describing the distribution as it provide the complete understanding about the parameters, their nature, their functions, failure rate (or hazard) and how the parameters become fully survive in probability distribution and according to its survival analysis. In the past few years, there are multiple lifetime models developed and significant amount of research are done in order to determine risk rates and risk rate variations. Adamidis and Loukas (1998) reported exponential geometric (EG) distribution is a distribution that consists on two-parameter lifetime that focused on the compounding exponential and geometric distributions according to their decreased rate of failure. "On the other hand, Kus (2007) and Tahmasbi and Rezaei (2012) introduced the Exponential Poisson (EP) and exponential logarithmic distributions (2008). The exponential power series (EPS) distributions created by Chahkandi and Ganjali (2009) represent an entirely new class of distributions".

Burr (1942) was the first to bring the Burr type XII distribution into the literature, and it has grown in popularity in the last twenty years or more. This is due to its wide range of applications in sectors such as dependability, failure time modelling, and acceptability sampling plans, to mention a few. Burr made twelve alternatives for $\varphi(x)$ resulting in twelve distributions that completely used for fitting data. "To determine point and interval estimates of the Burr XII distribution parameter values, Wang and Keats (1996) employed the maximum likelihood method (MLM). Abdel-Ghaly et al. (1997) utilised the Burr type XII distribution to evaluate software dependability". For the Burr type XII distribution, Wu and Yu (2005) gave key values that were utilised to evaluate the shape and confidence range for the shape parameters. A technique for estimating the empirical reliability of the Burr type XII distribution using maximum likelihood was first proposed by Li et al. in 2007, but it has since been improved by Li et al. utilising the maximum likelihood approach to build variable point estimators (2007). The ratio and inverse moments of the Marshall Olkin distribution, an enlarged type XII Burr distribution, are shown to rely on lower generalised order statistics, according to Kumar (2017)'s study of the statistical properties of the type XII Burr distribution.

Abdel-Hamid (2009) and Singh and Shukla (2009) examined Burr XII (B- XII) distribution with type II filtering with the help of partially accelerated life tests (ALT) that remains constant (2017). In 2018, research by Afify et al. studied the Weibull BXII distribution. Cordeiro et al. in 2018 proposed a double BXII model with forty special cases. Topp Leone produced B-XII distribution was suggested by Yousof and colleagues (2017). Weibull-DAL distributions with five parameters are studied in Shahbaz et al. (2021). "It was found that the maximum likelihood estimation strategy and two Bayesian estimation procedures were used on constant-stress ALT samples from the doubly B-XII distribution with three parameters”. Two Bayesian approaches have been presented to determine the models under normal circumstances for lifespan parameters and quantiles since the maximum likelihood estimation method is computationally intensive.

Weibull distribution is a continuous probability distribution that was pioneered and is an appropriate distribution to a particular field (Weibull, 1951). The purpose of the study is to determine the usage of BURRXII-DAL and WEIBULL-DAL distributions to investigate bank customers waiting time. Previous studies described that Weibull distribution easily implemented to version physical characteristics of structures including failure times, hazard rate, fluctuation in functions and tolerates reduction, decrease-increase reducing and inverted bathtub shapes are highly beneficial for complex system or conditions. Still, the modified five parameters BurrXII-DAL and Weibull-DAL (WDAL) distribution are not used so this study focus on their application as an appealing substitute for modeling distributions, along with other applications for investigating bank customers waiting time as it complex to determined.

Method and Material:

Burr XII-DAL distribution

“Evaluation of the parameters of the model is carried out using the maximum likelihood technique and four metrics of goodness-of-fit”. The suggested Burr XII-DAL model's performance was compared to that of the Kw GPW, BENH, EGPW, GPW, and NH distributions. The BXII-DAL model's probability was estimated using a real-world dataset. To examine the usefulness of the model, we explored its AIC in comparison with all the other models and their goodness of fit which can be determined by inserting the parameters that can provide maximum likelihood of function. Thus, MLE has a larger significance while standard errors are estimated for each parameter value to examine variation among original and fitted values.

