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Multi-Objective Portfolio Optimization Strategy Using the SPEA-II Algorithm

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ABSTRACT

In the finance world, the precise selection and optimization of stock portfolios are of paramount importance. This study explores the application of intelligent algorithms, particularly the multi-objective of Strength Pareto Evolutionary Algorithm II (SPEA-II), alongside traditional methods to determine optimal portfolios. Using the monthly stock prices, the Markowitz model is developed, focusing on the return and semi-variance criteria. Realistic constraints are applied to formulate a multi-objective optimization problem. SPEA-II and traditional multi-objective optimization methods are used to solve this problem, resulting in a set of optimal portfolios. The results show that the SPEA-II algorithm can generate portfolios with higher returns and lower risks compared to the Markowitz model and traditional methods, taking into account the complex and nonlinear conditions of the capital market. In addition, the SPEA- II algorithm showed significant efficiency and stability across different frequencies and time periods. The study highlights that the SPEA-II algorithm can serve as an effective and efficient method for stock portfolio selection and optimization, helping investors to identify portfolios with lower risk and higher return.

Keywords: Multi-objective strategy, Pareto front, SPEA-II optimization, Markowitz model.

INTRODUCTION

Optimizing stock portfolios has garnered significant attention from both researchers and practitioners due to its critical role in achieving favorable investment outcomes (Zein et al., 2024). The complex and multi-dimensional nature of portfolio optimization has led to the exploration of various models and advanced methodologies to improve its efficiency and applicability. Building on the extensive research in this area, this paper delves into the optimization of stock portfolios through the integration of evolutionary multi-objective algorithms, reliability-based models and a synthesis of different approaches such as neural networks and inverse clustering. By using these advanced techniques, we aim to increase the efficiency and robustness of portfolio optimization strategies.

The foundation of our study is based on the extensive research of Guarino et al. (2024) on multi-objective optimization (MOO) methods, who introduced an interactive genetic algorithm for portfolio selection. In addition, the pioneering work of Liu et al. (2024) on sustainable energy development through multi-objective decision optimization in shale gas investments serves as a cornerstone for integrating investment decisions with sustainable development goals. Gunjan and Bhattacharyya (2023) highlights different optimization approaches, including risk-based, factor-based and machine learning-based techniques, and shows their respective advantages and limitations. Their comparative analysis highlights the importance of tailoring optimization methods to investors' preferences and data characteristics. By drawing inspiration from innovative methods, such as the stochastic leap method proposed by Freitas and Junior (2023), which uses random walk techniques and artificial neural networks for better portfolio forecasting, and the distributed differential evolution algorithm for multi-objective portfolio optimization presented by Song et al. (2023), we seek to push the boundaries of portfolio optimization research. Furthermore, the incorporation of Bayesian predictions by Butler and Kwon (2023) considering uncertainty factors and the dynamic portfolio optimization method using inverse correlation clustering by Wang and Aste (2023) emphasize the importance of adaptive and responsive strategies under changing market conditions.

Finally, the study of portfolio optimization under market bubble conditions by Daryabor et al. (2020) and the evaluation of the effectiveness of bird swarm algorithms in portfolio optimization by Bayat and Asadi (2017) provide valuable insights into addressing unique challenges and improving performance under dynamic market scenarios. The existing literature has predominantly focused on conventional models such as the Markowitz mean-variance framework (Mba et al., 2022; Du, 2022; Chaweewanchon & Chaysiri, 2022; Board et al., 2008), which, while foundational, has limitations in handling non-linear constraints and adapting to market fluctuations. Recent advancements in evolutionary algorithms, machine learning, and MOO offer promising solutions (Zhang et al., 2024). However, integrating these methodologies with practical constraints and tailoring them to specific market conditions, especially in emerging markets, remains underexplored.

