Computers & Industrial Engineering 189 (2024) 109967

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An extended-directional mix-efficiency measure: Performance evaluation of OECD countries considering NetZero

Mushtaq Taleb [a], Ali Emrouznejad [b], Vincent Charles [c,*], Ruzelan Khalid [d], Razamin Ramli [d]

[a] College of Administration and Economics, University of Anbar, Ramadi, Iraq
[b] Surrey Business School, University of Surrey, Guildford, UK
[c] Queen's Business School, Queen's University Belfast, Belfast BT9 5EE, UK
[d] Institute of Strategic Industrial Decision Modelling (ISIDM), School of Quantitative Sciences, Universiti Utara Malaysia, Sintok, Kedah 06010 UUM, Malaysia

A R T I C L E I N F O

Keywords:
Data envelopment analysis
Eco-efficiency
NetZero
CO_2 emissions
Mix-efficiency

A B S T R A C T

Conventional data envelopment analysis (DEA) models make the assumption of controllable inputs and desirable outputs. However, in many real-world applications, there are two major issues facing the management of decision-making units. The first one is how to deal with uncontrollable inputs whose levels are determined by exogenous fixed factors. The second is how to deal with undesirable outputs that are accompanied by desirable outputs. The effect of the operating environment is frequently captured by uncontrollable inputs and undesirable outputs. The modulation of these two factors into a directional DEA model is still in its infancy in the DEA literature. This paper proposes new directional mix-efficiency measure and slacks-based measure models. These two efficiency models are proposed in the context of uncontrollable inputs and undesirable outputs. The new metric looks at how well the input and/or output mix should change to achieve a fully efficient status by decreasing controllable inputs and undesirable outputs and/or increasing desirable outputs while keeping uncontrollable inputs constant. The new mix-efficiency measure is based on the directional distance function and the slacks-based measure. The usefulness and applicability of the proposed models are assessed by measuring the eco-efficiency of the Organization for Economic Co-Operation and Development (OECD) countries. The aim of the application is to measure efficiency in the context of NetZero, with a specific focus on reducing CO_2 emissions. The findings reveal that six countries—France, Luxembourg, Germany, Norway, Sweden, and the UK—have achieved eco-efficiency; therefore, these countries function in the constant returns-to-scale (CRS) region.

1. Introduction

The eco-efficiency of countries, particularly developed nations, is imperative for achieving carbon neutrality and sustainable development. Sustainable development necessitates the efficient use of scarce resources, contributing to the realisation of eco-efficiency and overcoming environmental degradation. Environmental degradation can occur when scarce resources are used inefficiently. To control and mitigate environmental degradation, green technology should be developed. Developing green technology involves the use of renewable energy sources, leading to a further reduction in CO_2 emissions and the achievement of NetZero (Mandel et al., 2023). In the context of achieving NetZero emissions, a low-carbon economy is urgently required in many developed countries that consume a large amount of energy, such as OECD countries. Various activities have been developed and implemented by academics and policymakers in OECD countries to mitigate CO_2 emissions and achieve NetZero. As a result, achieving NetZero has recently become a complex target and a multidisciplinary task that can be addressed using Multi-Criteria Decision-Making (MCDM) problems (Taleb et al., 2023). One of the most common techniques of MCDM that does not require the imposition of subjective weights on inputs and outputs from decision-makers is Data Envelopment Analysis (DEA).

Charnes et al. (1978) proposed DEA, a non-parametric approach based on linear programming. DEA evaluates the relative efficiency of a peer set of entities called decision-making units (DMUs), which consume multiple inputs to produce multiple outputs. In the field of performance measurement, DEA has emerged as a reliable technique for assessing

* Corresponding author at: Queen's Business School, Queen's University Belfast, Belfast BT9 5EE, UK.
E-mail address: c.vincent@qub.ac.uk (V. Charles).

https://doi.org/10.1016/j.cie.2024.109967
Received 6 September 2023; Received in revised form 11 December 2023; Accepted 4 February 2024
Available online 8 February 2024

efficiency and establishing targets by identifying benchmarks (Charles et al., 2016; Charles et al., 2018). In this sense, also, DEA can be positioned as a prescriptive analytics-oriented technique (Charles et al., 2022). The relative efficiency of DMUs can be measured without prior weights on the inputs and outputs. This ability has made DEA a popular data-enabled efficiency evaluation approach (Charles et al., 2021; Zhu, 2022; Zhu & Charles, 2021) when compared to other frontier approaches, such as multiple regression analysis and stochastic frontier analysis. Furthermore, DEA takes into account two types of efficiency measures: radial and non-radial. The efficiency score of a radial model reflects the proportional extension of outputs or the proportional reduction of inputs, depending on whether the model is output-oriented or input-oriented. The oriented models ignore the existence of input and/or output slacks in their efficiency scores (Taleb et al., 2023). Therefore, the obtained efficiency scores do not reflect all the inefficiency of an inefficient DMU, which may mislead decision-makers. The excesses of each input and/or shortfalls of each output can be identified by projecting the inefficient DMU onto the efficient frontier. Thus, the radial models enhance the inefficient DMU to decrease its inputs while preserving outputs at a given level (input-oriented) and/or increase its outputs while preserving inputs at their given levels (output-oriented).

To address the problem, Färe and Knox Lovell (1978) proposed the Russell efficiency measure that simultaneously deals with inputs and outputs. Later, Pastor et al. (1999) developed a new version of the Russell model that combines inputs and outputs into a ratio form known as the Enhanced Russell-graph efficiency measure (ERGM). Chambers et al. (1996, 1998) proposed a non-radial directional distance function (DDF) model that does not require discrimination between input-oriented and output-oriented models to allow simultaneous input reduction and output augmentation. Subsequently, Tone (2001) proposed a novel non-radial slacks-based measure (SBM) model under the assumption that inputs and outputs can be allowed to decrease and increase at different rates by simultaneously dealing with input and output slacks. This characteristic has been identified as a significant advantage of non-radial models over radial models (Taleb et al., 2019). SBM has three variations: input-oriented, output-oriented, and non-oriented. A detailed comparison of radial and non-radial models can be found in Avkiran et al. (2008). Therefore, it assesses the efficiency of the output or input mix as well as the aggregate efficiency, where 'mix' refers to the proportions in which outputs or inputs are produced or consumed, respectively.

The efficiency scores generated by an input- or output-oriented model of Charnes et al. (1978), termed the CCR model, and an oriented SBM model (i.e., input- or output-oriented SBM) are used to evaluate mix-efficiency. As a result, Cooper et al. (2006) introduced output mix-efficiency and input mix-efficiency measures using the output-oriented and input-oriented versions of the CCR model and SBM model, respectively. In fact, the mix-efficiency measure has been considered by several studies, such as Herrero et al. (2006), Puri and Yadav (2013), Saranga (2009), Taleb (2023), and Visbal-Cadavid et al. (2017). However, these studies ignored the impact of uncontrollable inputs (*i.e.*, inputs subject to exogenous fixed factors, such as average precipitation, airport apron capacity, and soil characteristics) and undesirable outputs (i.e., a bad output that can be abated by reducing its level in a production process) (Kuosmanen, 2005). This argument holds true in a wide range of real-world applications of efficiency measures, such as CO_2 emissions, waste water, and the number of delayed flights at an airport (Lozano & Gutiérrez, 2011). Furthermore, all of these studies examined mix-efficiency from either an input-oriented or an output-oriented perspective.

Consequently, the main questions addressed in this research are:

1. How can uncontrollable inputs and undesirable outputs be modelled into non-radial DDF and SBM models?
2. How can the infeasibility issue that may arise from uncontrollable inputs be rectified?
3. How can the directional mix-efficiency be measured from a non-oriented perspective?
4. How can the effect of uncontrollable inputs and undesirable outputs on the eco-efficiency of OECD countries be measured?
5. How can the returns-to-scale be identified in the context of the proposed DDF and SBM?

To address the gap, we propose a new directional mix-efficiency measure, denoted MIX-NCIUO, that simultaneously considers both the inputs used and the outputs produced from the perspectives of inputs and outputs. This is achieved by introducing non-oriented DDF and SBM models in the presence of uncontrollable inputs and undesirable outputs. The newly proposed measure incorporates uncontrollable inputs and undesirable outputs into their respective input and output constraints, as well as the target function of the standard non-oriented DDF and SBM. The goal of using non-oriented DDF and SBM to measure mix-efficiency is that each of these models combines both output-oriented and input-oriented models into a non-oriented model, resolving the infeasibility issue that arises from integrating uncontrollable inputs into an output-oriented model. Additionally, the proposed non-oriented models are more applicable in addressing real-life situations that require reductions in controllable inputs and undesirable outputs, along with an augmentation of desirable outputs, such as eco-efficiency. To illustrate the practicality of the proposed efficiency measures, MIX-NCIUO is employed to measure and analyse the efficiency of 25 countries from the Organization for Economic Co-Operation and Development (OECD). This research introduces several novel contributions to the DEA literature:

- Both input- and output-oriented DDF and SBM models have been extended to incorporate uncontrollable inputs and undesirable outputs.
- A directional mix-efficiency measure from a non-oriented perspective is proposed to address uncontrollable inputs and undesirable outputs under constant returns-to-scale (CRS) and variable returns-to-scale (VRS).
- The proposed DDF and SBM models ensure the feasibility condition under CRS and VRS technologies, marking this as their prominent feature.
- Returns-to-scale in the context of uncontrollable inputs and undesirable outputs of the DDF and SBM models have been measured.
- Given that the proposed models handle uncontrollable inputs and undesirable outputs, reflecting many real-life situations, they provide a platform for a comprehensive quantitative approach to measure and improve system efficiency. This aspect helps decision-makers better understand and evaluate their systems.

The paper is organised into seven sections. Section 2 presents a review of previous efficiency studies involving undesirable outputs and uncontrollable inputs. Section 3 provides background information on existing models relevant to the development of MIX-NCIUO. Section 4 outlines the MIX-NCIUO research methodology. Section 5 examines data from 25 OECD countries and presents the efficiency results as measured by the proposed models. Section 6 considers the academic and managerial implications of the proposed methodology. Finally, Section 7 concludes with a summary, concluding remarks, and future research directions.

2. Literature review

Anthropogenic pollution can be globally balanced by eliminating CO_2 emissions over a specified time period; therefore, achieving net-zero CO_2 emissions is possible. The overlapping concepts of net-zero CO_2 emissions and carbon neutrality can be applied at different levels

(Jeudy-Hugo et al., 2021). Human activities contributing to net-zero CO_2 emissions include not only industrial production and energy services, but also agriculture and land use, all of which must be entirely eliminated to achieve this goal (Davis et al., 2018). Furthermore, the realisation of net-zero can be affected by the energy systems in use (Pye et al., 2021). The discourse on net-zero has rapidly evolved in recent years, prompting countries, especially OECD members, to strive for the net-zero CO_2 emissions target by balancing emission reductions and carbon utilisation in the coming years.

In an effort to combat global warming, an increasing number of nations, including the OECD, aim to achieve net-zero CO_2 emissions by 2050, eliminating as much CO_2 as they produce. To effectively limit global warming and mitigate the worst effects of climate change, OECD countries must take responsibility for all their undesirable (negative) environmental impacts, considering various scientific measures that can be adopted to reduce them efficiently. Attaining carbon neutrality is crucial for meeting the goal of keeping global temperature increases below 1.5 °C, necessitating the achievement of net-zero CO_2 emissions. Balancing CO_2 emissions through effective measures can lead to net-zero emissions (Wang et al., 2022; Zheng, 2023). In 2021, a climate change conference (COP26) was held in Glasgow to discuss the achievement of net-zero emissions. Among the various frontier approaches explored for assessing eco-efficiency, such as stochastic frontier analysis (SFA) (Aigner et al., 1977) and free disposal hull (FDH) (Tulkens, 1993), a powerful non-parametric approach proposed by Charnes et al. (1978), data envelopment analysis (DEA), has been effectively used to evaluate the eco-efficiency of DMUs. In the DEA literature, the evaluation of net-zero CO_2 emissions using DEA models is still relatively new and has been explored by a few studies (e.g., Azadi et al., 2022; Emrouznejad et al., 2023; Taleb et al., 2023; Xiao et al., 2021).

DEA considers two efficiency models: radial and non-radial. The radial models focus on the proportionate reduction of an input increase (input-oriented) or the proportionate expansion of an output decrease (output-oriented) (Debnath et al., 2008; Taleb et al., 2022). Therefore, oriented models can force evaluated DMUs to decrease (increase) their inputs (outputs) at a fixed rate to a maximum proportion obtained for the inputs and/or outputs (Cooper et al., 1999). Non-radial models (*i.e.*, non-oriented models), on the other hand, effectively deal with input and output slacks. As a result, inputs and outputs can be decreased and increased disproportionally at the same time. This is a distinguishing feature of non-oriented models over oriented models because decreasing inputs and/or increasing outputs are independent of one another (Rashidi et al., 2015).

The SBM model, proposed by Tone (2001), is one of the most popular non-radial models for evaluating the efficiency of DMUs in various settings. The model deals with input excesses and output shortfalls simultaneously and effectively discriminates the inefficiency of inefficient DMUs (Taleb et al., 2018; Zhou et al., 2007). Moreover, its target function is unit-invariant and a monotone function of input and output slacks. To improve inputs and outputs, SBM computes the ratio of the average input reduction to the average output augmentation (Lozano & Gutiérrez, 2011). As a result, its target function value is regarded as the product of input and output inefficiencies (Taleb et al., 2023). Tone (2001) considered the procedure of oriented SBM to produce an oriented SBM model in terms of input-oriented or output-oriented. For this, the efficiency scores resulting from input- or output-oriented SBM and CCR models can be used to calculate the mix-efficiency measure, as proposed by Cooper et al. (2006). The mix-efficiency is a metric that identifies the inefficiency caused by incorrect input or output composition. However, all of the studies on mix-efficiency assumed that all inputs and outputs are discretionary (*i.e.*, inputs and outputs can be controlled by a DMU's management) and desirable (*i.e.*, good outputs, whose levels should be increased appropriately). These assumptions do not hold true in many real-world applications. Hence, ignoring both uncontrollable and undesirable factors may lead to inaccurate efficiency measures.

In real-world settings, the outputs of a DMU may be accompanied by undesirable outputs such as CO_2 emissions in industries, aeroplane delay time in airports, and waste water. Numerous efficiency studies with undesirable outputs have been conducted to evaluate DMU performance using radial and non-radial DEA models (see Liu et al., 2010, p. 180). Radial models are widely used in a variety of settings to assess efficiency when undesirable outputs are present. For example, Färe et al. (1989) used a radial model to assess the efficiency of 30 US mills. They assumed weak disposability[1] for undesirable outputs. The findings revealed that the performance of DMUs is sensitive to the presence of undesirable outputs. A radial model was also used by Camarero et al. (2013) to measure the eco-efficiency of 22 OECD countries. Emrouznejad (2003) proposed an alternative dynamic efficiency model for measuring the efficiency of the OECD countries. To explore carbon emission abatement (CEA) in Chinese manufacturing industries, Li et al. (2020) proposed an integrated game DEA approach. Moreover, Färe and Grosskopf (2004), Scheel (2001), Seiford and Zhu (2002), and Tyteca (2016), among others, introduced radial models with undesirable outputs. Additionally, Mandal (2010), Watanabe and Tanaka (2007), Yang and Pollitt (2009) introduced various efficiency studies in the energy and industrial sectors. These studies emphasised the critical role of incorporating undesirable outputs into different-radial DEA models in order to avoid overestimation of efficiency measures.