“Table 1 lists the parameter;s MLEs values of the model, whereas Table 2 lists the goodness of fit values for fitted models”. The densities of the competing models are calculated as follows:

$$f_{KwGPW}(x) = ab\alpha\beta\lambda x^{\beta-1}(1 + \lambda x^{\beta})^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left(1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right)^{\alpha-1} \times \left[1-\{1-\exp\left(1-[1+\lambda x^{\beta}]^{\alpha}\right)\}^{a}\right]^{b-1} \quad ; \ a,b,\alpha,\beta,\lambda > 0. \quad \ldots\ldots\ldots(1)$$

$$f_{BENH}(x) = \frac{\alpha\beta\lambda}{\beta(a,b)}\left(1+\lambda x^{\beta}\right)^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\alpha\beta-1} \times \left[1-\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\beta-1}\right]^{b-1} ss \quad ; \quad a,b,\alpha,\beta,\lambda > 0. \quad \ldots\ldots.(2)$$

$$l(\theta) = n\log(\alpha\lambda) - n\log[B(a,b)] + \left(1-\frac{1}{\alpha}\right)\sum \log((1+\lambda x)^{\alpha}) + (a-1)\sum \log\left(1-e^{1-(1+\lambda x)^{\alpha}}\right) + b\sum 1-(1+\lambda x)^{\alpha}$$

The MLE for all parameters are included below by differentiating above equation.

$$U_a = \frac{\partial l}{\partial a} = -n\psi(a) + n\psi(a+b) + \sum \log\,(1-e^{1-(1+\lambda x)^{\alpha}})$$

$$U_b = \frac{\partial l}{\partial b} = -n\psi(b) + n\psi(a+b) + \sum 1-(1+\lambda x)^{\alpha}$$

$$U_{\alpha} = \frac{n}{\alpha} - \frac{1}{\alpha^2}\sum \log((1+\lambda x)^{\alpha}) - \left(1-\frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha}}{(1+\lambda x)^{\alpha}} + (1-\alpha)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha} e^{1-(1+\lambda x)^{\alpha}}}{1-e^{1-(1+\lambda x)^{\alpha}}} + b\sum (1-(1+\lambda x)^{\alpha})^{\alpha}$$

$$U_{\lambda} = \frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \left(1-\frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\lambda}}{(1+\lambda x)^{\alpha}} + (1-a)\sum \frac{(1-(1+\lambda x)^{\alpha}) e^{1-(1+\lambda x)^{\alpha}}}{1-e^{1-(1+\lambda x)^{\alpha}}} + b\sum (1-(1+\lambda x)^{\alpha})^{\lambda}$$

$$f_{EGPW}(x) \quad = \alpha\beta\lambda\theta x^{\beta-1}\left(1+\lambda x^{\beta}\right)^{\alpha} \exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\theta-1}$$

$$a, b, \alpha, \beta, \lambda > 0. \qquad \ldots\ldots\ldots(3)$$

$$f_{GPW}(x) \quad = \quad \alpha\beta\lambda x^{\beta-1}(1+\lambda x^{\beta})^{\alpha} \exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad ; \quad x, \alpha, \beta, \sigma > 0 \quad \ldots\ldots\ldots(4)$$

$$f_{NH}(x) \quad = \quad \alpha\lambda(1+\lambda x)^{\alpha} \exp(1-[1+\lambda x]^{\alpha}) \quad ; \quad x, \alpha, \lambda > 0 \quad \ldots\ldots\ldots(5)$$

Data: Customers Waiting time at a Bank

Based on a data set of customer wait times from Aldeni and colleagues, 2017 the suggested model was evaluated. Customers of 100 banks were analysed using the following parameters:

0.79	0.81	1.29	1.48	1.76	1.89	1.88	2
2.5	2.45	2.88	3.12	3.1	3.4	3.7	3.8
4.2	4.2	4.3	4.4	4.2	4.2	4.5	4.4
4.7	4.8	4.79	4.9	4.8	4.71	5.2	5.1

5.5	5.7	5.7	6.1	6.0	6.1	6.1	6.4
6.6	6.9	7.1	7.3	7.2	7.2	7.3	7.3
7.9	8.2	8.3	8.4	8.4	8.5	8.5	8.6
8.8	8.9	9.4	9.5	9.5	9.7	10.0	10.3
10.6	10.8	10.9	11.0	11.1	11.2	11.3	12.5
12.6	12.8	13.0	13.2	13.4	13.6	13.6	13.1
14.9	15.3	15.1	17.1	17.1	18.3	18.1	18.7
18.8	19.2	19.5	20.3	21.4	21.8	21.0	28
28.9	30.9	32.4	36.9.				

