

Modelling the Error Term of Australia Gross Domestic Product

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Abstract: The main aim of this study is to model the Gross Domestic Product (GDP) with the new Combine White Noise (CWN) Model and compare the results with the Vector Autoregressive (VAR) Model and Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) Model which are the existing models. The CWN model estimation yields best results with least information criteria and high log likelihood values. While the EGARCH model estimated yields better results with least information criteria and high log likelihood values when compared with VAR model. CWN has the least forecast errors which are indications of best results when compare with the EGARCH and VAR models, dynamic evaluation forecast errors. The minimum forecast error values indicate forecast accuracy. The determinant of the residual of the covariance matrix value indicates that CWN is efficient, while the determinant of the residual of the covariance matrix value indicates that VAR is not efficient. The total results testify that CWN is the most right model. To model the data that exhibit conditional heteroscedasticity with leverage effect in Australia and other societies in the world efficiently, CWN is recommended.

Keywords: Combine White Noise, Determinant of the Residual of Covariance, Efficient, Forecast Accuracy, Log Likelihood

Introduction

The main aim of this study is to model the Gross Domestic Product (GDP) with the new Combine White Noise (CWN) Model which is considered as the most suitable for data that exhibits stochastic time series errors (heteroscedastic errors). GDP is the total value of all the goods and services produced within a country's borders in a given time. Thus, GDP is an indicator of the economic health of a country and is a gauge of a country's standard of living (Hubbard and O'Brien, 2012). When the standard of living is high, it determines the economic health of the nation and reflects the well-being of the citizens. Economic growth is the increase in the market value of the goods and services produced by an economy over time. Economic growth, measured as a change in the GDP as defined in Hubbard and O'Brien (2012).

In order to have a good measure of the standard of living in a society, there is a need for a suitable model that will yield better results for suitable forecast and policy making. Error term described the errors exhibited by the empirical model (Qin and Gilbert, 2001). The actions of the error term in the stochastic time series rely mostly on the data size and high data frequencies. The error term named white noise or heteroscedasticity which rely on the type of data. The error term in VAR model reveals white noise errors, while GARCH family models show heteroscedastic errors.

Sims (1980) introduces Vector Autoregressive (VAR) model for modeling the white noise errors appropriately and overcome the weaknesses of simultaneous system of equations model for unsuitable evaluation of the error term (white noise errors). VARs have implements that are easy for estimation, structure inference and forecasting that serves suitably in policy

$$= Q\varepsilon_t \\ = U_t \quad (2.7)$$

It can be written as:

$$Y_t = U_t, (U_t \sim N(0, \sigma_c^2)) \quad (2.8)$$

where, $A(L) + B(L) + \dots = Q$ which are the matrix polynomial, U_t is the error term of combine white noise model and σ_c^2 is the combination of equal variances.

The combine variances of the combine white noise is:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \dots \quad (2.9)$$

Considering the best two variances in the best two models produced by the Bayesian model averaging output. The combine variance follows:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \quad (3.0)$$

The variance of errors, σ_c^2 in the combine white noise can be written:

$$\sigma_c^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2\rho W\sigma_1(1-W)\sigma_2 \quad (3.1)$$

where the balanced weight specified for the model is W . The least of σ_c^2 appearing, when the equation is differentiated with respect to W and equate to zero, obtaining:

$$W = \frac{\sigma_c^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (3.2)$$

where, ρ is the correlation; intra-class correlation coefficient is used for a reliable measurement.

Results

The data time plot reveals a behavior of non-stationary trending. The data is transformed in returns series to examine the volatility clustering, long tail skewness and excess kurtosis which are the features of heteroscedasticity. The graph exhibits unequal variances that suggest volatility.

Table 1 reveals that there is right tail skewness, excess kurtosis and Jarque-beta test is significant with the signification of non-normality. The standard deviation is slightly greater than one.

In Table 1 ARCH LM tests show that F-Statistic and Obs*R-squared is significant; there is no ARCH effect in the data.

Table 2 shows that EGARCH model is choosing among the GARCH family because it has the least

values of AIC, BIC and HQ with high log-likelihood values, this connotes that EGARCH is the best model for further computation.

To overcome the challenges of heteroscedasticity with leverage effect, the standardized residuals graph of the EGARCH model (EGARCH errors) with unequal variances and zero mean is computed into equal variances series (white noise series). Graphs of equal variances (white noise series) with mean zero are obtained from the graph of EGARCH errors. Then, white noise series are fitted into the regression model to get white noise models (Agboluaje *et al.*, 2015).

The performance of Bayesian model averaging reveals two best models from the first grouped best models (Asatryan and Feld, 2014). For authentications, fit linear regression with autoregressive errors and the number of observation, with zero mean and variance one (Higgins and Bera, 1992). The outcome reveals that the best two models are the white noise models.