Undesirable outputs have also been incorporated into non-radial models. Tone (2003), for example, integrated undesirable outputs into Tone's (2001) SBM model to propose an SBM model that deals with undesirable outputs. In Zhou et al. (2006), the undesirable output of CO_2 emissions was integrated into the SBM model to evaluate the ecological efficiency of 30 OECD countries. Choi et al.'s (2012) study took CO_2 emissions as an undesirable output into account. Later, Lee et al. (2014) examined the efficiency of port cities using an SBM model with undesirable outputs. Their undesirable outputs were CO_2, sulphur oxide (SO_x), and nitrogen oxide (NO_x). Similarly, many studies, such as Pang et al. (2015), Zhang and Choi (2013), and Zhou et al. (2006) have introduced SBM models with undesirable outputs in various real-life situations. Chambers et al. (1996) proposed a DDF model that demonstrated the possibility of simultaneously decreasing inputs and increasing outputs. The DDF model was then improved to deal with undesirable outputs and environmental factors, as demonstrated by studies such as Daraio and Simar (2014), Ramli et al. (2013), Singh and Gundimeda (2021), and Watanabe and Tanaka (2007). However, all of the studies that looked at radial or non-radial DEA models in the context of undesirable outputs ignored the impact of uncontrollable inputs on efficiency measures.

DMU inefficiencies are likely to occur due to poor management. This assumption was made by classical DEA models that did not account for the effect of uncontrollable inputs and/or outputs on DMU performance evaluation. Many efficiency studies have been conducted to investigate the effects of uncontrollable inputs and/or outputs on efficiency measures, including Banker and Morey (1986), Estelle et al. (2010), Lotfi et al. (2007), Patel and Pande (2013), and Taleb et al. (2019). Despite their prominent features, these studies did not take into account the effect of integrating uncontrollable inputs and undesirable outputs into efficiency measures at the same time. Some efficiency studies have taken both factors into account. For example, Yang and Pollitt (2009) integrated uncontrollable inputs and undesirable outputs into a four-stage radial model to evaluate the efficiency of coal-fired power plants in China. Noorizadeh et al. (2014) conducted a study that classified uncontrollable factors into two categories: permanent and temporary. The uncontrollable and undesirable outputs were incorporated into a radial super efficiency model for supplier ranking. These studies, however, did

[1] Any decrease in undesirable outputs and/or increase in undesirable inputs will result in a proportional reduction and/or increase in desirable outputs and/or inputs (Lozano et al., 2013; Taleb, Khalid, Emrouznejad et al., 2023; Taleb et al., 2023).

not examine the effects of both factors on the efficiency measures of a non-radial model. Because of the radial model's limitations, researchers have integrated both factors into non-radial models.

Yahia et al. (2018) combined undesirable outputs and uncontrollable inputs to propose a new DDF production possibility set (PPS). Lozano and Gutiérrez (2011) proposed an SBM model to measure the efficiency of 39 Spanish airports while simultaneously dealing with uncontrollable inputs and undesirable outputs. Rashidi et al. (2015) investigated the efficiency of 25 OECD countries. They developed a non-radial measure by combining the range-adjusted measure (RAM) model of Cooper et al. (1999) and the SBM model of Tone (2001) with uncontrollable inputs and controllable and uncontrollable desirable and undesirable outputs in order to simultaneously decrease controllable inputs and controllable undesirable outputs, as well as increase controllable desirable outputs, while keeping uncontrollable inputs and outputs at their fixed levels. Hua et al. (2007) proposed a non-radial model with both factors for measuring and analysing the eco-efficiency of paper mills in China. They also considered the impact of uncontrollable inputs on the returns-to-scale (RTS) of DMUs. The RTS is an economic measure that examines the proportionate augmentation of outputs obtained from inputs to determine the efficiency level of a DMU (Taleb et al. 2019). The DMU falls into one of three RTS regions: constant returns-to-scale (CRS), decreasing returns-to-scale (DRS), or increasing returns-to-scale (IRS). The CRS reflects that when inputs are increased, outputs can be increased proportionally. As a result, RTS is constant for each efficient DMU. IRS or DRS, on the other hand, reflects whether outputs have increased proportionally more or less than inputs (Taleb et al., 2022).

In the presence of undesirable outputs, the RTS technology has evolved to include environmental assessment, as introduced by Sueyoshi and Goto (2011, 2013). They proposed a new technology known as Damages-to-Scale (DTS), designed to examine the RTS in DEA models with undesirable outputs. Although the mathematical concepts of RTS and DTS are similar, the economic implications of these techniques are diametrically opposed. For example, if an increase in inputs results in a proportionally higher increase in undesirable outputs, the RTS functions under increasing DTS (IDTS). Consequently, the operational size of an evaluated DMU may increase, and the DMU will produce additional damage (*i.e.*, undesirable outputs). To avoid this and improve the environmental efficiency of the DMU, its operational size should be reduced. Conversely, the DTS decreases when an increase in inputs results in a proportionally smaller increase in undesirable outputs (*i.e.*, less damage). Decreasing DTS (DDTS) implies that increasing inputs can proportionally result in a smaller increase in undesirable outputs. Consequently, it is acceptable for the DMU to increase its operational size to enhance its environmental efficiency.

It is worth mentioning that the DTS technology of Sueyoshi and Goto (2011, 2013) was introduced under two different concepts of disposability, which are natural disposability[2] and managerial disposability.[3] This research considers undesirable outputs under the weak disposability technology, while inputs and desirable outputs are considered under the strong disposability.[4] Therefore, RTS for uncontrollable inputs and undesirable outputs is identified based on Seiford and Zhu's (1999) study. A summary of the literature review on relevant DEA studies integrating uncontrollable inputs and/or undesirable outputs and their limitations is reported in Table 1.

To the best of our knowledge, no studies have considered the directional mix-efficiency measure from the perspectives of both inputs and outputs in the presence of uncontrollable inputs and undesirable

[2] Any decrease in the inputs of a DMU will be accompanied by a decrease in undesirable outputs.

[3] A DMU increases an input, but decreases undesirable outputs (Sueyoshi & Goto, 2012).

[4] A term that refers to any reduction in controllable inputs that can most likely occur without any reduction in desirable outputs.

Table 1
A selected review on uncontrollable inputs and/or undesirable outputs in DEA models.

Author(s)	Topic or field of evaluation	DEA model used	Limitation(s)
Apergis et al. (2015)	Evaluating the eco-efficiency of OECD countries	SBM model with undesirable outputs	• Uncontrollable inputs were not taken into consideration. • The directional proportions in which outputs or inputs are produced or consumed were not measured.
Diabat et al. (2015)	Evaluating the efficiency of Information Technology firms that operate in India	Non-radial range DDF model	• It considers DDF undesirable outputs. • It did not measure the RTS of the evaluated DMUs.
Emrouznejad (2003)	Measuring the efficiency of OECD countries	Dynamic efficiency DEA model	• It is a radial model. • It did not consider environmental factors, such as undesirable outputs and uncontrollable inputs.
Fukuyama and Weber (2009)	Measuring the efficiency of financial services provided by Japanese banks	Directional slacks-based measure model	• The proposed model assumes that all inputs are controllable, and outputs are desirable.
Iram et al. (2020)	Measuring the environmental and energy efficiency of 26 OECD countries	SBM model considering undesirable outputs	• It only considers undesirable outputs as environmental factors. • It did not consider the directional mix-efficiency measure.
Ramli et al. (2013)	Measuring the eco-efficiency of the manufacturing sector in Malaysia	Radial DDF model	• It considers the scale DDF model as a radial measure. • It did not evaluate uncontrollable inputs and undesirable outputs. • It did not consider both non-radial SBM and DDF models.
Rashidi et al. (2015)	Assessing the eco-efficiency of OECD countries	SBM model and range-adjusted measure (RAM) considering uncontrollable inputs and undesirable outputs	• It did not consider the directional proportional rate of decreasing inputs and increasing outputs concurrently. • It did not measure the RTS in uncontrollable inputs and undesirable outputs.
Taleb (2023)	Evaluating the environmental and energy efficiency of land transportation system in China	Radial efficiency measures with mixed integer-value data and undesirable outputs	• It considers output-oriented SBM and BCC efficiency measures. • It did not consider uncontrollable

(continued on next page)

Table 1 (*continued*)

Author(s)	Topic or field of evaluation	DEA model used	Limitation(s)
			inputs and undesirable outputs simultaneously. • It did not consider the directional mix-efficiency measure.
Yahia et al. (2018)	Measuring the efficiency of education	DDF model with uncontrollable inputs and undesirable outputs	• The study did not explicitly state that a DDF model considers uncontrollable inputs and undesirable outputs since it only considers the PPS of DDF under different disposability assumptions. • The study did not consider the directional mix-efficiency measure. • RTS based on the DDF model, with uncontrollable inputs and undesirable outputs, was not measured.
Yang and Pollitt (2009)	Measuring the efficiency of Chinese coal-fired power plants	Radial DEA with uncontrollable inputs and undesirable outputs	• It considers a radial model. • It did not consider the directional mix-efficiency measure.
Zhou et al. (2006)	Measuring the environmental and energy efficiency of OECD countries	SBM model with undesirable outputs of CO_2 emissions	• It only considers SBM with undesirable outputs. • The study did not consider the non-radial mix-efficiency measure.

outputs. To address this gap, this paper incorporates both factors into the standard non-oriented DDF and SBM models before developing new non-oriented DDF and SBM models. For the first time, the proposed models are used to propose a new directional mix-efficiency measure from both input and output perspectives in order to evaluate environmental efficiency while assuming net-zero CO_2 emissions. The preliminaries of the existing models are discussed in the following section, playing a crucial role in proposing the methodology of this paper, as shown in Section 4.

3. Background

3.1. Directional distance function (DDF) model

DDF is a generalisation of the radial input-oriented and output-oriented models proposed by Chambers et al. (1996) to assess the efficiency of a set of J DMUs (DMU$_j$, $j = 1, \ldots, J$) (Ray, 2008). Each DMU consumes m inputs that can be observed by x_{ij}, $i = 1, \ldots, m$ to produce s outputs that can be observed by y_{rj}, $r = 1, \ldots, s$. Let x_{ij} denote the positive amount of the ith input used by the jth DMU, and y_{rj} denote the positive amount of the rth output produced by the jth DMU. The jth DMU can also be denoted by DMUo, which represents the evaluated unit under the DDF efficiency model. The DDF model measures the distance from a particular combination of input–output $(x, y) \in \Re^{m+s}$ to the efficient frontier of the technology set T in a direction vector determined by formula (1) $g = (g_x, g_y) \in \mathbb{R}^{m+s}$, $g \neq 0$,

$$\overrightarrow{D_T}(x,\ y;\ g) = \max\left[\beta \in [0,1) \middle| (x - \beta g_x,\ y + \beta g_y) \in T\ \right] \tag{1}$$

Färe and Grosskopf (2000) proposed the DDF model of DMUo along the direction vector $g = (g_x, g_y)$ under the VRS technology, which is expressed in model (2). Färe and Charles (2018) showed that DDF is free from non-Archimedean estimation.

$$\max \beta_o \tag{2}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j \leqslant x_{io}(1-\beta_o) \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{J} y_{rj}\eta_j \geqslant y_{ro}(1+\beta_o) \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$$\eta_j \geqslant 0 \qquad j = 1, \ldots, J$$

$$\beta_o \in [0,\ 1),$$

$$x_{io},\ y_{ro} \in \mathbb{R}_+^{m+s}$$

By setting the direction vector $g_x(g_y) = 0$, an input (output) oriented model can be obtained. A proportional rate to decrease the input and increase the output concurrently in the ith input and the rth output of DMUo is released by β_o. Therefore, the β_o value is maximised in the target function of model (2). The non-zero directional vector of g is given as $(g_x, g_y) = (g_{x_1}, \ldots, g_{x_m}, g_{y_1}, \ldots, g_{y_s})$ along with the inputs to be decreased and the extended outputs (Toloo et al., 2018). For example, if $\beta_o = 0.15$, we decrease all inputs by 0.15, while increasing all outputs by 0.15. A non-negative intensity vector serving to construct the convex combination of inputs and outputs of evaluated DMUs is denoted by η_j. Model (2) is a straightforward linear programming (LP) problem that can be solved easily.

Since the values of both inputs and outputs are positive, the value of the proportional rate is $0 \leqslant \beta_j < 1$, $j = 1, \ldots, J$. The proportional value of the direction vector is equal to zero for each efficient DMU, whereas it is either greater than zero or less than one for each inefficient DMU. Thus, based on the proportional rate computed by model (2), the relative efficiency score computed by DDF model (2) is equal to one for each efficient DMU; otherwise, the DMU is DDF inefficient. Therefore, to compute the relative efficiency score of the evaluated DMUo using a non-oriented version of model (2), the model's target function should be changed to that stated in Eq. (3).

$$\min \theta_o^{\mathrm{DDF}} = \frac{1-\beta_o}{1+\beta_o} \tag{3}$$

Proposition 1. *DMUo is said to be fully efficient if and only if it satisfies $\beta_o = 0$. This condition is equivalent to all directional vectors of inputs and outputs being zero, thus guaranteeing Pareto-Koopmans efficiency. The directional vectors of inputs and outputs reflect the slacks of input excesses and output shortfalls of DMUo $(\beta_o x_{io},\ \beta_o y_{ro})$, respectively. However, decreasing inputs and increasing outputs are proportional because they are dependent on a single directional value (i.e., β_o), as stated in the preceding example.*

There are two special cases that can be derived from the oriented

DDF model: the directional input distance function (DIDF) model and the directional output distance function (DODF) model. The DIDF is calculated by assuming that the direction vector of outputs is equal to zero (i.e., $g_y = 0$), whereas the DODF is calculated by assuming that the direction vector of inputs is equal to zero (i.e., $g_x = 0$). The technology sets that generate DIDF and DODF are formulated in (4) and (5), respectively (Yahia et al., 2018).

$$\overrightarrow{D}(x,\ y;\ g_x) = \max\left[\beta|(x-\beta g_x,\ y)\in T\right] \tag{4}$$

$$\overrightarrow{D}(x,\ y;\ g_y) = \max\left[\beta\middle|(x,\ y+\beta g_y)\in T\right] \tag{5}$$

In an oriented DDF model, the efficiency score depends on the proportional reduction of inputs (input-oriented), or the proportional expansion of outputs (output-oriented). Thus, the efficiency score of DMUo under the DIDF model or the DODF model can be computed using model (6) or model (7).

$$\min \theta^{DIDF} = 1-\beta_o \tag{6}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j \leqslant x_{io}(1-\beta_o) \quad i = 1,\ ...,\ m,$$

$$\sum_{j=1}^{J} y_{rj}\eta_j \geqslant y_{ro} \quad r = 1,\ ...,\ s,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$$\eta_j \geqslant 0 \qquad j = 1,\ ...,\ J$$

$$\beta_o \in [0,\ 1),$$

$$x_{io},\ y_{ro} \in \mathbb{R}_+^{m+s}$$

$$\min\ \theta_o^{\mathrm{DODF}} = \frac{1}{1+\beta_o} \tag{7}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j \leqslant x_{io} \quad i = 1,\ ...,\ m,$$

$$\sum_{j=1}^{J} y_{rj}\eta_j \geqslant y_{ro}(1-\beta_o) \quad r = 1,\ ...,\ s,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$$\eta_j \geqslant 0 \qquad j = 1,\ ...,\ J$$

$$\beta_o \in [0,\ 1),$$

$$x_{io},\ y_{ro} \in \mathbb{R}_+^{m+s}$$

Based on the technology set T in the direction vector proposed by Chambers et al. (1996), as stated in (1), Chung et al. (1997) improved the main concept of the DDF model to include undesirable outputs, as outlined in the technology set (8).

$$\overrightarrow{D_T}(x,\ y^G,\ y^B;\ g_x,\ g_{y^G},\ -g_{y^B}) = \max\big[\beta \in [0,1)\big|\left(x-\beta g_x,\ y^G+\beta g_{y^G},\ y^B-\beta g_{y^B}\right) \in T\big] \tag{8}$$

The distance function on the technology set (8) determines the reduction in inputs and undesirable outputs, as well as the extension in desirable outputs, by considering their directions in g_x, g_y^G, and g_y^B. The direction vector g measures increases in desirable outputs and decreases in inputs and undesirable outputs, stated as $(x, y^G, -y^B)$. The direction value β proportionally seeks to increase desirable outputs and decrease inputs and undesirable outputs (Ramli et al., 2013). To state the DDF model with undesirable outputs, some notations are introduced. Let $x \in \mathbb{R}_+^m$ represent an input vector, $y^G \in \mathbb{R}_+^{s_1}$ denote a desirable output vector, $y^B \in \mathbb{R}_+^{s_2}$ stand for an undesirable output vector. The DDF model in the presence of undesirable outputs under VRS can be expressed as in model (9) (Diabat et al., 2015).

$$\max \beta_o \tag{9}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j \leqslant x_{io}(1-\beta_o) \quad i = 1,\ ...,\ m,$$

$$\sum_{j=1}^{J} y_{r_1 j}\eta_j \geqslant y_{r_1 o}^G(1+\beta_o) \quad r_1 = 1,\ ...,\ s_1,$$

$$\sum_{j=1}^{J} y_{r_2 j}\eta_j \leqslant y_{r_2 o}^B(1-\beta_o) \quad r_2 = 1,\ ...,\ s_2,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$$\eta_j \geqslant 0 \qquad j = 1,\ ...,\ J$$

$$\beta_o \in [0,\ 1),$$

where β_o is as previously defined, the parameters η_j and the input variable x_{ij} are the same as those defined in model (2), $y_{r_1}^G$ and $y_{r_2}^B$ are the desirable and undesirable outputs of the jth DMU. Based on the direction value β_o, the efficiency scores of efficient and inefficient DMUs are calculated using the target function 1-β_o.