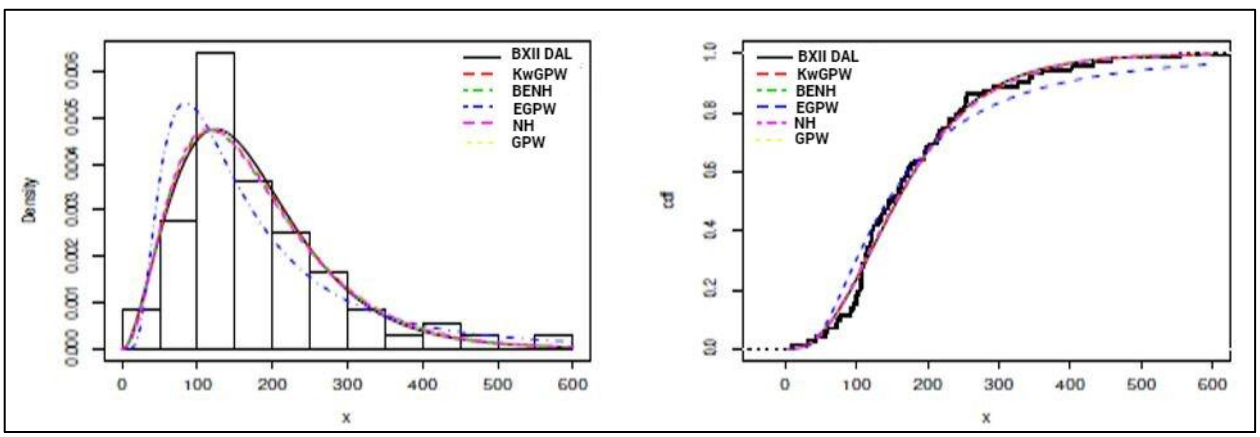
Table 1: Parentheses for MLEs and its standard errors:

Distributions	**a**	**b**	α	β	λ	**θ**
BXII-DAL	0.0888	0.5542	2.4542	2.0268	3.5318	-
	(-)	(0.6113)	(0.0039)	(0.2130)	(0.0096)	-
Kw GPW	0.8100	0.9414	5.4669	0.9465	0.6390	-
	(0.8084)	(1.7563)	(1.4121)	(0.2669)	(0.0041)	-
BENH	2.1615	0.5271	1.3655	1.4763	0.0096	-
	(1.7161)	(0.8944)	(0.8190)	(1.3146)	(0.0036)	-
EGPW	-	-	0.0071	0.9065	0.0939	.9928
	-	-	(0.0246)	(4.2118)	(0.0039)	(6.8421)
GPW	-	-	1.84544	0.16970	1.6492	-
	-	-	(0.0037)	(8.6113)	(-)	-
NH	-	-	0.6034	-	1.9987	-
	-	-	(0.2130)	-	(0.0001)	-

Table 2: "W*, A*, K-S, p-value, AIC, BIC and l^statictics":

Distribution	W*	A*	K-S	p-value(K-S)	AIC	BIC	Î
BXII-DAL	.05711	.0787	0.0878	0.6351	801. 606	812. 817	425.6653
Kw GPW	0.1290	0.7799	0.0744	0.482	827.1284	832.8167	411.5642
BENH	0.3591	0.7425	0. 222	0.5 2 2	892.78	884.2554	439.394
EGPW	0.5558	0.1361	3.959	0. 7941	942.608	942.051	471.804
GPW	2.6703	14.568	1	0.0000	1113.71	1105.15	559.8552
NH	0.4115	2.6060	0.7675	0.0000	859.1497	860.8533	296.074

Figure 1: BXII-DAL Plots:



(a) (b)