In light of these advances, this paper aims to contribute to the ongoing discourse on stock portfolio optimization by bringing together different methodologies and employing state-of-the-art techniques to deliver robust and effective optimization solutions tailored to the evolving needs of investors and stakeholders. Therefore, researchers have attempted to eliminate or reduce these limitations by presenting new models and methods (Moradi et al., 2022). This research attempts to present an innovative model in the field of financial risk management by combining two different approaches in portfolio management: Value at Risk (VAR) algorithm and the MOO algorithm SPEA-II. The use of the SPEA-II algorithm as an optimization tool alongside the VAR, which is commonly used for risk assessment, represents the main innovation of this study. This unique combination offers the possibility of a more precise analysis of risk and return and enables investors to make data-driven decisions with a better understanding of the trade-off between risk and return. Furthermore, this research aims to provide practical and context-specific solutions by evaluating the application of the aforementioned algorithms in Iranian financial markets and adapting them to the unique conditions of these markets. This can help fill the existing gap in the regional finance literature and contribute new perspectives to the existing body of knowledge.

In this paper, a multi-objective portfolio optimization model using the SPEA-II algorithm is presented. In this model, in addition to the usual criteria such as the mean and variance of returns, other criteria such as value at risk, conditional value at risk and mean absolute deviation are also considered. To evaluate the performance of the model and the algorithm, adjusted data from the TSE market is used, which shows that by selecting portfolios that are on the Pareto front, the desired risk and return can be achieved. In addition, the best portfolio is selected from the non-dominated solutions based on risk and return is selected using classical methods and the investment weights and the level of risk and return in each stock are determined. Then, using the SPEA-II algorithm after validating the algorithm, the risk and return of the portfolio are calculated and the optimal portfolio is determined. The models and algorithms presented in this study can contribute to further research and evaluation in the field of portfolio and investment.

LITERATURE REVIEW

A portfolio is a series of financial assets selected by investors. These can include stocks, bonds, foreign exchange, gold and real estate. The weighting of each asset in the portfolio represents its share of the total investment (Khadempour Arani et al., 2022). The goal of forming a portfolio is to achieve a desirable balance between return and risk. The return of a portfolio is defined as the weighted average return of the assets, and its risk is defined as the weighted standard deviation of the returns of the assets (Koochaki et al., 2022). Various evaluation criteria are used to assess the efficiency and quality of a portfolio. These criteria can include average return, variance, liquidity, skewness and diversification. The average return and variance represent the profit and risk associated with the portfolio respectively. Liquidity indicates how easily assets can be converted into cash, while skewness determines the deviation of the return distribution from the normal distribution. Diversification refers to the distribution of risk in the portfolio (Tehrani-Pour et al., 2023).

There are mathematical and statistical models for estimating and predicting the returns and risks of assets and portfolios. These models include single-factor, multi-factor, Markowitz, Sharpe and CAPM models. The single-factor model relates asset returns to market returns, while the multi-factor model links returns to several risk factors. The Markowitz model is used to optimize portfolios on the basis of the mean and variance of returns. The Sharpe model measures portfolio performance relative to a risk-free portfolio, and the CAPM model determines the expected return of an asset based on its systematic risk.

Guarino et al. (2024) dealt with optimal portfolio selection - the formation of a set of stocks/shares with appropriate risk and high returns - through multi-objective and evolutionary optimization methods. In particular, they introduced a method for optimal portfolio selection based on evolutionary multi-objective algorithms. In particular, they have developed an improved Non-Dominated Sorting Genetic Algorithm II (NSGA-II) that aims to maximize returns and minimize risk. The algorithm was tested on stock returns over a three-year period, varying the settings/operators of the NSGA-II. The proposed algorithm is interactive and allows users to incorporate insights and preferences into stock selection. The results show that the proposed algorithm significantly reduces the risk of stock-only portfolios while achieving high performance (in terms of returns). In addition, the effective consideration of investor preferences in portfolio composition increases the attractiveness of the portfolio to end users. Similarly, Liu et al. (2024)