3.2. Non-oriented SBM model

Tone (2001) proposed the SBM model, which is a powerful non-oriented DEA model that considers both input excesses and output shortfalls simultaneously while dealing with their slacks. It projects an inefficient DMU onto the efficient frontier. In addition, the SBM efficiency score leaves no output or input uncalculated because the target function takes into account all potential improvements to outputs and inputs (Lozano & Gutiérrez, 2011). The SBM model evaluates the efficiency of DMU$_j$ ($j = 1, ..., J$) by solving the fractional programme presented in model (10) (see Lo & Lu, 2009, p. 345).

$$\rho_O = \min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{a_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{b_r^+}{y_{ro}}} \tag{10}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j = x_{io} - a_i^- \quad i = 1,\ ...,\ m,$$

$$\sum_{j=1}^{J} y_{rj}\eta_j = y_{ro} + b_r^+ \quad r = 1,\ ...,\ s,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$\eta_j \geqslant 0 \qquad j = 1, \ldots, J,$

$a_i^- \geqslant 0,\ b_r^+ \geqslant 0,$

$x_{io},\ y_{ro} \in \mathbb{R}_+^{m+s}$

where a_i^- and b_r^+ are the non-radial slacks of input excesses and output shortfalls, and η_j is as defined in model (2). The SBM's target function calculates the ratio of the average input contraction to the average output extension by computing the relative contraction rate of input *i* by the term $\frac{a_i^-}{x_{io}}$; the term $\frac{1}{m}\sum_{i=1}^{m}\frac{a_i^-}{x_{io}}$ computes the average reduction rate of the input *i*. On the other hand, the relative extension rate of the output *r* is computed by the term $\frac{b_r^+}{y_{ro}}$; the term $\frac{1}{s}\sum_{r=1}^{s}\frac{b_r^+}{y_{ro}}$ computes the average extension rate of the output *r*. Thus, the target function is a monotone decreasing function with respect to input excesses and output shortfalls, as well as unit-invariant since it does not depend on the measurement unit for inputs and outputs. To ensure that both inputs and outputs improve, the computed ratio of the target function should be appropriately minimised; therefore, the model is non-radial.

Proposition 2. *DMUo is efficient in the SBM model* (10) *if and only if its efficiency score is equal to one* ($\rho_O = 1$). *This condition is equivalent to all input and output slacks being zero* (*i.e.*,$a_i^- = b_r^+ = 0$). *If the efficiency score is equal to one* ($\rho_O = 1$), *and some input and/or output slacks are positive, then DMUo is weak-efficient. If the efficiency score is less than one, then DMUo is SBM inefficient.*

Model (10) is a non-oriented SBM. Tone (2001) was the first to lay out the idea of an oriented SBM model (i.e., input- or output-oriented). The oriented SBM model was then improved by Cooper et al. (2006, p. 142) to calculate the mix-efficiency measure. The input-oriented SBM (I-SBM) model and output-oriented SBM (O-SBM) model can be expressed as in (11) and (11.1).

$$\rho_O^{\text{I - SBM}} = \min 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{a_i^-}{x_{io}} \tag{11}$$

or

$$\rho_O^{\text{O - SBM}} = \min \left[1 + \frac{1}{s}\sum_{i=1}^{m}\frac{b_r^+}{y_{ro}}\right]^{-1} \tag{11.1}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j = x_{io} - a_i^- \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{J} y_{rj}\eta_j = y_{ro} + b_r^+ \quad r = 1, \ldots, s,$$

$\eta_j \geqslant 0, \quad j = 1, \ldots, J,$
$a_i^-,\ b_r^+ \geqslant 0,$

$x_{io},\ y_{ro} \in \mathbb{R}_+^{m+s}$

Definitions of the input and output data, input and output slack variables, and the intensity vector of model (11) are the same as those defined in model (10).

Proposition 3. *DMUo is said to be input-oriented SBM efficient after running model* (11) *if and only if* (*i*) *its efficiency score is equal to one* ($\rho_O^{\text{I - SBM}} = 1$), *and* (*ii*) *all input and output slacks are zero* (*i.e.*, $a_i^- = b_r^+ = 0$). *If condition* (*ii*) *is not satisfied, then DMUo is weakly efficient. If neither condition* (*i*) *nor condition* (*ii*) *is satisfied, then DMUo is I-SBM inefficient.*

Tone's (2001) non-oriented SBM model, presented in (10), considers all inputs and outputs as desirable factors, which may conflict with many real-world applications. To overcome this limitation, Tone (2003) developed a new SBM model that deals with undesirable outputs. In considering Tone's (2003) SBM model, we first examine the model's technology set, as outlined in (12).

$$T = \left\{ \left(y^G, y^B\right) \middle| x \text{ can } produce \left(y^G, y^B\right),\ X\eta \leqslant x,\ Y^G\eta \geqslant y^G,\ Y^B\eta \leqslant y^B,\ \eta \geqslant 0 \right\} \tag{12}$$

The parameters and variables of the technology set (12) are the same as those in model (9). Tone's (2003) SBM incorporates undesirable outputs into the target function and the relevant undesirable output constraint. Hence, the SBM model dealing with undesirable outputs can be presented as follows:

$$\tau_o = \min \frac{1 - \left(\frac{1}{m}\right)\left(\sum_{i=1}^{m}\frac{a_i^-}{x_{io}}\right)}{1 + \left(\frac{1}{s_1+s_2}\right)\left(\sum_{r_1=1}^{s_1}\frac{b_{r_1}^{G+}}{y_{r_1o}^G} + \sum_{r_2=1}^{s_2}\frac{b_{r_2}^{B-}}{y_{r_2o}^B}\right)} \tag{13}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}\eta_j = x_{io} - a_i^- \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{J} y_{r_1j}^G\eta_j = y_{r_1o}^G + b_{r_1}^{G+} \quad r_1 = 1, \ldots, s_1,$$

$$\sum_{j=1}^{J} y_{r_2j}^B\eta_j = y_{r_2o}^B - b_{r_2}^{B-} \quad r_2 = 1, \ldots, s_2,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

$\eta_j \geqslant 0 \qquad j = 1, \ldots, J,$

$a_i^-,\ b_{r_1}^{G+},\ b_{r_2}^{B-} \geqslant 0,$

$x_{io},\ y_{r_1}^G,\ y_{r_2}^B \in \mathbb{R}_+^{m+s_1+s_2}$

where m is the number of inputs, s_1 and s_2 are the number of desirable and undesirable outputs, respectively, s_i^- represents the potential reduction of inputs (input excesses), $s_{r_1}^{G+}$ represents the potential enhancement of desirable outputs (good output shortfalls), and $\boldsymbol{s}_{\boldsymbol{r_2}}^{\boldsymbol{B}-}$ represents the potential reduction of undesirable outputs (bad output excesses).

Due to the salient features of both oriented DDF and SBM models in inputs and outputs, considered in DIDF model (6), DODF model (7), I-SBM model (11), and O-SBM model (11.1), the efficiency scores generated by these efficiency measures can be incorporated to identify the mix-efficiency measure in input- or output-oriented cases.

3.3. Mix-efficiency

The mix-efficiency (MIX) is a measure that estimates the level and mix of inputs or outputs required to efficiently produce or consume a given level of outputs or inputs (*i.e.*, it reflects the degree to which the input mix or output mix should change to achieve the efficient status) (Puri & Yadav, 2013). In order to define the MIX of DMUo, the ratio of the oriented SBM model (input- or output-oriented SBM) (see Cooper et al., 2006, p. 142) to the oriented DDF model (DIDF or DODF model) (see Chung et al., 1997, p. 231; Ramli et al., 2013, p. 387) should be calculated. Therefore, the directional input or output mix-efficiency (DIMIX or DOMIX) of DMUo is defined as:

$$\psi_o^{\mathrm{DIMIX}} = \frac{\rho_o^{\mathrm{I\text{-}SBM}}}{\theta_o^{\mathrm{DIDF}}} \tag{14}$$

or

$$\psi_o^{\mathrm{DOMIX}} = \frac{\rho_o^{\mathrm{O\text{-}SBM}}}{\theta_o^{\mathrm{DODF}}} \tag{15}$$

Since the values of inputs and outputs in the data set are assumed to be positive, the efficiency scores obtained from the DIDF and I-SBM in models (6) and (11) are $0 < \theta_O^{\mathrm{DIDF}} \leqslant 1$ and $0 < \rho_O^{\mathrm{ISBM}} \leqslant 1$ for $o = 1, \ldots, J$. Note that $\rho_O^{\mathrm{I\text{-}SBM}} \leqslant \theta_O^{\mathrm{DIDF}}$ reflects that the value resulted by applying Eq. (14) or Eq. (15) is less than or equal to one (*i.e.*, $0 < \psi_o^{\mathrm{DIMIX}} \leqslant 1$). The directional input mix-efficiency measure can achieve unity ($\psi_O^{\mathrm{DIMIX}} = 1$) if and only if ($\theta_o^{\mathrm{DIDF}} = \rho_o^{\mathrm{I\text{-}SBM}}$). This implies that $\mathrm{DMU_o}$ has the most efficient combination of inputs, but it may be technically inefficient.

Due to the key role of the standard efficiency measures presented in model (2), model (6), model. (7), model (11), as well as Eq. (14) and Eq. (15), in proposing new efficiency measures, an illustration of these standard efficiency measures is given by considering an example. The data set for our considered example was retrieved from a study conducted by Färe and Charles (2018). Two inputs and one unique output of five DMUs are considered, as follows: A (1, 2, 1), B (1, 1, 1), C (2, 1, 1), D (2, 2, 1), and E (2, 4, 1). Now, to demonstrate DIMIX and DOMIX, the DIDF and DODF in (6) and (7), as well as I-SBM and O-SBM in (11) and (11.1), should be run. The I-SBM and O-SBM models are run using target functions (11) and (11.1), as input- or output-oriented SBM models, subject to the same combination of input and output constraints. The aim of considering I-SBM and O-SBM models under the same set of input and output constraints is to avoid the infeasibility issue that may occur in the case of removing the output slack from the output constraint of I-SBM or the input slack from the input constraint of O-SBM. All of the considered efficiency measures are run under VRS to evaluate these five DMUs, whose results are tabulated in Table 2.

As observed in Table 2, the efficiency scores of all the evaluated units resulting from the I-SBM model (column 4) are less than or equal to those obtained from the DIDF model (column 2). In the same context, the efficiency scores resulting from the O-SBM model (column 5) are equal to those obtained using the DODF model (column 3). Besides, by using Eq. (14) and Eq. (15), the DIMIX and DOMIX efficiency measures were calculated, as shown in columns 6 and 7, respectively. Since all the efficiency scores obtained from DODF are equal to one and are the same as those obtained from the O-SBM model, the efficiency scores obtained from DIMIX in Eq. (13) are smaller than or equal to those resulting from DOMIX in Eq. (15). As a result, we can deduce that all the efficiency measures have been appropriately run and calculated. In terms of DIMIX, only DMUs B and D efficiently produce their given levels of output based on the input mix consumed, while all of the DMUs are efficient under DOMIX since they efficiently consume the given levels of their inputs based on the output mix produced. In order to demonstrate that the directional mix-efficiency measure obtained by Eq. (14) or Eq. (15) is less than or equal to one, theorem 1 is introduced.

Table 2
Results of DMUs.

DMU	DIDF	DODF	I-SBM	O-SBM	DIMIX	DOMIX
A	1	1	0.750	1	0.750	1
B	1	1	1	1	1	1
C	1	1	0.750	1	0.750	1
D	0.500	1	0.500	1	1	1
E	0.500	1	0.375	1	0.750	1

Theorem 1. *The optimal efficiency scores of both DIMIX ψ_o^{DIMIX} and DOMIX ψ_o^{DOMIX} are less than or equal to one.*

Proof*. Suppose that $\mathrm{DMU}(x_o, y_o)$ is DIDF inefficient, then we have $\beta x_{io} \neq 0$. In the same context, suppose that $\mathrm{DMU}(x_o, y_o)$ is I-SBM inefficient, then we have at least one slack of inputs that has a positive value (*i.e.,* $a_i^- \neq 0$). The equality of $\theta_o^{\mathrm{DIDF}} = \rho_o^{\mathrm{I\text{-}SBM}}$ holds if and only if the reduction rate of inputs obtained from the I-SBM model is the same as that of the DIDF model since βx_{io} of DIDF is equivalent to input slack a_i^- of I-SBM. Therefore, the input mix-efficiency is equal to one. On the other hand, the DIMIX will be less than one if and only if $\rho_o^{I-SBM} < \theta_o^{DIDF}$. Since the definitions of inefficient and efficient are mutually exclusive, theorem 1 is proven.*□

The optimality condition $\rho_o^{O-SBM} < \theta_o^{DODF}$ of DOMIX can be proven in the same manner as DIMIX, but the proof is omitted here for brevity.