Weibull-DAL distribution

It's important to note that the W-DAL model outperforms several other popular ones, including Weibull Dagum and the generalised power Weibull as well as the Nadarajah Haghighis, the Beta

Weibull and the Kumaraswamy Weibull distributions. This is done to show how useful the W-DAL model is. According to and, respectively, the densities of the two opposing models.

$$f_{WDa}(x) = ab\alpha\beta\lambda x^{\beta-1}\left[\frac{\left(1+\lambda x^{-\beta}\right)^{-ab-1}}{\{1-(1+\lambda x^{-\beta})^{-\alpha}\}^{b+1}}\right]\exp\left[-a\left\{\frac{\left(1+\lambda x^{-\beta}\right)^{-\alpha}}{1-(1+\lambda x^{-\beta})^{-\alpha}}\right\}^{b}\right] \quad \ldots\ldots\ldots..(6)$$

$$a, b, \alpha, \beta, \lambda, x > 0.$$

$$f_{WPF}(x) = ab\alpha^{-\beta}\beta x^{\beta-1}\left[\frac{\left\{\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b-1}}{\left\{1-\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b+1}}\right]\exp\left[-a\left\{\frac{\left(\frac{x}{\alpha}\right)^{\beta}}{1-\left(\frac{x}{\alpha}\right)^{\beta}}\right\}^{b}\right] \quad , \ a, b, \alpha, \beta, x > 0. \ldots\ldots(7)$$

$$f_{WLx}(x) = ab\alpha\beta^{-1}(1+\frac{x}{\beta})^{-\alpha-1}\left[\frac{\left\{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right\}^{b-1}}{\left\{\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\right\}^{b+1}}\right]\exp\left[-a\left\{\frac{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}{\left(1+\frac{x}{\beta}\right)^{-\alpha}}\right\}^{b}\right] \quad , \ldots.(8)$$

$$a, b, \alpha, \beta, x > 0.$$

$$f_{GPW}(x) = \alpha\beta\lambda x^{\beta-1}\left(1+\lambda x^{\beta}\right)^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad , \quad x, \alpha, \beta, \lambda > 0 \ldots\ldots\ldots(9)$$

$$f_{NH}(x) = \alpha\lambda(1+\lambda x)^{\alpha}\exp(1-[1+\lambda x]^{\alpha}) \quad , \quad x, \alpha, \lambda > 0. \ldots\ldots\ldots(10)$$

$$f_{BW}(x) = \frac{\alpha\beta x^{\beta-1}\exp\left(-\alpha x^{\beta}\right)}{\beta(a, b)}\left[1-\exp\left(-\alpha x^{\beta}\right)\right]^{\alpha-1}\left[\exp\left(\alpha x^{\beta}\right)\right]^{b-1} \quad , \ldots\ldots(11)$$

$$x, a, b, \alpha, \beta, \lambda > 0$$

$$f_{KwW}(x) = ab\alpha\beta x^{\beta-1}\exp\left(-\alpha x^{\beta}\right)\left(1-\exp\left[-\alpha x^{\beta}\right]\right)^{a-1}\left[1 - \left\{1-\exp\left[-\alpha x^{\beta}\right]\right\}^{a}\right]^{b-1} \quad , \ldots\ldots\ldots(12)$$

$$x, a, b, \alpha, \beta > 0$$

Data: Customer's Waiting time at a Bank

This data represents the bank data was retrieved from a research paper of Farooq et al., (2021).

Table 3 includes the model parameter's ML estimates for the variables, as well as the related SEs (in parenthesis) and goodness of fit statistics.

Table 3: "Parentheses of MLEs along standard errors":