investigated the optimization of investment decisions in multi-objective shale gas development for sustainable energy. Their study combines innovative investment decisions with sustainable development. They presented a multi-objective decision optimization model for investment in shale gas development portfolios to solve the optimization of shale gas block portfolios and allocate investment amounts for exploration and development under different competitive objectives and constraints. The results show that preferred shale gas blocks with low development costs, high initial production, low methane emissions and low internal environmental costs are selected for the optimal portfolio boundaries and the optimal investment portfolio functions have a wide range. If emissions reduction is paramount in determining the final plan, the final investment plans are indifferent to the order of economic benefit and environmental pollution; if economic benefit or environmental pollution is paramount, the order of the other two objectives influences the selection of the final investment plan.

Gunjan and Bhattacharya (2023) conducted a comprehensive study on portfolio optimization methods in which they examined common techniques based on risk, factor-based and machine learning. They evaluated the advantages and disadvantages of each of these methods and compared their performance with real data. They concluded that the choice of optimization method depends on the individual preferences of investors and the characteristics of the data. Freitas and Junior (2023) introduced an innovative method for portfolio optimization using random walk in stock networks and predictive analysis. By creating a correlation-based stock network and using random walk algorithms for stock selection and artificial neural networks for predicting future return, they achieved better results with Brazilian stock data than Markowitz's model and other benchmarks. Song et al. (2023) presented a distributed differential evolution algorithm for multi-objective portfolio optimization that is capable of managing multiple objectives and constraints and can be run on parallel computing platforms. This algorithm has shown superior performance compared to other advanced algorithms in various tests with real data. Butler and Kwon (2023) integrated Bayesian predictions into the unified mean-variance portfolio optimization model and provided analytical solutions for optimal portfolios considering uncertainties. They also proposed a dynamic method for updating weights based on new information that outperforms traditional mean-variance models with US stock data.

Wang and Aste (2023) presented a method for dynamic portfolio optimization using inverse correlation clustering. They obtained results with higher returns and lower risk compared to the Markowitz model and other clustering methods with US and UK stock data by using hierarchical clustering and assigning portfolio weights proportional to inverse correlation matrices, along with periodic weight updates based on market changes. Gonzalez-Salazar et al. (2023) investigated portfolio optimization in district heating and compared skill ordering methods with mixed-integer linear programming for the practical optimization of the operation of district heating plants. They formulated a MOO problem taking into account technical, economic and environmental factors. The results of the case study in Germany showed that the mixed-integer linear programming approach yielded better solutions than the competence ordering method. Emamat and Hanafizadeh (2021) introduced a reliability-based approach for the optimization of stock portfolios based on the desired confidence level of investors. Using evolutionary optimization algorithms, this approach finds portfolios with a lower probability of falling below a desired return and thus provides better results than the Markowitz model and the CVaR model. Daryabor et al. (2018) evaluated the application of bird swarm algorithm in the calculation of optimal portfolios.

Heidari Heratameh (2020) optimized the portfolio using the CVaR criterion and the gamma variance process (VG) process and identified portfolios with lower risk and higher returns by modeling the stock returns with the VG distribution and differential evolution optimization. Taghizadegan et al. (2023) compared the performance of Markowitz and VaR models considering liquidity risk and using the Capula distribution and DCC method and obtained better results. Zamanpour et al. (2022) identified effective factors in stock portfolio optimization using fuzzy network analysis and ranked them. They presented a multi-criteria stock evaluation and selection model to help investors select stocks with better performance. These studies show that the optimization of stock portfolios is a multi-dimensional and complex issue that requires the use of various advanced models and methods. To provide a clearer comparision of the existing studies, a summary of key research works, their methodologies, and their respective advantages and disadvantages is presented in Table 1.