4. Methodology

To build the mathematical formula for the proposed non-oriented DDF and SBM models in the presence of uncontrollable inputs and undesirable outputs, we consider a production system comprised of J DMUs. Each DMU has four factors: controllable inputs, uncontrollable inputs, desirable outputs, and undesirable outputs. The vectors of the controllable and uncontrollable inputs are described as $x^C \in \mathbb{R}_+^{m_1}$, $x^{NC} \in \mathbb{R}_+^{m_2}$, while the vectors of the desirable (good) and undesirable (bad) outputs are described as $y^G \in \mathbb{R}_+^{s_1}$, $y^B \in \mathbb{R}_+^{s_2}$. The matrices of these four vectors are defined as:

$$X^C = \left[x_{ij}^C\right] = \left[x_1^C, \ldots, x_J^C\right] \in \mathbb{R}_+^{m_1 \times J},\ X^{NC} = \left[x_{lj}^{NC}\right] = \left[x_1^{NC}, \ldots, x_J^{NC}\right] \in \mathbb{R}_+^{m_2 \times J},\ Y^G = \left[y_{r_1 j}^G\right] = \left[y_1^G, \ldots, y_J^G\right] \in \mathbb{R}_+^{s_1 \times J},$$
$$Y^B = \left[y_{r_2 j}^B\right] = \left[y_1^B, \ldots, y_J^B\right] \in \mathbb{R}_+^{s_2 \times J} \tag{16}$$

All the values of the controllable and uncontrollable inputs and desirable and undesirable outputs are assumed to be positive (*i.e.*,$X^C > 0,\ X^{NC} > 0,\ Y^G > 0,\ Y^B > 0$). Because non-oriented efficiency measures have a higher discrimination power in assessing the efficiency of the DMUs, this paper proposes a new mix-efficiency measure based on non-oriented DDF and SBM models with uncontrollable inputs and undesirable outputs. The new DDF and SBM models seek to decrease controllable inputs and undesirable outputs, as well as increase desirable outputs, while preserving uncontrollable inputs at their fixed levels, as defined by the empirical PPS of the DDF and the SBM in (17) and (18). In order to consider the technology of DDF with uncontrollable inputs and undesirable outputs, assume $(g_{x^C} \neq 0, g_{x^{NC}} = 0, g_{y^G} \neq 0, g_{y^B} \neq 0)$ yields the directional distance function in the existence of uncontrollable inputs and undesirable outputs, as formulated in (17). We set the directional vector of uncontrollable inputs to zero because their levels are beyond management's control.

$$\overrightarrow{D_T}(x^C, x^{NC}, y^G, y^B; g) = \max\left[\beta \in [0,1) \middle| \left(x^C - \beta g_{x^C}, x^{NC}, y^G + \beta g_{y^G}, y^B - \beta g_{y^B}\right) \in T\right] \tag{17}$$

The DDF technology in (17) allows the controllable inputs to be decreased in the direction of g_x^C, while the uncontrollable inputs remain constant in the direction of g_x^{NC} since the direction is zero. In contrast, it seeks to increase desirable outputs in the g_y^G direction while decreasing undesirable outputs in the g_y^B direction. In other words, a proportional rate β attempts to decrease controllable inputs and undesirable outputs, while symmetrically increasing desirable outputs. This measurement reduces controllable inputs and undesirable outputs and increases desirable outputs by the direction vector of g. The technology set representing all feasible combinations of controllable and uncontrollable inputs, as well as desirable and undesirable outputs, is denoted by T.

The proposed empirical technology set (T) of DDF measures the distance from a particular combination of controllable and uncontrollable inputs, as well as desirable and undesirable outputs (x^C, x^{NC}, y^G, y^B) $\in \mathbb{R}_+^{m_1+m_2+s_1+s_2}$, to a point located on the efficient frontier in a directional vector determined by $g = (g_{x^C} \neq 0,\ g_{x^{NC}} = 0,\ g_{y^G} \neq 0,\ g_{y^B} \neq 0) \in \mathbb{R}_+^{m_1+m_2+s_1+s_2}$.

The properties of the DDF technology set are the same as those of SBM since the two models seek to reduce controllable inputs and undesirable outputs, as well as increase desirable outputs simultaneously, while preserving uncontrollable inputs at their fixed levels. Hence, the technology set of the two proposed models depends on identifying the vector of input and output combinations and matrices of the data set (see Tone, 2001, p.499). Thus, the technology set of the proposed DDF and SBM models considers a convex linear combination of the inputs and outputs, as formulated in (17) and (18). However, a difference between DDF and SBM is that the former depends on proportional changes in the directional vector of inputs and outputs to determine efficiency measures (see Chambers et al., 1998), while the latter depends on disproportional changes in the input and output slacks to identify efficiency measures.

$$T = \left[\left(x^C,\ x^{NC},\ y^G,\ y^B\right) \in \mathbb{R}_+^{m_1+m_2+s_1+s_2} \,\middle|\, X^C\eta \leqslant x^C,\ X^{NC}\eta = x^{NC},\ Y^G\eta \geqslant y^G,\ Y^B\eta \leqslant y^B,\ \eta = 1,\ \eta \geqslant 0 \right] \tag{18}$$

In the PPS of DDF and SBM, η is defined as in model (2). Controllable inputs are formulated as inequality, implying that these inputs are strongly disposable. Because uncontrollable inputs are beyond the management's control, their levels are considered fixed, leading to the formulation of the uncontrollable input constraint as an equality. However, to prevent potential issues of infeasibility, the uncontrollable input constraint was formulated as an inequality. The linear combination of DMUs with controllable inputs $X^C\eta$ is less than or equal to the actual level of its related inputs x^C, but equal to the actual level of its related uncontrollable inputs of DMUo (i.e., $X^C\eta = x^{NC}$). The weak disposability assumption,[5] on the other hand, has been imposed on both desirable and undesirable outputs. Therefore, the inequalities between desirable and undesirable outputs are consistent with the assumption that these outputs are null-joint[6] (Li & Hu, 2012). The linear combination of desirable outputs ($Y^G\eta$) is greater than or equal to the actual level of its related factor of DMUo. By contrast, the linear combination of undesirable outputs ($Y^B\eta$) is formulated similarly to that of controllable inputs because the feature of undesirable outputs in decreasing their levels is the same as that of controllable inputs. Reference sets, in general, reveal the actual level of controllable and uncontrollable inputs, as well as desirable and undesirable outputs, when compared to their linear combinations (a set of all efficient DMUs, which can be a benchmark for inefficient DMUs). It should be noted that in (18) is proposed under the VRS technology.

The following are the mathematical nomenclatures of data and variables in the two new models proposed:

4.0.1. Parameters

i: 1, …, m_1 index of controllable (*i.e.*, discretionary) inputs.
m_1: number of controllable inputs.
l: 1, …, m_2 index of uncontrollable (*i.e.*, non-discretionary) inputs.
m_2: number of uncontrollable inputs.
r_1: 1, …, s_1 index of desirable (*i.e.*, good) outputs.
s_1: number of desirable outputs.
r_2: 1, …, s_2 index of undesirable (*i.e.*, bad) outputs.
s_2: number of undesirable outputs.
j: 1, …, J index of evaluated DMUs.
o: subscript factor revealing a specific DMU whose efficiency is being measured.
J: number of DMUs whose efficiency is being measured.
x_{io}^C: positive amount of controllable input i of DMUo.
x_{lo}^{NC}: positive amount of uncontrollable input l of DMUo.
$y_{r_1o}^G$: positive amount of desirable output r_1 of DMUo.
$y_{r_2o}^B$: positive amount of undesirable output r_2 of DMUo.

4.0.2. Variables

$(\eta_1, \ldots, \eta_J)$: non-negative multipliers used for calculating a reference set of evaluated DMUs in the data set.
a_{io}^{C-}: controllable input slack (*i.e.*, potential reduction) of controllable input i of DMUo.
a_{lo}^{NC-}: uncontrollable input slack of uncontrollable input l of DMUo.
$b_{r_1o}^{G+}$: desirable output slack (*i.e.*, potential expansion) of desirable output r_1 of DMUo.
$b_{r_2o}^{B-}$: undesirable output slack (*i.e.*, potential reduction) of undesirable output r_2 of DMUo.

4.1. Improved non-oriented DDF model

By relying on the theoretical concepts of the DDF model introduced in Chambers et al. (1998), Diabat et al. (2015), Ramli et al. (2013), and Yahia et al. (2018), we derive a non-oriented DDF model in the presence of uncontrollable inputs and undesirable outputs (DDF-NCIUO) for evaluating DMUo as formulated in (19):

$$\min\ \tau_o^{\mathrm{NCIUO}} = \frac{1-\beta_o}{1+\beta_o} \tag{19}$$

subject to:

$$\sum_{j=1}^{J} x_{ij}^C \eta_j \leqslant x_{io}^C (1-\beta_o) \quad i = 1, \ldots, m_1, \tag{19a}$$

$$\sum_{j=1}^{J} x_{lj}^{NC} \eta_j \leqslant x_{lo}^{NC} \quad l = 1, \ldots, m_2, \tag{19b}$$

$$\sum_{j=1}^{J} y_{r_1j}^G \eta_j \geqslant y_{r_1o}^G (1+\beta_o) \quad r_1 = 1, \ldots, s_1, \tag{19c}$$

$$\sum_{j=1}^{J} y_{r_2j}^B \eta_j \leqslant y_{r_2o}^B (1-\beta_o) \quad r_2 = 1, \ldots, s_2, \tag{19d}$$

$$\sum_{j=1}^{J} \eta_j = 1 \quad (19e)$$

[5] The undesirable outputs have two facets under the disposability technology. On the one hand, some undesirable outputs, such as CO_2 emissions from a coal-fired power generation, can only be formulated under the weak disposability technology. On the other hand, the strong disposability technology can be imposed on DEA models dealing with some undesirable outputs, such as SO_2 emissions (Yang & Pollitt, 2009). For simplicity, this paper makes the assumption of weak disposability on the constraint of undesirable outputs.

[6] It reveals that if desirable outputs of a production process are produced, then some undesirable outputs should be produced as well (Arabi et al., 2014).

$$\eta_j \geqslant 0, \qquad j = 1, \ldots, J \tag{19f}$$

$$\beta_o \in [0, 1)$$

$$x^C_{io}, x^{NC}_{lo}, y^G_{r_1o}, y^B_{r_2o} \in \mathbb{R}^{m_1+m_2+s_1+s_2}_+.$$

Model (19) is a non-oriented DDF model that takes uncontrollable inputs and undesirable outputs into account. A DMU_o assessed by model (19) guarantees efficient status if and only if $\beta_o = 0$, and all slacks are zero (see Proposition 1). Otherwise, DMU_o is inefficient. The DDF in model (19) depends on a direction vector value (i.e., β_o) to identify the proportionate decrease in controllable inputs and undesirable outputs and the increase in desirable outputs. The uncontrollable input constraint (19b) should not incorporate the directional value since the levels of these inputs are beyond the control of the DMU's management. Despite this, constraint (19b) is formulated as an inequality to preempt potential infeasibility issues (refer to Rashidi et al., 2015, p.5; Taleb et al., 2018, p.17). Such efficiency minimisation should only be identified based on the directional vectors of controllable inputs, as well as desirable and undesirable outputs. However, the constraint of uncontrollable inputs should be considered in the efficiency model to ensure fair evaluations (Saati et al., 2011, p. 47). The proposed DDF model satisfies the feasibility condition under the CRS and VRS technologies because it simultaneously takes into account the reduction in controllable inputs and undesirable outputs, as well as the increase in desirable outputs, by considering their directional vectors.

Further, it is costly to reduce undesirable outputs without a reduction in desirable outputs. Therefore, desirable outputs should also be reduced to make sure that the new vector of controllable inputs, uncontrollable inputs, desirable outputs, and undesirable outputs (x^C, x^{NC}, y^G, y^B) is feasible. In particular, each reduction in undesirable outputs cannot occur freely (Yahia et al., 2018, p.122; Ramli et al., 2013, p.287; Toloo et al., 2018, p.3). Thus, the assumption of weak disposability has been imposed on the undesirable output constraint (19d). As a result, this constraint is formulated as an inequality. Since the DDF model (19) is a generalised version of the DIDF and DODF models, whose target functions are considered in models (6) and (7), the efficiency score is the product of the distance in the DIDF model and the distance in the DODF model (*i.e.*, models (6) and (7)).

Proposition 4. *DMU_o is Pareto-Koopmans efficient in the DDF-NCIUO model if and only if its directional input and output vector values are zero* (i.e., $\beta_o = 0$). *This condition is equivalent to the efficiency score of DMU_o being equal to one* (i.e., $\tau^{\text{NCIUO}}_o = 1$). *If the directional vector value lies in the interval* $(0, 1)$, *then DMUo is inefficient.*

4.2. Improved non-oriented SBM model

We modify the SBM model of Tone (2003) by incorporating uncontrollable inputs into a specific input constraint of the model. The slack of uncontrollable inputs is omitted from the model's (20) target function because the efficiency evaluation only depends on controllable variables, whereas the slack of uncontrollable inputs can be considered in its relevant constraint to avoid the infeasibility issue (see Esmaeili, 2009, p.4823). Thus, under the VRS technology, an SBM for the case of uncontrollable inputs and undesirable outputs (SBM-NCIUO) is proposed for evaluating DMU_o, as follows:

$$\delta^{\text{NCIUO}}_o = \min \frac{1 - \frac{1}{m_1}\left(\sum_{i=1}^{m_1} \frac{a^{C-}_i}{x^C_{io}}\right)}{1 + \frac{1}{s_1+s_2}\left(\sum_{r_1=1}^{s_1} \frac{b^{G+}_{r_1}}{y^G_{r_1o}} + \sum_{r_2=1}^{s_2} \frac{b^{B-}_{r_2}}{y^B_{r_2o}}\right)} \tag{20}$$

subject to:

$$\sum_{j=1}^{J} x^C_{ij}\eta_j = x^C_{io} - a^{C-}_i \qquad i = 1, \ldots, m_1, \tag{20a}$$

$$\sum_{j=1}^{J} x^{NC}_{lj}\eta_j = x^{NC}_{lo} - a^{NC-}_l \qquad l = 1, \ldots, m_2, \tag{20b}$$

$$\sum_{j=1}^{J} y^G_{r_1j}\eta_j = y^G_{r_1o} + b^{G+}_{r_1} \qquad r_1 = 1, \ldots, s_1, \tag{20c}$$

$$\sum_{j=1}^{J} y^B_{r_2j}\eta_j = y^B_{r_2o} - b^{B-}_{r_2} \qquad r_2 = 1, \ldots, s_2, \tag{20d}$$

constraints (19e and 19f)

$$a^{C-}_i \geqslant 0,\ a^{NC-}_l \geqslant 0,\ b^{G+}_{r_1} \geqslant 0,\ b^{B-}_{r_2} \geqslant 0,$$

$$x^C_{io}, x^{NC}_{lo}, y^G_{r_1o}, y^B_{r_2o} \in \mathbb{R}^{m_1+m_2+s_1+s_2}_+.$$

Model (20) computes the ratio of the average reduction rate of controllable inputs to the average of desirable output expansion and undesirable output reduction based on the slacks of controllable input excesses (a^{C-}_i), desirable output shortfalls ($b^{G+}_{r_1}$), and undesirable output excesses ($b^{B-}_{r_2}$) to determine whether DMU_o is efficient or not. The slacks of controllable inputs and undesirable outputs consider how much controllable inputs and undesirable outputs can be decreased, while the slacks of desirable outputs consider how much desirable outputs can be increased to achieve efficient status. Note that the VRS technology has been imposed on model (20) by adding the convexity constraint (19e). Model (20), which is a fractional programme, must be converted into a linear programme to obtain the optimal efficiency measures. The optimality of the efficiency measures can be obtained by solving the linear programme presented in model (A.2) in Appendix A.