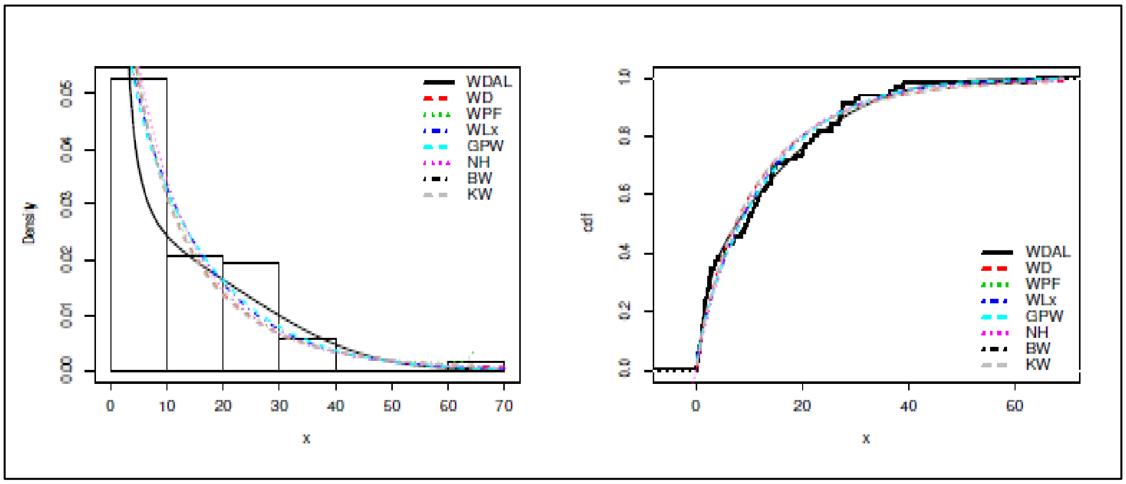
Distribution	α	β	**a**	**b**	λ
W-DAL	.61069	.1069	0.9999	0. 9792	0.9999
	(0.217)	(….)	(0.2766)	(7.5075)	(0.2892)
BW	0.0495	1.2931s	0.5518	0.4976	-
	(0.1235)	(0.4504)	(0.2766)	(0.6416)	-
KwW	1.1833	0.8240	1.3778	0.1196	-
	(0.4547)	(0.0408)	(0.3988)	(0.0020)	-
WDa	0.1908	2.8688	0.1487	0.2999	1.0119
	(0.0025)	(0.2557)	(0.3122)	(0.4283)	(7.9938)
WPF	65	10.8374	4.5937	0.0820	-
	(-)	(9.7206)	(0.7439)	(0.0723)	-
WLx	1.2757	3.1889	0.2320	0.7920	-
	(1.2514)	(3.4718)	(0.6218)	(0.3270)	-
GPW	2.9113	0.7511	-	-	0.0403
	(4.0503)	(0.1359)	-	-	(0.0540)
NH	0.8412	-	-	-	0.1094
	(0.2600)	-	-	-	(0.0597)

Table 4: "W^*, A^*, K-S, p-value, AIC, BIC and Î statistics".

Distribution	**W^***	**A^***	**K-S**	**p-value (K-S)**	**AIC**	**BIC**	**Î**
W-DAL	0.25761	.0348	1	0.000	869.302	860.746	937.651
BW	.103	0.634	0.107	0.376	509.959	519.066	250.980
KwW	0.147	0.816	0.093	0.567	509.878	518.984	250.939
Wda	.	.	1.0162	0.000	851.20	859.75	422.60
WPF	0.131	0.754	0.103	0.434	506.469	513.299	250.610

WLx	0.126	0.740	0.107	0.377	510.792	519.899	251.396
GPW	0.106	0.638	0.106	0.393	507.831	514.661	250.916
NH	0.144	0.817	0.124	0.215	507.975	512.528	251.987

Figure 2: Plots of estimated pdf (a) and cdf (b) of W-DAL



(a) (b)

Result:

The study focused on the applications of BXII-DAL model and W-DAL distribution in determining the waiting time of bank customer along with its distribution and comparison with other models.

Table (I) shows the MLE and standard error of BXII-DAL model with other comparative models. Based on the table, which BXII-DAL performs better than the other models. This is because of it estimates larger values of parameters which in turn produces larger value of log likelihood function that predicts any model as best fit. The parameters value obtained in Table I indicated that the waiting time of customers at the bank would be maximized for larger values of parameters which are then used for comparison of goodness of fit of the models. The comparison was made using Kolmogrov Smirnov test. In addition, waiting time of customers in banks has found to have smaller variation or in other words the customers have almost similar waiting time or in other words no demographic factor will than cause an impact.