Table 1

Comparision of Related Studies on Portfolio Optimization

Study	Method/Model Used	Advantages	Disadvantages
Guarino et al. (2024)	Improved NSGA-II algorithm	High performance, user interaction, reduce risk	Limited to three-year data, lacks real-time adaptability
Liu et al. (2024)	Multiple-objective optimization in shale gas	Integration with sustainability, comprehensive approach	Specific to energy sector, not directly applicable to stock portfolios
Gunjan and Bhattacharyya (2023)	Risk-based and ML-based techniques	Flexibility, data-driven insights	Requires extensive computational resources
Freitas and Junior (2023)	Random walk and neural networks	Improved forecasting and portfolio selection accuracy	Lack of consideration for emerging markets
Song et al. (2023)	Distributed Differential Evolution Algorithm	Handles multiple objectives and constraints effectively	Requires parallel computing infrastructure
Butler and Kwon (2023)	Bayesian Mean-Variance Optimization	Accounts for uncertainty, dynamic updates	Complexity in parameter estimation
Wang and Aste (2023)	Inverse Correlation Clustering	Achieves higher returns with lower risks	Dependent on periodic updates; computationally intensive

As observed in Table 1, while various models offer significant contributions, their limitation highlight the need for more robust, adaptable, and context-specific approaches, particularly in emerging markets.

Multi-Objective Optimization (MOO)

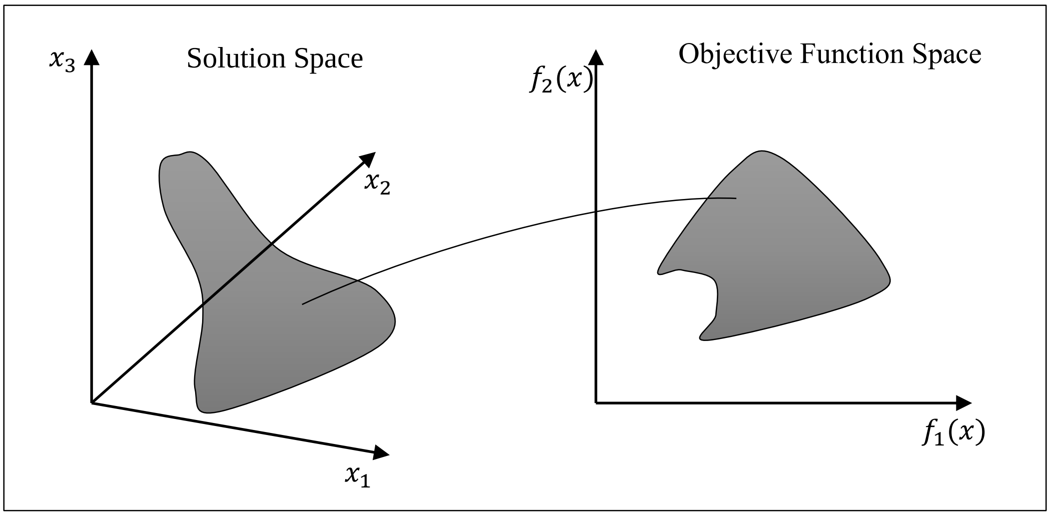
In MOO, multiple objective functions are optimized simultaneously, whereby the existing constraints must be observed. The MOO problem involves multiple objective functions that are maximized or minimized by weighted combinations (Nateghian et al., 2022). The use of MOO is significant due to its simplicity as it avoids complex equations and simplifies the problem. Decision-making in MOO allows for trade-offs on conflicting issues. Each objective function vector represents a solution vector. Thus, in MOO, there is not a unique solution for all objectives, but multiple solutions. Mathematically, the equation for the MOO problem is as in Equation 1 (Ehrgott, 2005).

$$max/\min \ f_1(x), f_2(x), \dots, f_n(x) \quad (1)$$
$$subject\ to: x \in U$$

Where *x* stands for the solution, *n* for the number of objective functions and *U* for the feasible set. In addition, *f*(*x*) stands for the objective function and max and min are combined objective operations. In MOO, there is a multi-dimensional space of objective function vectors and solution vectors in the decision variable space. For each solution x in the decision variable space, there is a point in the objective function space. The mapping between the solution space and the objective function space can be illustrated in Figure 1 (Deb, 2001).