Theorem 2. *The proposed SBM model with uncontrollable inputs and undesirable outputs always achieves feasibility conditions under CRS and VRS technologies.*

Proof. *Radial models do not concurrently impose input and output slacks in their relevant input and output constraints. As a result, the efficiency scores derived from these models do not reflect all the inefficiencies of inefficient DMUs, leading to increased infeasibility, especially in some cases of VRS. In contrast,* Tone (2001) demonstrated that the SBM model (10) is always feasible under CRS and VRS. Therefore, we assert that the proposed non-radial SBM model (20) also achieves feasibility under CRS and VRS because its input and output constraints concurrently address both controllable and uncontrollable input slacks, as well as desirable and undesirable output slacks. Simultaneous consideration of input and output slacks ensures that the right-hand side of each constraint equals the left-hand side of the relevant constraint. This implies that $\sum_{j=1}^{J} x^C_{ij}\eta_j = x^C_{io} - s^{C-}_i, i = 1, \ldots, m_1, s^{C-}_i = \left(x^C_{io} - \sum_{j=1}^{J} x^C_{ij}\right) \geq 0, \sum_{j=1}^{J} x^{NC}_{lj}\eta_j = x^{NC}_{lo} - s^{NC-}_l, l = 1, \ldots, m_2, s^{NC-}_l = \left(x^{NC}_{lo} - \sum_{j=1}^{J} x^{NC}_{lj}\right) \geq 0, \sum_{j=1}^{J} y^G_{r_1j}\eta_j = y^G_{r_1o} + s^{G+}_{r_1}, r_1 = 1, \ldots,$

$$s_1, s^{G+}_{r_1} = \left(\sum_{j=1}^{J} y^G_{r_1j}\eta_j - y^G_{r_1o}\right) \geq 0, \sum_{j=1}^{J} y^B_{r_2j}\eta_j = y^B_{r_2o} - s^{B-}_{r_2}, r_2 = 1, \ldots, s_2, s^{B-}_{r_2} = \left(y^B_{r_2o} - \sum_{j=1}^{J} y^B_{r_2j}\eta_j\right) \geq 0.$$

□Hence, $\left(\eta, s^{C-}_i, s^{NC-}_l, s^{G+}_{r_1}, s^{B-}_{r_2}\right)$ is a feasible solution to the proposed SBM model.

Proposition 5. *DMU_o is efficient under SBM-NCIUO* (20) *if and only if its efficiency score is equal to one* ($\delta^{\text{NCIUO}}_o = 1$). *This condition is satisfied if all the slacks of controllable inputs, desirable outputs, and undesirable outputs are zero, while the slack of uncontrollable inputs is not required to be zero because its mathematical term is not considered in the model's* (20) *target function. Otherwise, DMU_o is inefficient.*

At this point, the efficiency scores using DDF-NCIUO (19) and SBM-NCIUO (20) can be calculated. The computed ratio of efficiency scores resulting from model (20) to DDF-NCIUO efficiency scores represents a new directional mix-efficiency measure with uncontrollable inputs and undesirable outputs (MIX-NCIUO). MIX-NCIUO of DMUo (ψ_o^{NCIUO}) is calculated as in (21):

$$\psi_o^{\text{NCIUO}} = \frac{\delta_o^{\text{NCIUO}}}{\tau_o^{\text{NCIUO}}}, \quad \tau_o^{\text{NCIUO}} \neq 0 \tag{21}$$

5. Numerical results and discussion of net-zero

5.1. Data and variables

To illustrate the applicability and usefulness of the proposed efficiency measures, data on the eco-efficiency of 25 OECD countries were used. The data set was retrieved from Rashidi et al.'s (2015) study and is presented in Table B1 in Appendix B. Each country was assigned the status of DMU and its efficiency measures were calculated. To measure the OECD countries' directional mix-efficiency, six significant factors related to controllable and uncontrollable inputs, as well as desirable and undesirable outputs, were chosen (Fig. 1). The controllable inputs are the labour force, coal consumption, and petroleum consumption, while the uncontrollable input is the average precipitation. The desirable and undesirable outputs are gross domestic product (GDP) and CO_2 emissions. Table 3 presents the characteristics of the data set for the 25 OECD countries.

To effectively discriminate between efficient DMUs, the number of evaluated DMUs in a DEA model should be, according to a common rule of thumb, at least three times larger than the total number of inputs and outputs. Otherwise, the problem of discrimination may arise. Furthermore, DEA approaches are based on the assumption that the relationship between inputs and outputs is linear (Lu, 2012; Taleb et al., 2019). The relationship between GDP, which is commonly used in macroeconomics, and the labour force was positive, *i.e.*, 0.629. As a result, the labour force plays a crucial role in economic prosperity. Furthermore, the relationship between GDP and CO_2 emissions was strongly positive, *i.e.*, 0.920. Färe et al. (2004) obtained a comparable result. Average precipitation is one of the most important environmental factors in pollution reduction. Furthermore, because their relationship was positive, *i.e.*, 0.631, both petroleum and coal consumption are relevant environmental factors for measuring eco-efficiency.

5.2. Eco-efficiency analysis of OECD countries

Table 4 reports the efficiency scores obtained from models (19), (20), and (21). The efficiency scores of models (19) and (20) under the CRS technology are presented in columns 2 and 3, while their efficiency scores under the VRS technology are presented in columns 5 and 6. The efficiency scores were then used to compute the directional mix-efficiency measure under both technologies, which is shown in columns 4 and 7. Lingo software version 14 was used to obtain all of the optimal efficiency score values.

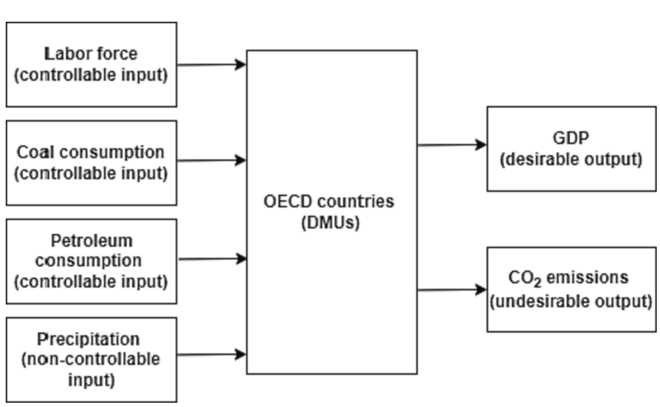


Fig. 1. Production model of OECD countries.

According to Propositions 4 and 5 under the CRS technology, only six countries achieved eco-efficiency (efficiencies of 100 %) in the DDF-NCIUO (19) and SBM-NCIUO (20) models, as reported in columns 2 and 3. As a result, they achieved directional mix-efficiency under a full-efficient status. This indicates that these are the countries in the sample with the most effective environmental performance. It can also be seen that the efficiency scores of the majority of eco-inefficient countries produced by model (19) under CRS and VRS were rather high. In contrast, among the eco-inefficient countries yielded by model (20) under CRS, 11 inefficient countries have efficiency scores of less than 40 % (*i.e.*, Australia, Canada, the Czech Republic, Greece, Hungary, The Netherlands, Poland, Portugal, Slovakia, South Korea, and Turkey). In the same context as model (19), 10 eco-inefficient countries achieved low efficiencies (below 40 %) under VRS. As a result, inefficient countries that scored high on efficiency under models (19) and (20) also scored high on directional mix-efficiency.

According to columns 5 and 6 of Table 4 under the VRS technology, 14 countries achieved the eco-efficient status in model (19), while 10 countries achieved the eco-efficient status in model (20) because it was considered that Australia, Canada, Hungary, and Slovakia were eco-inefficient. The efficiency scores of the DDF model (19) were observed to be greater than or equal to those of the SBM model (20), indicating the main relationship between the DDF and SBM models (see Färe & Grosskopf, 2010, p. 321). Column 7 shows that there are six directional mix-inefficient countries with efficiency scores obtained from Eq. (21) under VRS that are less than 40 % (*i.e.*, Australia, the Czech Republic, Hungary, Poland, Slovakia, and Turkey). These countries were low-efficient under model (20), while their efficiency scores obtained from model (19) under VRS were relatively high, causing low mix-efficiency.

Under the RTS technology, the CRS efficiency scores of models (19) and (20) were less than or equal to the VRS efficiency scores of both models. In contrast, not all directional mix-inefficiency scores under CRS were lower than those under VRS, as shown in columns 4 and 7. The main reason for that is that the difference between the efficiency scores obtained from the SBM model (20) and the DDF model (19) (*i.e.*, the numerator and denominator of Eq. (21)) under VRS is greater than that obtained from CRS. For example, the difference between the SBM and DDF efficiency scores of Australia under CRS is 0.0928, while it is equal to 0.6265 under VRS for the same DMU. Therefore, we deduce that the efficiency scores of input or output mix-efficiency under VRS need not be greater than those under CRS (Taleb, 2023).

Spearman's rank correlation[7] was used to ensure that the efficiency scores obtained from models (19), (20), and Eq. (21) under the CRS and VRS technologies were consistent. The correlations between the efficiency scores of models (19) and (20) under CRS and VRS were strongly positive, *i.e.*, 0.9017 with a *p*-value of zero and 0.7364 with a *p*-value of 0.00003, respectively. More specifically, the correlation between the efficiency scores of model (19) under the two technologies was 0.6475 with a *p*-value of 0.0004. The efficiency scores of model (20) under CRS and VRS were also strongly correlated, i.e., 0.8967 with a *p*-value of zero. Similarly, the correlation coefficient resulting from Eq. (21) between the CRS and VRS directional mix-efficiency scores was strongly positive, *i.e.*, 0.8094 with a *p*-value of zero. Up to this point, the CRS and

[7] A statistical measure that analyses the linear relationship between two variables; thus, it is suitable for use in DEA. The value of correlation ranges between +1 and –1. The value of +1 indicates that the relationship between the two variables is positive, implying that an increase in one variable will result in an increase in the other variable. In contrast, the value of –1 indicates that the relationship between the two variables is negative, implying that an increase in one variable will result in a decrease in the other variable.

Table 3
Characteristics of the data set of the 25 OECD countries.

Factors	Mean	Std. dev.	Unit	Category	Notation
Labour force[1]	186.618	208.630	One hundred thousand workers	Controllable input	x_{1j}^{C}
Coal[2] consumption	57.933	71.956	Million tons/year	Controllable input	x_{2j}^{C}
Petroleum[3] consumption	102.256	113.018	Ten thousand barrels/day	Controllable input	x_{3j}^{C}
Average precipitation[4]	876.64	359.802	Millimetre/year	Uncontrollable input	x_{1j}^{NC}
GDP[5]	1003.727	1135.677	Billion US$	Desirable output	y_{1j}^{G}
CO_2 emissions[6]	269.554	290.201	Million tons/year	Undesirable output	y_{1j}^{B}

[1] It refers to everyone who meets the requirements to be counted among the employed.
[2] It represents one of the main energy sources used in the industrial, transportation, education, and residential sectors.
[3] It refers to both crude oil occurring unprocessed and petroleum products consisting of refined oil.
[4] Condensation of atmospheric water that falls under clouds contributes to producing precipitation, rain, drizzle, sleet, and snow are the main forms of precipitation. Therefore, precipitation and temperature are obvious instances of uncontrollable factors in the environment (Rashidi et al., 2015).
[5] A monetary measure of economic activities related to final goods and services production.
[6] They involve carbon dioxide emissions produced during the burning of fossil fuels, as well as consumption of liquid, solid, and gas fuels.

Table 4
Results of various efficiency measures.

DMU	Efficiency scores under CRS technology			Efficiency scores under VRS technology		
	τ^{NCIUO} Model (19)	δ^{NCIUO} Model (20)	ψ^{NCIUO} Model (21)	τ^{NCIUO} Model (19)	δ^{NCIUO} Model (20)	ψ^{NCIUO} Model (21)
Australia	0.4595	0.3667	0.7980	1	0.3735	0.3735
Austria	0.7645	0.4347	0.5686	0.7827	0.4523	0.5778
Belgium	0.8698	0.4469	0.5137	0.9176	0.4534	0.4941
Canada	0.8755	0.3851	0.4398	1	0.4425	0.4425
The Czech Republic	0.5090	0.1724	0.3387	0.6511	0.2181	0.3349
Denmark	0.9853	0.6952	0.7055	1	1	1
Finland	0.7928	0.4004	0.5050	1	1	1
France	1	1	1	1	1	1
Germany	1	1	1	1	1	1
Greece	0.5638	0.2329	0.4130	0.6242	0.2776	0.4447
Hungary	0.5024	0.1810	0.3602	1	0.3211	0.3211
Iceland	0.9525	0.5700	0.5984	1	1	1
Italy	0.9452	0.7196	0.7613	0.9501	0.7196	0.7573
Japan	0.7276	0.4120	0.5662	1	1	1
Luxembourg	1	1	1	1	1	1
The Netherlands	0.5788	0.2751	0.4752	0.5953	0.2778	0.4666
Norway	1	1	1	1	1	1
Poland	0.5018	0.1794	0.3575	0.6192	0.1913	0.3089
Portugal	0.4693	0.2218	0.4726	0.5294	0.2685	0.5071
Slovakia	0.5949	0.1576	0.2649	1	0.2385	0.2385
South Korea	0.4428	0.1953	0.4410	0.4439	0.2070	0.4663
Spain	0.7124	0.4509	0.6329	0.9870	0.4526	0.4585
Sweden	1	1	1	1	1	1
Turkey	0.5795	0.2397	0.4136	0.7115	0.2450	0.3443
UK	1	1	1	1	1	1
Average	**0.7530**	**0.5094**	**0.6250**	**0.8724**	**0.6055**	**0.6614**

VRS efficiency scores generated by the three proposed models for the 25 OECD countries have been calculated and interpreted. In addition, the correlation coefficients between the resulting efficiency scores were investigated. In order to illustrate the effect of proposing new factors on the efficiency scores of standard models, the overall performance of the proposed new efficiency models must be evaluated.

5.3. Evaluation of the overall performance of the proposed models

Because the proposed directional mix-efficiency measure in (21) is dependent on the proposed non-oriented DDF in model (19) and the proposed SBM in model (20), Spearman's rank correlation is examined between the efficiency scores of model (19) and those of the standard DDF model in (2), as well as the efficiency scores of model (20) and those of the standard SBM model in (10), under the CRS and VRS technologies. This comparison aims to ensure that uncontrollable inputs and undesirable outputs have been appropriately integrated into the standard models by achieving a strong positive correlation. In this case, the overall performance of the two proposed efficiency models over the evaluation period has been achieved. The efficiency scores of the standard models and the proposed models under CRS and VRS are reported in Table 5.

Table 5 reveals the efficiency scores of the standard models in (2) and (10) under CRS and VRS in columns 2 to 5, while the efficiency scores of the proposed models in (19) and (20) under CRS and VRS, retrieved from Table 4, are presented in columns 6 to 9. To be noted that the efficiency levels resulting from a DEA model are sensitive to different integrated factors (Taleb, Khalid, Emrouznejad et al., 2023). In particular, because DEA is a non-parametric approach, the linear relationship between the efficiency scores obtained from two compared DEA models could be measured using the non-parametric measure of Spearman's correlation coefficient. Therefore, it is imperative to establish whether the resultant rankings from models (19) and (20), as well as models (2) and (10), are similarly affected. In doing so, Spearman's rank correlation coefficients between the efficiencies of the standard models and the proposed models were examined, as reported in Table 6.

Table 6 reports the rank correlation coefficients between the standard model in (2) and the proposed model in (19), as well as the

Table 5
The efficiency scores of the standard and proposed models.