The value of AIC, A^* and W^* for BXII-DAL statistic are displaying in Table II. These are the smallest among the fitted models which predicted accuracy of the model is high. AIC of BXII-DAL model was reduced from the earlier value of 1113.71 to the final value of 801.606. Moreover, BIC of BXII-DAL model was reduced from the earlier value of 1105.15 to the final value of

812.817, which indicated accuracy of the model. Hence it is concluded that the proposed BXII-DAL model is best model among the fitted models.

The probability and cumulative distribution function of the proposed model, BXII-DAL was described in graphical representation in Figure 1 showed that The BXII-DAL follows real data set as compared to other models. This proved that the proposed model is effective. Furthermore, for cumulative distribution, the curve of BXII-DAL shows a little variation in the data of waiting time of the customers at the banks as compared to the other models while it follows an increasing pattern estimating the original waiting time of the customers.

The MLE and standard error of W-DAL model with other comparative models according to Table (III) showed that the which shows that W-DAL is better than the other models because of the parameters estimated using MLE method produced larger value of log-likelihood function.

The value of W-DAL statistic for AIC, $A^{*}$and W^{*}in Table IV displayed that AIC, $A^{*}$and W^{*} is the smallest among the fitted models that leads to model accuracAIC of W-DAL model was reduced from the earlier value of 511.399 to the final value of 505.395. Moreover, BIC of this model was reduced from the earlier value of 522.783 to the minimum value of 516.778, which indicate significant improvement in the model fit. This is because of it has the least value. Hence it is concluded that the proposed W-DAL model is best model among the fitted models.

Similarly; Figure 2 describe the graphical probability and cumulative distribution function of proposed model W-DAL. The waiting time of customers at a bank is well observed by W-DAL model because the fitted values by this distribution are nearly equal to the observed values in best way as compared to the other models. Furthermore, for cumulative distribution, the curve W-DAL predicts an increasing pattern estimating the data set of waiting time of customers at a bank.

Discussion:

In the current study, one real-life data set which is waiting time of customers at a bank are compared to the suggested five-parameter Burr XII-DAL and Weibull-DAL models for illustration purposes. For each model, maximum likelihood estimates (MLEs) are obtained. As goodness of fit measurements, the statistics l(.), AIC, BIC, A*,W*, K-S, and p-value (K-S) are utilised. The MLEs are estimated using the iterative method BFGS (Broyden–Fletcher–Goldfarb–Shanno) using the R-language and R-package Max Lik and Adequacy Model.

Here, current study suggest five Burr XII-Dal and Weibull-Dal distributions. There are a few unique cases discussed. In order to analyse structural components features in complete generality, we propose a mixed representation in terms of Burr and Weibull distributions. Current study results can be readily adapted for any baseline distribution to obtain the major structural aspects of the produced distribution.. Zhang et al. (2016) reported that these models are highly useful in determining the behavior of characteristics, failure data along with increased and decreased bathtub shape rate of failure. This support our study as the models efficiently describe the parameters with greater efficiency due to its flexible nature (Zhang ,2016).

The technique of maximum likelihood is used to calculate the model parameters, and the bootstrap method is used to calculate the biases and standard errors of the MLEs. Nawaz el at. (2021) concluded that Weibull-DAL distribution and modified form are highly be used for the estimation of MLEs as it is necessary to gerenarted for every modle as it helps in better analysis of distribution. Study also reported that MLE authentically assessed by utilizing likelihood method. This study highly supported current study results as likelihood providing better estimation of each parameter (Nawaz, 2021).

Thus, for data set, Burr XII-DAL and Weibull-DAL appeared to be more effective distributions. Hence, we can say that these two distributions are relatively better than the previous distributions. The usefulness of these models appeared for this data set because they predict the failure rate more appropriately than other distributions due to presence of scale parameter which was lacking in previous distributions. In addition, its hazard rate functions are much improved as compared to previous distributions due to various shapes that may be modeled in complex situations. Another important reason behind using these models is that they are more flexible and variation in reliability curve.

Conclusion:

BurrXII-DAL and Weibull-DAL models are excellent across competitive models to the data sets, according to the current study. Both recommended distributions have the capability to be used in a broader variety of reliability analysis techniques.

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