CWN has the least information criteria with high log likelihood values to get the best results when compare with EGARCH and VAR model estimation. The estimation of EGARCH model and CWN model with their forecasting values is stated in Table 4.

Table 3 testifies that an independent samples test is experimented to test whether a data set of the two white noise models have equal variances or not. The test reveals that the inconsistency in the distribution of the two data sets is no significantly different value which is greater than the p-value 0.05. As a result the two models have equal variances (Lim and Loh, 1996; Boos and Brownie, 2004; Bast *et al.*, 2015).

In Table 4: Stability test reveals that CWN and EGARCH are stable, but VAR is not stable. The three models are stationary. CWN and EGARCH have no autocorrelation, but autocorrelation exists in VAR. In Histogram-Normality tests, CWN and VAR are not normal, while EGARCH appear normal. There is no ARCH effect on the models. In Dynamic Forecast Evaluation: CWN has least forecast error value in Root Mean Standard Error (RMSE) when compare with EGARCH and VAR. Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) values have the least forecast error in CWN when compare with EGARCH and VAR forecast error values. Therefore, CWN has the best forecast ability among the models under discussion. Ramey RESET tests show that CWN and EGARCH are stable but VAR is not stable. The determinant of the residual of the covariance matrix value indicates that CWN estimation is efficient, since the value is approximately zero. VAR estimation is not efficient. The CWN is efficient and has suitable forecast when compare with the existing models.

Table 1. Histogram-Normality and ARCH Tests of transformed data

| | Coefficient/value | probability |
|--------------------|-------------------|-------------|
| Normal test | | |
| Standard deviation | 1.055567 | |
| Skewness | 0.364743 | |
| Kurtosis | 3.949680 | |
| Jarque-Bera | 13.085650 | 0.001440 |
| ARCH Tests | | |
| F-Statistic | 4.908379 | 0.000300 |
| Obs*R-squared | 22.576580 | 0.000400 |

Table 2. ARCH, EGARCH, VAR, CWN models

| | α | β | δ | γ | AIC | BIC | HQ | LL |
|--------|-----------------|-----------------|----------------|----------------|---------|----------|----------|----------|
| ARCH | 0.13645 (0.000) | 0.31623(0.006) | | | 2.90733 | 2.96942 | 2.93241 | -312.89 |
| EGARCH | -0.0462 (0.448) | -0.0157 (0.811) | 0.02031(0.422) | 1.0106 (0.000) | 2.65324 | 2.76191 | 2.69713 | -282.20 |
| VAR | | | | | 11.2244 | 11.27090 | 11.24320 | -1226.10 |
| CWN | | | | | -6.3362 | -6.2433 | | 699.813 |

Note: α is the coefficient of the mean equation, β and δ are the coefficients of the variance equations, while γ is the coefficient of the log of variance equation. In the parentheses are the probability values (PV).

Table 3. Levene's test for equal variances

Independent Samples Test

| | Levene's test for equality of variances | | t-test for Equality of Means | | | | | 95% Confidence Interval of the Difference | |
|-----------------------------|---|-------|------------------------------|---------|----------------|-----------------|----------------------|---|---------|
| | F | Sig. | t | df | Sig.(2-tailed) | Mean Difference | Std.Error Difference | Lower | Upper |
| | | | | | | | | | |
| B Equal variances assumed | 0.045 | 0.833 | -2.993 | 438 | 0.003 | -0.01409 | 0.00471 | 0.02334 | -0.0048 |
| Equal variances not assumed | | | -2.993 | 424.759 | 0.003 | -0.01409 | 0.00471 | -0.02335 | -0.0048 |

Table 4. The summary of CWN, EGARCH and VAR models estimation and forecasting evaluation

| | CWN | EGARCH | VAR |
|---------------------------------|-----------------------|-----------------------|-----------------------|
| Estimation Residual Diagnostic | | | |
| Stability Test(Lag structure) | Stable | Stable | Not Stable |
| Correlogram (square) residual | covariance stationary | Stationary | covariance stationary |
| Portmanteau Tests | No autocorrelation | No autocorrelation | Autocorrelation |
| Histogram-Normality Tests | Not Normal | Appear Normal | Not Normal |
| ARCH Test | No ARCH effect | No ARCH effect | No ARCH effect |
| Dynamic Forecast Evaluation | | | |
| RMSE | 0.0333325 | 0.489917 | 53253.79 |
| MAE | 0.007404 | 0.366493 | 46226.78 |
| MAPE | 1.233974 | 107.6098 | 15.61704 |
| Residual Diagnostic | | | |
| Correlogram (square) residual | Stationary | Stationary | Stationary |
| Histogram-Normality Tests | Not Normal | Appear Normal | Not Normal |
| Serial Correlation LM Tests | No Serial Correlation | No Serial Correlation | Serial Correlation |
| Heteroscedasticity Test | No ARCH effect | No ARCH effect | ARCH effect |
| Stability Diagnostic | | | |
| Ramsey Reset Tests | Stable | Stable | Not Stable |
| Determinant residual covariance | 0.001923 | | 12.86549 |

Discussion

Chuffart (2015) reveals that wrong specifications can be the use of only Bayesian Information Criteria (BIC). The Logistic Smooth Transition GARCH and Markov-Switching GARCH models were employed to confirm

the weakness of BIC. Obtaining the right model specification, CWN model employs Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and log likelihood for specification of the model.