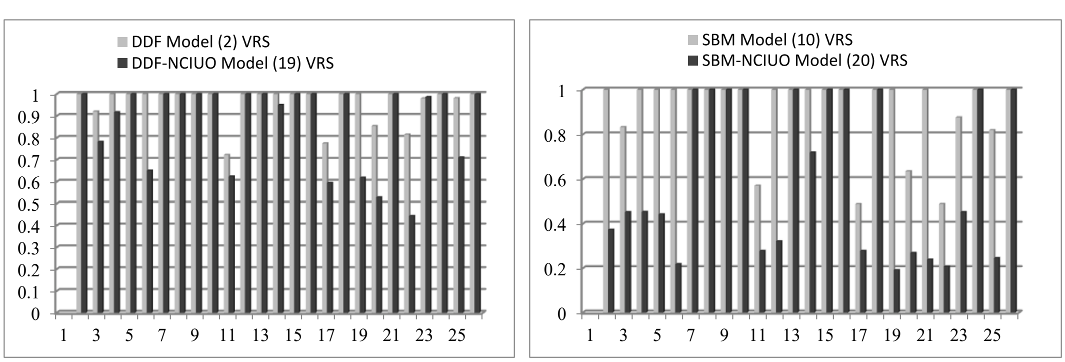
Country	Efficiency scores of the standard models				Efficiency scores of the proposed models			
	DDF Model (2) CRS	DDF Model (2) VRS	SBM Model (10) CRS	SBM Model (10) VRS	DDF-NCIUO Model (19) CRS	DDF-NCIUO Model (19) VRS	SBM-NCIUO Model (20) CRS	SBM-NCIUO Model (20) VRS
Australia	1	1	1	1	0.4595	1	0.3667	0.3735
Austria	0.8981	0.9211	0.7061	0.8326	0.7645	0.7827	0.4347	0.4523
Belgium	0.9979	1	0.8993	1	0.8698	0.9176	0.4469	0.4534
Canada	1	1	1	1	0.8755	1	0.3851	0.4425
The Czech Republic	1	1	1	1	0.5090	0.6511	0.1724	0.2181
Denmark	0.9961	1	0.8163	1	0.9853	1	0.6952	1
Finland	1	1	1	1	0.7928	1	0.4004	1
France	1	1	1	1	1	1	1	1
Germany	1	1	1	1	1	1	1	1
Greece	0.6981	0.7223	0.4554	0.5707	0.5638	0.6242	0.2329	0.2776
Hungary	0.9859	1	0.5306	1	0.5024	1	0.1810	0.3211
Iceland	0.7152	1	0.4295	1	0.9525	1	0.5700	1
Italy	1	1	1	1	0.9452	0.9501	0.7196	0.7196
Japan	0.9232	1	0.8289	1	0.7276	1	0.4120	1
Luxembourg	1	1	1	1	1	1	1	1
The Netherlands	0.6830	0.7757	0.4311	0.4886	0.5788	0.5953	0.2751	0.2778
Norway	1	1	1	1	1	1	1	1
Poland	1	1	1	1	0.5018	0.6192	0.1794	0.1913
Portugal	0.7362	0.8543	0.4859	0.6356	0.4693	0.5294	0.2218	0.2685
Slovakia	1	1	1	1	0.5949	1	0.1576	0.2385
South Korea	0.8005	0.8156	0.4814	0.4889	0.4428	0.4439	0.1953	0.2070
Spain	0.8349	0.9805	0.7454	0.8762	0.7124	0.9870	0.4509	0.4526
Sweden	0.9380	1	1	1	1	1	1	1
Turkey	0.9280	0.9815	0.7177	0.8195	0.5795	0.7115	0.2397	0.2450
UK	1	1	1	1	1	1	1	1

Table 6
Matrix correlations of standard and proposed efficiencies.

	DDF model (19)-CRS	DDF model (19)-VRS	SBM model (20)-CRS	SBM model (20)-VRS
DDF model (2)-CRS	0.3647	–	–	–
DDF model (2)-VRS	–	0.7527	–	–
SBM model (10)-CRS	–	–	0.3181	–
SBM model (10)-VRS	–	–	–	0.5098

standard model in (10) and the proposed model in (20), under CRS and VRS. The obtained efficiencies are positive and relatively highly correlated, especially those obtained under the VRS technology. Therefore, the overall performance of the proposed models has been achieved. This indicates that uncontrollable inputs and undesirable outputs have been properly integrated into the standard models and confirms that the efficiency scores of the proposed models have been calculated efficiently. Additionally, varying levels of efficiency indicate that the rankings are significant and remarkable (Johnes & Li, 2008, p.689).

To visually illustrate the overall performance evaluation of the proposed models, graphical representations are displayed in Fig. 2. The figure showcases the efficiency scores resulting from both the standard and the proposed DDF models in (a) and those generated by the standard and proposed SBM models in (b). In both figures, it is evident that the efficiency scores generated by the proposed DDF and SBM models, depicted by black bars, are either less than or equal to those generated by the corresponding standard models, depicted by grey bars, for all evaluated OECD countries. Lower efficiency scores coincide with more substantial decreases in inputs and increases in outputs, contributing to the accurate efficiency performance of the DMUs by eliminating bias in efficiency measures. Consequently, uncontrollable inputs and undesirable outputs significantly impact the proposed DDF and SBM models.



(a) Efficiency scores of the DDF models. (b) Efficiency scores of the SBM models.

Fig. 2. Evaluating the performance of the proposed DDF and SBM models.

Table 7
Classifications of RTS for the 25 OECD countries.

Country	Efficiency measures of model (19)				Efficiency measures of model (20)			
	τ_{CRS}^{NCIUO}	τ_{VRS}^{NCIUO}	$\sum\eta$	RTS	δ_{CRS}^{NCIUO}	δ_{VRS}^{NCIUO}	$\sum\eta$	RTS
Australia	0.4595	1	0.6297	Increasing	0.3667	0.3735	0.4577	Increasing
Austria	0.7645	0.7827	0.7914	Increasing	0.4347	0.4523	0.7862	Increasing
Belgium	0.8698	0.9176	0.6192	Increasing	0.4469	0.4534	0.6226	Increasing
Canada	0.8755	1	0.5786	Increasing	0.3851	0.4425	0.5337	Increasing
The Czech Republic	0.5090	0.6511	0.4815	Increasing	0.1724	0.2181	0.3238	Increasing
Denmark	0.9853	1	0.5036	Increasing	0.6952	1	0.5046	Increasing
Finland	0.7928	1	0.3859	Increasing	0.4004	1	0.3865	Increasing
France	1	1	1	Constant	1	1	1	Constant
Germany	1	1	1	Constant	1	1	1	Constant
Greece	0.5638	0.6242	0.4726	Increasing	0.2329	0.2776	0.4696	Increasing
Hungary	0.5024	1	0.4175	Increasing	0.1810	0.3211	0.2542	Increasing
Iceland	0.9525	1	0.0490	Increasing	0.5700	1	0.0518	Increasing
Italy	0.9452	0.9501	0.8615	Increasing	0.7196	0.7196	0.8513	Increasing
Japan	0.7276	1	1.9355	Decreasing	0.4120	1	1.6792	Decreasing
Luxembourg	1	1	1	Constant	1	1	1	Constant
The Netherlands	0.5788	0.5953	1.1635	Decreasing	0.2751	0.2778	0.6233	Increasing
Norway	1	1	1	Constant	1	1	1	Constant
Poland	0.5018	0.6192	0.4463	Increasing	0.1794	0.1913	0.4319	Increasing
Portugal	0.4693	0.5294	0.6778	Increasing	0.2218	0.2685	0.4594	Increasing
Slovakia	0.5949	1	0.2680	Increasing	0.1576	0.2385	0.1562	Increasing
South Korea	0.4428	0.4439	0.9430	Increasing	0.1953	0.2070	0.9521	Increasing
Spain	0.7124	0.9870	0.6273	Increasing	0.4509	0.4526	0.6172	Increasing
Sweden	1	1	1	Constant	1	1	1	Constant
Turkey	0.5795	0.7115	0.4554	Increasing	0.2397	0.2450	0.4375	Increasing
UK	1	1	1	Constant	1	1	1	Constant

5.4. Estimating the returns-to-scale for OECD countries

The proposed models (19) and (20) were used to compute the efficiency scores of 25 OECD countries under the CRS and VRS technologies. Each of these models is employed to identify the RTS region of the evaluated countries.

As a result, the RTS region for each country is identified, whether it is CRS, IRS, or DRS. Table 7 shows the classification of their regions. The three conditions proposed by Zhu and Shen (1995) are used to estimate the nature of RTS. These conditions depend on the value of the intensity variable,η_j, which corresponds to the CRS efficiency scores. Thus, the nature of RTS can be defined as follows:

i. If the efficiency scores of DMU_o obtained from the DDF-NCIUO model (19) under CRS and VRS are equal (i.e., $\tau_{CRS}^{NCIUO} = \tau_{VRS}^{NCIUO}$), and the intensity factor value corresponding to the CRS efficiency score is one (i.e.,$\sum_{j=1}^{J}\eta_j = 1$), then the returns-to-scale of DMU_o function under a CRS region.
ii. If the efficiency scores of DMU_o resulting from model (19) are not equal under CRS and VRS (i.e., $\tau_{CRS}^{NCIUO} \neq \tau_{VRS}^{NCIUO}$), and the intensity factor value corresponding to the CRS efficiency score is less than one ($\sum_{j=1}^{J}\eta_j < 1$), then DMU_o functions under an IRS region.
iii. If ($\tau_{CRS}^{NCIUO} \neq \tau_{VRS}^{NCIUO}$) of DMU_o obtained from model (19) are not equal under CRS and VRS, and the intensity factor value corresponding to the CRS efficiency score is greater than one ($\sum_{j=1}^{J}\eta_j > 1$), then DMU_o functions under a DRS region.

The three conditions of RTS are also used to identify the RTS region based on the SBM-NCIUO model (20), as displayed in columns 6 to 9 in Table 7. An inefficient DMU can be projected onto the efficient frontier to estimate its RTS classification (Seiford & Zhu, 1999). Here, it is worth noting that RTS provides a clear meaning only if DMU_o can be projected onto the efficient frontier constructed by the VRS technology.

Columns 2, 3, 6, and 7 contain the efficiency scores of models (19) and (20) under CRS and VRS, retrieved from Table 4. Using the three RTS conditions, the identified RTS regions by each of the proposed models in (19) and (20) are shown in columns 5 and 9. The findings reveal that six countries—France, Luxembourg, Germany, Norway, Sweden, and the UK—function in the CRS region in models (19) and (20) because they achieved efficient status under CRS and VRS. This implies that their production frontier is operating under the optimal condition. The remaining 19 countries are CRS-inefficient and are classified in the IRS or DRS region.

On the IRS side, the majority of the CRS-inefficient countries, which are 17 countries in model (19) and 18 countries in model (20), function in the IRS region. This indicates that they were not operating at optimal levels during the time period under consideration. They had excesses in their controllable inputs and/or undesirable outputs, and any additional amount of desirable output would result in higher returns. Therefore, to increase their environmental performance and achieve the most productive scale size, these countries must focus on their controllable inputs and undesirable outputs (Ahn et al., 1989).

On the DRS side, among the 19 CRS-inefficient countries, two countries—Japan and the Netherlands—function in the DRS region in model (19), while only Japan functions in the DRS region in model (20) since the DRS of the Netherlands changed to the IRS in model (20). The reason behind that change is that Zhu and Shen's (1995) RTS conditions depend on identifying the sum of intensity variables regarding the CRS technology of DDF and SBM, as well as the CRS and VRS efficiency scores resulting from the DDF and SBM models. Thus, the Netherlands was classified as DRS in DDF and IRS in SBM. The DRS reflects that any additional amounts of controllable inputs and/or undesirable output will decrease the returns for these countries (*i.e.*, Japan and the Netherlands). This suggests that these inefficient countries should increase their desirable outputs in order to achieve better environmental performance. Italy's inefficiency scores in model (20) were the same under both technologies. However, it did not function in the CRS region because the sum of its intensity variable values was not equal to one and its efficiency score was less than one (*i.e.*, inefficient status).

6. Academic and managerial implications

Both the academic and managerial implications of this research are considered in this section. From an academic standpoint, the need for a more effective environment has become increasingly urgent in this age of economic competition. In this regard, this study contributes to the establishment of the main concept revealing the environmental efficiency of OECD countries considering net-zero. This research proposes a new directional mix-efficiency measure considering the impacts of both uncontrollable inputs and undesirable outputs on environmental efficiency. Thus, this research proposes a methodology that incorporates both environmental factors into the standard DDF and SBM efficiency measures. The proposed efficiency measures contribute to the examination of the impacts of uncontrollable inputs and undesirable outputs on environmental efficiency. This research can thus lead to further application and refinement of the proposed environmental efficiency measures. Many DEA studies have been conducted to assess the environmental efficiency of OECD countries (*e.g.*, Emrouznejad, 2003; Emrouznejad & Thanassoulis, 2010; Färe et al., 2004; Rashidi et al., 2015; Zhou & Ang, 2008; Ziolo et al., 2020). However, these studies did not account for the simultaneous effect of uncontrollable inputs and undesirable outputs on the mix-efficiency measure and returns-to-scale, as well as net-zero. Therefore, this research is meaningful and can provide new avenues for future research in another area of ecological economics. The ability of the proposed methodology to measure and analyse the environmental efficiency of OECD countries makes it an effective methodology for assessing the impact of CO_2 emissions on the realisation of net-zero.

From another perspective, this research has several managerial implications. First, the findings from the proposed efficiency measures confirm that among the ten eco-efficient countries, six of them (i.e., France, Germany, Luxembourg, Norway, Sweden, and the UK) have the capacity to achieve net-zero CO_2 emissions during the examined period, as these six countries have achieved the eco-efficient status under all of the proposed efficiency measures and have been operating in the CRS region. Second, the classification of inputs as controllable and uncontrollable, as well as outputs as desirable and undesirable, enables the management of OECD countries to determine sources of inefficiency, allowing for the improvement of eco-inefficient countries. Third, based on the proposed efficiency measures, the OECD management can determine the returns-to-scale for efficient and inefficient countries in order to identify the impact of a proportional increase in inputs that leads to a proportional increase or decrease in outputs. Fourth, the measure of the directional output mix-efficiency can be used to determine the degree to which a desirable or undesirable output should change in order to achieve eco-efficient status. Fifth, achieving net-zero CO_2 emissions can support the management of eco-efficient OECD countries in order to achieve prosperity in various sectors.

7. Conclusions, limitations, and directions for future research

In real-life production processes, some inputs are uncontrollable, and some outputs are undesirable. Conventional DEA models cannot handle these situations because they assume that all inputs are controllable, and all outputs are desirable. As a result, the models' reference targets may be inaccurate, leading decision-makers to be misled. To obtain a more accurate efficiency measure, this paper proposes a new directional mix-efficiency measure with uncontrollable inputs and undesirable outputs, called MIX-NCIUO. MIX-NCIUO is based on two proposed non-oriented models, which are DDF and SBM. Furthermore, the efficiency measures of the proposed models have been obtained under the two RTS technologies (*i.e.*, CRS and VRS).

MIX-NCIUO was measured based on the newly proposed non-oriented models to assess the ecological efficiency of 25 OECD countries. Each country is treated as an independent DMU, with controllable and uncontrollable inputs producing desirable and undesirable outputs. The efficiency measures have been obtained under the CRS and VRS technologies. Under CRS, six countries achieved directional mix-efficiency. Meanwhile, ten countries achieved directional mix-efficiency under VRS. This implies that, when compared to other OECD countries, these countries have achieved effective environmental performance. Additionally, the efficiency scores obtained by the proposed non-oriented DDF model were greater than or equal to those obtained by the proposed non-oriented SBM model under CRS and VRS. This indicates that the main relationship between the DDF and SBM models has been achieved. The Spearman's rank correlation between the efficiency scores resulting from the proposed and standard DDF models, as well as between the proposed and standard SBM models under CRS and VRS, was examined to ensure that uncontrollable inputs and undesirable outputs have been properly integrated into the standard models.