Mutunga *et al.* (2015) emphasizes that the EGARCH model has the minimum mean square error and mean

absolute error when compare with Glosten-Jagannathan-Runkle GARCH model; this reveals that EGARCH forecast is more precise. Conversely, CWN model has least information criteria and minimum forecast errors that indicate the performance of CWN to be better than the EGARCH model.

Chang *et al.* (2015) extend reasonable conditions of strict stationary and ergodicity in favor of three nonlinear models of Self-Exciting Threshold Autoregressive (SETAR) -GARCH process, multiple-regime logistic Transition AutoRegressive (STAR) model by GARCH errors and Exponential STAR-GARCH model. The STAR-GARCH model estimation results are regarded as essential in financial Econometrics. The GARCH family errors are disintegrated into CWN model. CWN is examined by employing different countries' data set, having better performance when compare with family GARCH model (EGARCH) which Mutunga *et al.* (2015) show as appropriate.

McAler (2014) argues that the asymmetry and leverage are identical. The challenge is that leverage effect has no statistical properties to check the EGARCH estimation. The positivity restriction of the parameters cannot estimate the leverage effect. CWN model estimation has the available statistical properties of maximum likelihood estimation to get efficient estimation, which provides a better estimation than the existing models.

McAler and Hafner (2014) show one line derivation of EGARCH to model the asymmetric leverage effect, but no resolution for stationarity and invertibility conditions. This makes it impossible to model the leverage effect. But as for CWN model stationarity and invertibility are possible. CWN estimation is more efficient.

Riposo and Bianaca (2015) show the distinctiveness of ARCH (1)-M and GARCH(1;1)-M with the continuous limit of a time-discrete processes. They show that there is more volatility variation in the time-discrete GARCH(1;1) when compared with the time-discrete ARCH(1). Whereas, the EGARCH error is decomposed for the formulation of CWN processes to dealt with different types of heteroscedastic error behaviors in the data including the leverage effect that none of GARCH family model can tackle effectively.

Agboluaje *et al.* (2015) employs US GDP data to show that CWN is efficient and gives suitable results when compare with existing models. Equally, tests have been conducted using different data from different countries to show the suitability of the CWN. This CWN can handle other financial/economic data that possesses asymmetry and the leverage effect (Agboluaje *et al.*, 2015). When the standardized residuals of EGARCH are disintegrated into equal variances, few points are zeros on the graph which can cause some errors. There is no package to take care of the zero points when re-graphing the points.

Future research: To disintegrate the conditional standard deviation graph into equal graph series and model it.

Conclusion

The standardized residual GARCH errors are decomposed into Combine White Noise (CWN). CWN has proved to be more efficient and it takes care of GARCH weaknesses. The estimation of Combine White Noise model passes the stability condition, stationary, serial correlation, the ARCH effect tests and it also passes the Levene's test of equal variances.

The CWN model estimation yields best results with minimum information criteria and high log likelihood values. While the EGARCH model estimated yields better results with minimum information criteria and high log likelihood values when compared with VAR model.

CWN has the minimum forecast errors which are indications of best results when compare with the EGARCH and VAR models dynamic evaluation forecast errors (Ismail and Muda, 2006; Fildes *et al.*, 2011; Lazim, 2013). The minimum forecast error values reveal the forecast accuracy.

The determinant of the residual of the covariance matrix value indicates that CWN is efficient. But, the determinant of the residual of the covariance matrix value indicates that VAR is not efficient.

Based on the every result of the empirical analysis, CWN is the most appropriate model. For this reason, CWN is recommended for the modeling of data that exhibits conditional heteroscedasticity and the leverage effect in Australia and other societies in the world.

The contribution of this study to the scientific community is that the CWN gives suitable results that improve the weaknesses of the existing models. The CWN forecast output is more reasonable for effective policy making. Implementation of this CWN will boost the economy of the society.

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Author's Contributions

Ayodele Abraham Agboluaje: Analyzing, producing the results and writing the paper.

Suzilah Ismail: Supervising the contents and flow of the paper.

Chee Yin Yip: Offering ideas, encouragement, proof read text and equations.

Ethics

This manuscript is original; there will be no expectation of any ethical issues after the publication. The three authors have read and approved the manuscript.

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