The RTS regions of the proposed DDF and SBM models have also been identified. The identification process relied on the intensity variable values, which corresponded to the efficiency scores under the CRS technology of the proposed models. The proposed models under CRS and VRS technologies were used to achieve the eco-efficient status of RTS. The countries that operated in the CRS region are France, Germany, Luxembourg, Norway, Sweden, and the UK, all of which achieved ecological efficiency status. Thus, these countries operated at the optimal level of environmental performance, as well; therefore, they could achieve net-zero CO_2 emissions during the evaluation period. The remaining 19 countries are CRS-inefficient. Thus, they operated in the IRS or DRS region. They did not operate at the level of optimal scale because they had excesses in some of their controllable inputs and/or undesirable outputs, causing some of their desirable outputs to decrease.

This research not only contributes to the research methodology of DEA efficiency measurements, but it also examines the managerial implications necessary to achieve a sustainable economy and eco-efficiency. Our findings indicate that eco-efficiency must be implemented by introducing eco-efficiency technologies. Specifically, eco-efficiency can enhance industries and diversify ecological sources, particularly in developing OECD countries. Thus, the classification of inputs and outputs according to environmental and undesirable factors can effectively contribute to achieving net-zero CO_2 emissions by removing pollution sources and achieving eco-efficiency. Furthermore, the identification of RTS as a DEA approach helps OECD management determine the impact of increasing inputs on output expansion, thereby allowing inefficient countries to improve their inputs and outputs. RTS can be used to identify countries that are most likely to achieve net-zero CO_2 emissions. OECD countries should consider a balance in the relationship between eco-efficiency and sustainable economies when developing strategies based on these policies.

In summary, the proposed efficiency measures can be regarded as an improved combination of features of both uncontrollable inputs and undesirable outputs on non-radial DDF and SBM models. Under the proposed efficiency measures, (i) a new directional mix-efficiency measure considering uncontrollable inputs and undesirable outputs relying on the improved non-oriented DDF and SBM models has been proposed. The measure reflects the degree to which the directional mix-efficiency should change in order to achieve a fully efficient status by decreasing the controllable inputs of labour, coal consumption, and petroleum consumption, increasing desirable output of GDP and reducing undesirable output of environmental pollution from CO_2 emissions; (ii) the infeasibility issue that may occur under VRS and in some cases of CRS in terms of uncontrollable inputs of DDF and SBM models has been tackled; and (iii) the eco-efficiency of OECD countries in the presence of uncontrollable inputs and undesirable outputs has been assessed. To the best of our knowledge, such an assessment has not been considered by any study in the literature on directional mix-efficiency. In fact, no efficiency study has been conducted that considers net-zero CO_2 emissions using non-oriented DEA models in the presence of uncontrollable inputs and undesirable outputs. As a confirmation of the applicability and usefulness of methodical innovation, this paper reveals strong correlation coefficients between the efficiency scores of the conventional models and the proposed models. Thus, it can be deduced that uncontrollable inputs and undesirable outputs have been properly integrated into the standard DDF and SBM models.

This paper has some limitations. First, the proposed efficiency models do not account for the presence of fuzzy data, which may mislead the results. Second, the proposed efficiency measures treat the DMUs as a black box. The black box considers the DMU as a single process, disregarding its internal structure. Third, the data set only covered the year 2007 (*i.e.*, one-year cross-sectional data), and some data regarding the main sources of air pollutant emissions that belong to undesirable outputs, such as diesel particulate matter (PM), sulphur oxide (SO_2), and nitrogen oxide (NO_x), were not available. Fourth, the proposed efficiency measures did not simultaneously consider energy efficiency and eco-efficiency, despite the fact that these efficiencies can significantly contribute to achieving net-zero CO_2 emissions. Fifth, the proposed efficiency measures were evaluated using the full model, which entails the utilisation of a comprehensive combination of inputs and outputs. However, it is crucial to note that relying solely on the full model may be misleading for decision-makers, given the sensitivity of DEA to the use of an entire set of inputs and outputs.

As a result of the limitations raised, the following future research extensions have been proposed for each of them. First, a fuzzy directional mix-efficiency measure model with uncontrollable inputs and undesirable outputs can be proposed, given that the vast majority of data sets from real-world applications are uncertain or fluctuating. Second, a two-stage network system based on the proposed efficiency measures in which all outputs from the first stage are considered inputs to the second stage (*i.e.*, intermediate products) can be suggested. Third, to capture the extra-dynamic nature of the OECD countries, panel data should be collected and used. In order to investigate the mechanisms underlying changes in eco-efficiency, the proposed efficiency measures can be extended to evaluate changes over time. The Malmquist Productivity Index approach of DDF and SBM models can be proposed for this purpose. Fourth, on the basis of the proposed efficiency measures, it is possible to simultaneously measure energy efficiency and eco-efficiency by categorising inputs as energy, non-energy, controllable, and uncontrollable, and outputs as desirable and undesirable. Fifth, to prevent the potential misguidance of efficiency scores derived from the full model, it is essential to assess the internal validity through sensitivity analysis. We anticipate that these future extensions will be thoroughly documented in published papers.

CRediT authorship contribution statement

Mushtaq Taleb: Conceptualization, Data curation, Formal analysis, Methodology, Project administration, Validation, Visualization, Writing – original draft, Writing – review & editing. **Ali Emrouznejad:** Conceptualization, Formal analysis, Methodology, Project administration, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. **Vincent Charles:** Conceptualization, Formal analysis, Methodology, Project administration, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. **Ruzelan Khalid:** Conceptualization, Data curation, Funding acquisition, Validation, Visualization, Writing – original draft. **Razamin Ramli:** Conceptualization, Data curation, Funding acquisition, Validation, Visualization, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data set was retrieved from Rashidi et al.'s (2015) study.

Acknowledgements

The authors would like to thank the Editor-in-Chief, the Associate Editor, and the three anonymous reviewers for their valuable feedback on the previous version of this manuscript. This research was supported by Ministry of Higher Education (MoHE) of Malaysia through Fundamental Research Grant Scheme (FRGS/1/2019/STG06/UUM/03/1).

Appendix A

To calculate the efficiency score of each DMU using the proposed SBM model (20), its fractional programme should be converted into a linear programme by introducing a positive scalar variable (*i.e.*, $h > 0$), as shown in model (A1) (see Taleb et al., 2023).

$$\phi_o^{\text{NCIUO}} = \min h - \frac{1}{m_1}\left(\sum_{i=1}^{m_1}\frac{a_i^{C-}}{x_{io}^{C}}\right) \tag{A.1}$$

subject to:

$$h + \frac{1}{s_1 + s_2}\left(\sum_{r_1=1}^{s_1}\frac{b_{r_1}^{G+}}{y_{r_1 o}^{G}} + \sum_{r_2=1}^{s_2}\frac{b_{r_2}^{B-}}{y_{r_2 o}^{B}}\right) = 1,$$

$$\sum_{j=1}^{J} x_{ij}^{C}\eta_j = x_{io}^{C} - a_i^{C-} \qquad i = 1, \ldots, m_1,$$

$$\sum_{j=1}^{J} x_{lj}^{NC}\eta_j = x_{lo}^{NC} - a_l^{NC-} \qquad l = 1, \ldots, m_2,$$

$$\sum_{j=1}^{J} y_{r_1 j}^{G}\eta_j = y_{r_1 o}^{G} + b_{r_1}^{G+} \qquad r_1 = 1, \ldots, s_1,$$

$$\sum_{j=1}^{J} y_{r_2 j}^{B}\eta_j = y_{r_2 o}^{B} - b_{r_2}^{B-} \qquad r_2 = 1, \ldots, s_2,$$

$$\sum_{j=1}^{J} \eta_j = 1,$$

Now, we define:

$S_i^{C-} = ha_i^{C-},\ S_{r_1}^{G+} = hb_{r_1}^{G+},\ S_{r_2}^{B-} = hb_{r_2}^{B-},\ \Lambda_j = h\lambda_j$

Thus, the linear programme of model (A.1) in $S_i^{C-},\ S_{r_1}^{G+},\ S_{r_2}^{B-}$, and Λ_j is formulated in (A2) as follows:

$$\phi_o^{\text{NCIUO}} = \ \min h - \frac{1}{m_1}\left(\sum_{i=1}^{m_1} \frac{S_i^{C-}}{x_{io}^{C}}\right) \tag{A.2}$$

subject to:

$$h + \frac{1}{s_1 + s_2}\left(\sum_{r_1=1}^{s_1} \frac{S_{r_1}^{G+}}{y_{r_1 o}^{G}} + \sum_{r_2=1}^{s_2} \frac{S_{r_2}^{B-}}{y_{r_2 o}^{B}}\right) = 1$$

$$\sum_{j=1}^{J} x_{ij}^{C}\Lambda_j + S_i^{C-} = hx_{io}^{C} \qquad i = 1, \ldots, m_1,$$

$$\sum_{j=1}^{J} x_{lj}^{NC}\Lambda_j + S_l^{NC-} = hx_{lo}^{NC} \qquad l = 1, \ldots, m_2,$$

$$\sum_{j=1}^{J} y_{r_1 j}^{G}\Lambda_j - S_{r_1}^{G+} = hy_{r_1 o}^{G} \qquad r_1 = 1, \ldots, s_1,$$

$$\sum_{j=1}^{J} y_{r_2 j}^{B}\Lambda_j + S_{r_2}^{B-} = hy_{r_2 o}^{B} \qquad r_2 = 1, \ldots, s_2,$$

$$\sum_{j=1}^{J} \Lambda_j = h,$$

$$S_i^{C-},\ S_{r_1}^{G+},\ S_{r_2}^{B-},\ \Lambda_j \geqslant 0,\ h > 0.$$

We let an optimal solution of (A.2) be denoted by $S_i^{C-*},\ S_{r_1}^{G+*},\ S_{r_2}^{B-*},\ \Lambda_j^{*},\ h^{*}$. Hence, the optimal solution of (A.2) is generated by $\phi^{*} = \kappa^{*}, s_i^{C-*} = S_i^{C-*}/h^{*},\ s_{r_1}^{G+*} = S_{r_1}^{G+*}/h^{*},\ s_{r_2}^{B-*} = S_{r_2}^{B-*}/h^{*},\ \lambda_j^{*} = \Lambda_j^{*}/h^{*}$.

Appendix B

Table B1
The data set of the 25 OECD countries.

Country	Labour force (10^5)	Coal consumption (10^6 tons)	Petroleum consumption (10^4 barrels/day)	Precipitation (millimetre /year)	GDP 10^9 US$	CO_2 10^6 tons
Australia	111.12	156.53	97.6	534	850.32	381.36
Austria	42.14	6.17	29.3	1110	375.04	69.01
Belgium	47.66	7.33	64	847	459.62	102.53
Canada	179.46	63.62	228.3	537	1424.06	560.8
The Czech Republic	51.98	64.25	21.1	677	180.51	123.95
Denmark	28.93	8.75	19	703	311.42	49.87
Finland	26.95	8.4	22.7	536	246.13	63.92
France	286.2	22.43	197.9	867	2582.39	375.68
Germany	415.9	281.44	241.6	700	3323.81	787.24
Greece	49.18	73.95	44.9	652	305.43	98.25
Hungary	42.38	13.02	16	589	136.1	55.86
Iceland	1.82	0.18	2	1940	20.43	2.34
Italy	247.28	27.95	172.8	832	2127.18	461.13
Japan	666.9	207.58	503.7	1668	4356.33	1251.17
Luxemburg	3.43	0.13	6	934	51.32	10.75
The Netherlands	877.96	14.86	111.1	778	782.57	171.77
Norway	25.07	1.32	23.1	1414	393.48	45.12
Poland	169.09	149.58	52	600	425.32	315.2
Portugal	56.18	5.23	30.8	854	231.74	60.87
Slovak	264.92	8.82	8.3	824	84.11	36.6
South Korea	242.16	98.23	224	1247	1049.24	495.84
Spain	221.9	46.15	161.1	636	1441.43	358.24
Sweden	48.38	3.83	35	624	462.51	48.06

(continued on next page)

Table B1 (*continued*)

Country	Labour force (10^5)	Coal consumption (10^6 tons)	Petroleum consumption (10^4 barrels/day)	Precipitation (millimetre /year)	GDP 10^9 US$	CO_2 10^6 tons
Turkey	252.76	108.92	68.9	593	647.16	284.66
UK	305.72	69.67	175.2	1220	2825.53	528.63

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cie.2024.109967.

References

Ahn, T., Arnold, V., Charnes, A., & Cooper, W. W. (1989). DEA and ratio efficiency analyses for public institutions of higher learning in Texas. *Research in Governmental and Nonprofit Accounting, 5*(2), 165–185.

Aigner, D., Lovell, C. A. K., & Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics, 6*(1), 21–37.

Apergis, N., Aye, G. C., Barros, C. P., Gupta, R., & Wanke, P. (2015). Energy efficiency of selected OECD countries: A slacks based model with undesirable outputs. *Energy Economics, 51*, 45–53.

Arabi, B., Munisamy, S., Emrouznejad, A., & Shadman, F. (2014). Power industry restructuring and eco-efficiency changes: A new slacks-based model in Malmquist-Luenberger Index measurement. *Energy Policy, 68*, 132–145.

Avkiran, N. K., Tone, K., & Tsutsui, M. (2008). Bridging radial and non-radial measures of efficiency in DEA. *Annals of Operations Research, 164*(1), 127–138.

Azadi, M., Kazemi Matin, R., Emrouznejad, A., & Ho, W. (2022). Evaluating sustainably resilient supply chains: A stochastic double frontier analytic model considering Netzero. *Annals of Operations Research*, 1–34.

Banker, R. D., & Morey, R. C. (1986). The use of categorical variables in data envelopment analysis. *Management Science, 32*(12), 1613–1627.

Camarero, M., Castillo, J., Picazo-Tadeo, A. J., & Tamarit, C. (2013). Eco-efficiency and convergence in OECD countries. *Environmental and Resource Economics, 55*(1), 87–106.

Chambers, R. G., Chung, Y., & Färe, R. (1996). Benefit and distance functions. *Journal of Economic Theory, 70*(2), 407–419.

Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *Journal of Optimization Theory and Applications, 98*(2), 351–364.

Charles, V., Emrouznejad, A., Gherman, T., & Cochran, J. (2022). Why data analytics is an art. *Significance, 19*(6), 42–45.

Charles, V., Gherman, T., & Zhu, J. (2021). Data Envelopment Analysis and Big Data: A Systematic Literature Review with Bibliometric Analysis. In J. Zhu, & V. Charles (Eds.), *Data-Enabled Analytics: DEA for Big Data* (pp. 1–30). Cham: Springer.

Charles, V., Peretto, C., & Gherman, T. (2016). Technical efficiency analytics for the public banks in Argentina. *ICPE Public Enterprise Half-Yearly Journal, 22*(1), 118–139.

Charles, V., Tsolas, I. E., & Gherman, T. (2018). Satisficing data envelopment analysis: A Bayesian approach for peer mining in the banking sector. *Annals of Operations Research, 269*(1–2), 81–102.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.

Choi, Y., Zhang, N., & Zhou, P. (2012). Efficiency and abatement costs of energy-related CO 2 emissions in China: A slacks-based efficiency measure. *Applied Energy, 98*, 198–208.

Chung, Y. H., Färe, R., & Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management, 51*(3), 229–240.

Cooper, W. W., Park, K. S., & Pastor, J. T. (1999). RAM: A range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity Analysis, 11*(1), 5–42.

Cooper, W. W., Seiford, L. M., & Tone, K. (2006). *Introduction to data envelopment analysis and its uses: With DEA-solver software and references*. Springer Science & Business Media.

Daraio, C., & Simar, L. (2014). Directional distances and their robust versions: Computational and testing issues. *European Journal of Operational Research, 237*(1), 358–369.

Davis, S. J., Lewis, N. S., Shaner, M., Aggarwal, S., Arent, D., Azevedo, I. L., Benson, S. M., Bradley, T., Brouwer, J., & Chiang, Y.-M. (2018). Net-zero emissions energy systems. *Science, 360*(6396).

Debnath, R. M., Shankar, R., & Kumar, S. (2008). Performance evaluation of technical institutions: An application of data envelopment analysis. *Education, Knowledge & Economy, 2*(1), 51–66.

Diabat, A., Shetty, U., & Pakkala, T. P. M. (2015). Improved efficiency measures through directional distance formulation of data envelopment analysis. *Annals of Operations Research, 229*(1), 325–346.

Emrouznejad, A. (2003). An alternative DEA measure: A case of OECD countries. *Applied Economics Letters, 10*(12), 779–782.

Emrouznejad, A., Marra, M., Yang, G., & Michali, M. (2023). Eco-efficiency considering NetZero and data envelopment analysis: A critical literature review. *IMA Journal of Management Mathematics, dpad002*.

Emrouznejad, A., & Thanassoulis, E. (2010). Measurement of productivity index with dynamic DEA. *International Journal of Operational Research, 8*(2), 247–260.

Esmaeili, M. (2009). A slacks-based measure of efficiency for the case of exogenously fixed factors. *Expert Systems with Applications, 36*(3 PART 1), 4822–4825.

Estelle, S. M., Johnson, A. L., & Ruggiero, J. (2010). Three-stage DEA models for incorporating exogenous inputs. *Computers and Operations Research, 37*(6), 1087–1090.

Färe, R., & Charles, V. (2018). A directional distance function approach to void the non-Archimedean in DEA. *Journal of the Operational Research Society, 69*(5), 772–775.

Färe, R., & Grosskopf, S. (2000). Theory and application of directional distance functions. *Journal of Productivity Analysis, 13*(2), 93–103.

Färe, R., & Grosskopf, S. (2004). Modeling undesirable factors in efficiency evaluation: Comment. *European Journal of Operational Research, 157*(1), 242–245.

Färe, R., & Grosskopf, S. (2010). Directional distance functions and slacks-based measures of efficiency. *European Journal of Operational Research, 200*(1), 320–322.

Färe, R., Grosskopf, S., & Hernandez-Sancho, F. (2004). *Environmental performance: An index number approach, 26*, 343–352. https://doi.org/10.1016/j.reseneeco.2003.10.003

Färe, R., Grosskopf, S., Lovell, C. A. K., & Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics*, 90–98.

Färe, R., & Knox Lovell, C. A. (1978). Measuring the technical efficiency of production. *Journal of Economic Theory, 19*(1), 150–162.

Fukuyama, H., & Weber, W. L. (2009). A directional slacks-based measure of technical inefficiency. *Socio-Economic Planning Sciences, 43*(4), 274–287.

Herrero, I., Pascoe, S., & Mardle, S. (2006). Mix efficiency in a multi-species fishery. *Journal of Productivity Analysis, 25*, 231–241.

Hua, Z., Bian, Y., & Liang, L. (2007). Eco-efficiency analysis of paper mills along the Huai River: An extended DEA approach. *Omega, 35*, 578–587.

Iram, R., Zhang, J., Erdogan, S., Abbas, Q., & Mohsin, M. (2020). Economics of energy and environmental efficiency: Evidence from OECD countries. *Environmental Science and Pollution Research, 27*(4), 3858–3870.

Jeudy-Hugo, S., Re, L. Lo, & Falduto, C. (2021). Understanding countries' net-zero emissions targets. *OECD/IEA Climate Change Expert Group Papers*, No. 2021/03, OECD Publishing, Paris.

Johnes, J., & Li, Y. U. (2008). Measuring the research performance of Chinese higher education institutions using data envelopment analysis. *China Economic Review, 19*(4), 679–696.

Kuosmanen, T. (2005). Weak disposability in nonparametric production analysis with undesirable outputs. *American Journal of Agricultural Economics, 87*(4), 1077–1082.

Lee, T., Yeo, G.-T., & Thai, V. V. (2014). Environmental efficiency analysis of port cities: Slacks-based measure data envelopment analysis approach. *Transport Policy, 33*, 82–88.

Li, F., Emrouznejad, A., Yang, G., & Li, Y. (2020). Carbon emission abatement quota allocation in Chinese manufacturing industries: An integrated cooperative game data envelopment analysis approach. *Journal of the Operational Research Society, 71*(8), 1259–1288.

Li, L.-B., & Hu, J.-L. (2012). Ecological total-factor energy efficiency of regions in China. *Energy Policy, 46*, 216–224.

Liu, W. B., Meng, W., Li, X. X., & Zhang, D. Q. (2010). DEA models with undesirable inputs and outputs. *Annals of Operations Research, 173*(1), 177–194.

Lo, S.-F., & Lu, W.-M. (2009). An integrated performance evaluation of financial holding companies in Taiwan. *European Journal of Operational Research, 198*(1), 341–350.

Lotfi, F. H., Jahanshahloo, G. R., & Esmaeili, M. (2007). Sensitivity analysis of efficient units in the presence of non-discretionary inputs. *Applied Mathematics and Computation, 190*(2), 1185–1197.

Lozano, S., & Gutiérrez, E. (2011). Slacks-based measure of efficiency of airports with airplanes delays as undesirable outputs. *Computers & Operations Research, 38*(1), 131–139.

Lozano, S., Gutiérrez, E., & Moreno, P. (2013). Network DEA approach to airports performance assessment considering undesirable outputs. *Applied Mathematical Modelling, 37*(4), 1665–1676.

Lu, W.-M. (2012). Intellectual capital and university performance in Taiwan. *Economic Modelling, 29*(4), 1081–1089.

Mandal, S. K. (2010). Do undesirable output and environmental regulation matter in energy efficiency analysis? Evidence from Indian cement industry. *Energy Policy, 38*(10), 6076–6083.

Mandel, T., Kranzl, L., Popovski, E., Sensfuß, F., Müller, A., & Eichhammer, W. (2023). Investigating pathways to a net-zero emissions building sector in the European

Union: What role for the energy efficiency first principle? *Energy Efficiency, 16*(4), 22.
Noorizadeh, A., Mahdiloo, M., & Farzipoor Saen, R. (2014). A new model for ranking suppliers in the presence of both undesirable and non-discretionary outputs. *International Journal of Services and Operations Management, 17*(3), 280–293.
Pang, R. Z., Deng, Z. Q., & Hu, J. L. (2015). Clean energy use and total-factor efficiencies: An international comparison. *Renewable and Sustainable Energy Reviews, 52*, 1158–1171.
Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). Enhanced DEA Russell graph efficiency measure. *European Journal of Operational Research, 115*(3), 596–607.
Patel, G. N., & Pande, S. (2013). *Measuring the performance of pharmacy stores using discretionary and non-discretionary variables, 50*(5), 25–41.
Puri, J., & Yadav, S. P. (2013). A concept of fuzzy input mix-efficiency in fuzzy DEA and its application in banking sector. *Expert Systems with Applications, 40*(5), 1437–1450.
Pye, S., Broad, O., Bataille, C., Brockway, P., Daly, H. E., Freeman, R., Gambhir, A., Geden, O., Rogan, F., & Sanghvi, S. (2021). Modelling net-zero emissions energy systems requires a change in approach. *Climate Policy, 21*(2), 222–231.
Ramli, N. A., Munisamy, S., & Arabi, B. (2013). Scale directional distance function and its application to the measurement of eco-efficiency in the manufacturing sector. *Annals of Operations Research, 211*, 381–398.
Rashidi, K., Shabani, A., & Saen, R. F. (2015). Using data envelopment analysis for estimating energy saving and undesirable output abatement: A case study in the Organization for Economic Co-Operation and Development (OECD) countries. *Journal of Cleaner Production, 105*, 241–252.
Ray, S. C. (2008). The directional distance function and measurement of super-efficiency: An application to airlines data. *Journal of the Operational Research Society, 59*(6), 788–797.
Saati, S., Hatami-Marbini, A., & Tavana, M. (2011). A data envelopment analysis model with discretionary and non-discretionary factors in fuzzy environments. *International Journal of Productivity and Quality Management, 8*(1), 45–63.
Saranga, H. (2009). The Indian auto component industry–Estimation of operational efficiency and its determinants using DEA. *European Journal of Operational Research, 196*(2), 707–718.
Scheel, H. (2001). Undesirable outputs in efficiency valuations. *European Journal of Operational Research, 132*(2), 400–410.
Seiford, L. M., & Zhu, J. (1999). An investigation of returns to scale in data envelopment analysis. *Omega, 27*(1), 1–11.
Seiford, L. M., & Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research, 142*(1), 16–20.
Singh, A., & Gundimeda, H. (2021). Impact of bad outputs and environmental regulation on efficiency of Indian leather firms: A directional distance function approach. *Journal of Environmental Planning and Management, 64*(8), 1331–1351.
Sueyoshi, T., & Goto, M. (2011). Methodological comparison between two unified (operational and environmental) efficiency measurements for environmental assessment. *European Journal of Operational Research, 210*(3).
Sueyoshi, T., & Goto, M. (2012). Returns to scale and damages to scale under natural and managerial disposability: Strategy, efficiency and competitiveness of petroleum firms. *Energy Economics, 34*(3), 645–662.
Sueyoshi, T., & Goto, M. (2013). Returns to scale vs. damages to scale in data envelopment analysis: An impact of U.S. clean air act on coal-fired power plants. *Omega, 41*(2), 164–175.
Taleb, M. (2023). Modelling environmental energy efficiency in the presence of carbon emissions: Modified oriented efficiency measures under polluting technology of data envelopment analysis. *Journal of Cleaner Production, 137743*.
Taleb, M., Khalid, R., Attallah, M., Kareem, Q. A., & Ramli, R. (2023). Assessing and ranking the performance of higher education institutions: A non-radial super efficiency DEA approach. *International Journal of Education Economics and Development, 14*(2), 213–230.
Taleb, M., Khalid, R., Attallah, M., Ramli, R., & Mohd Nawawi, M. K. (2023). Evaluating efficiency and ranking of suppliers using non-radial super-efficiency data envelopment analysis with uncontrollable factors. *International Journal of Computer Mathematics: Computer Systems Theory, 8*(2), 108–127.
Taleb, M., Khalid, R., Emrouznejad, A., & Ramli, R. (2023). Environmental efficiency under weak disposability: An improved super efficiency data envelopment analysis model with application for assessment of port operations considering NetZero. *Environment, Development and Sustainability, 25*, 6627–6656.
Taleb, M., Ramli, R., & Khalid, R. (2018). Developing a two-stage approach of super efficiency slack-based measure in the presence of non-discretionary factors and mixed integer-valued data envelopment analysis. *Expert Systems with Applications, 103*, 14–24.
Taleb, M., Khalid, R., & Ramli, R. (2019). Estimating the return to scale of an integer-valued data envelopment analysis model: Efficiency assessment of a higher education institution. *Arab Journal of Basic and Applied Sciences, 26*(1), 144–152.
Taleb, M., Khalid, R., Ramli, R., Ghasemi, M. R., & Ignatius, J. (2022). An integrated bi-objective data envelopment analysis model for measuring returns to scale. *European Journal of Operational Research, 296*(3), 967–979.
Taleb, M., Ramli, R., & Khalid, R. (2019). Measuring the efficiency of community colleges using super efficiency approach for the case of non-discretionary factors in data envelopment analysis with sensitivity analysis. *International Journal of Process Management and Benchmarking, 9*(2), 149–172.
Toloo, M., Allahyar, M., & Hančlová, J. (2018). A non-radial directional distance method on classifying inputs and outputs in DEA: Application to banking industry. *Expert Systems with Applications, 92*, 495–506.
Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research, 130*(3), 498–509.
Tone, K. (2003). Dealing with undesirable outputs in DEA: A slacks-based measure (SBM) approach. *GRIPS Research Report Series*, *2003*.
Tulkens, H. (1993). On FDH efficiency analysis: Some methodological issues and applications to retail banking, courts, and urban transit. *Journal of Productivity Analysis, 4*(1–2), 183–210.
Tyteca, D. (2016). *Linear Programming Models for the Measurement of Environmental Performance of Firms — Concepts and Empirical Results Author (s): DANIEL TYTECA Stable URL : http://www.jstor.org/stable/41770535 REFERENCES Linked references are available on JSTOR for this., 8*(2), 183–197.
Visbal-Cadavid, D., Martínez-Gómez, M., & Guijarro, F. (2017). Assessing the efficiency of public universities through DEA. A case study. *Sustainability, 9*(8), 1416.
Wang, X., Huang, J., & Liu, H. (2022). Can China's carbon trading policy help achieve Carbon Neutrality?—A study of policy effects from the Five-sphere Integrated Plan perspective. *Journal of Environmental Management, 305*, Article 114357.
Watanabe, M., & Tanaka, K. (2007). *Efficiency analysis of Chinese industry: A directional distance function approach, 35*, 6323–6331.
Xiao, H., Wang, D., Qi, Y., Shao, S., Zhou, Y., & Shan, Y. (2021). The governance-production nexus of eco-efficiency in Chinese resource-based cities: A two-stage network DEA approach. *Energy Economics, 101*, Article 105408.
Yahia, F. B., Essid, H., & Rebai, S. (2018). Do dropout and environmental factors matter? A directional distance function assessment of tunisian education efficiency. *International Journal of Educational Development, 60*, 120–127.
Yang, H., & Pollitt, M. (2009). Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants. *European Journal of Operational Research, 197*(3), 1095–1105.
Zhang, N., & Choi, Y. (2013). Environmental energy efficiency of China's regional economies: A non-oriented slacks-based measure analysis. *The Social Science Journal, 50*(2), 225–234.
Zheng, H. (2023). Analysis of nonlinear evolution mechanism of power technology progress under the constraints of net-zero carbon dioxide emissions in China. *International Journal of Climate Change Strategies and Management, 15*(2), 128–140.
Zhou, P., & Ang, B. W. (2008). Linear programming models for measuring economy-wide energy efficiency performance. *Energy Policy, 36*(8), 2911–2916.
Zhou, P., Ang, B. W., & Poh, K. L. (2006). Slacks-based efficiency measures for modeling environmental performance. *Ecological Economics, 60*(1), 111–118.
Zhou, P., Poh, K. L., & Ang, B. W. (2007). A non-radial DEA approach to measuring environmental performance. *European Journal of Operational Research, 178*(1), 1–9.
Zhu, J. (2022). DEA under big data: Data enabled analytics and network data envelopment analysis. *Annals of Operations Research, 309*, 761–783.
Zhu, J., & Charles, V. (2021). *Data-Enabled Analytics: DEA for Big Data*. Cham: Springer.
Zhu, J., & Shen, Z.-H. (1995). A discussion of testing DMUs' returns to scale. *European Journal of Operational Research, 81*(3), 590–596.
Ziolo, M., Jednak, S., Savić, G., & Kragulj, D. (2020). Link between energy efficiency and sustainable economic and financial development in OECD countries. *Energies, 13*(22), 5898.