Figure 1

Mapping Solution Space and Objective Function Space (Gunantara, 2018)



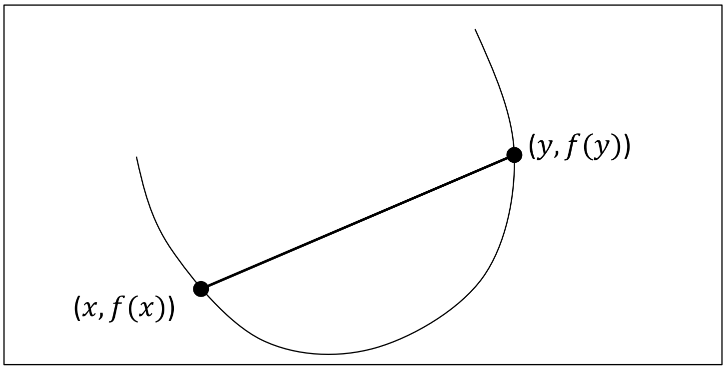
Given this mapping, the convexity of the solution space and the objective function space is crucial for determining the algorithm to solve the problem. If the MOO problem is convex, there are many algorithms that can be effectively used to solve the problem. If all objective functions and the solution space are also convex, the MOO problems are convex. However, an objective function is said to be convex if it satisfies Equation 2 (Boyd & Dan Vandenberghe, 2004).

$$f(\theta x + (1 - \theta) \leq \theta f(x) + (1 - \theta) f(y) \quad (2)$$

With *x*, *y* domain *f* and value 01. For better understanding, it is deduced that the line between (*x*, *f*(*x*)) and (*y*, *f*(*y*)), which runs from x to y, lies above the graph of *f*. This can be seen in Figure 2.

Figure 2

Convex Function (Gunantara, 2018)



To further clarify the role of convexity, its definition, calculation methods, and guarantees across the interval are discussed below. In MOO, convexity plays a crucial role in determining the feasibility and efficiency of solving the problem (Kováčová & Rudloff, 2022). A function is said to be convex if, for any two points $X_1, X_2 \ni U$ and any λ ∋ [0, 1] (where U represents the feasible set), the following inequality holds (Equation 3):

$$(\lambda f(x_2) - 1) + \lambda f(x_1) \geq (_2\lambda)x - 1)+_2 f\ (\lambda_x) \quad (3)$$

This property ensures that the line segment between any two points in the domain lies above or on the graph of the function, making optimization more tractable. Convexity simplifies the problem by guaranteeing that local minima are also global minima, which is particularly valuable in algorithms such as SPEA-II that rely on iterative evaluations (Lobato & Steffen, 2017). Convexity is typically verified by examining the second derivative (Hessian matrix) of the objective functions. For a scalar-valued function $f(x)$, convexity is ensured if the Hessian matrix $H(f(x))$ is positive semi-definite ($0 \leq H(f(x))$ for all $\ni xU$. In vector-valued MOO, the convexity of each individual objective function is assessed. In addition, the convexity of the feasible set U is examined by ensuring that all constraints defining the set are linear or convex (Efroni et al., 2024).

Convexity may not be guaranteed across the entire interval in all scenarios. For example, certain objective functions, particularly those involving non-linear terms, may exhibit regions of non-convexity. In such cases, the optimization problem becomes more challenging and may require heuristic or approximation methods to handle these non-convex regions. Specifically, the integration of non-linear constraints or criteria, such as Value at Risk (VaR) or Conditional Value at Risk (CVaR), can introduce non-convexity (Zhang & Zhang, 2019). To mitigate this, the SPEA-II algorithm is designed to handle non-convex Pareto fronts by iteratively refining solutions within the feasible domain.

In this study, the convexity of the primary objective functions (risk and return) and the feasible set were analyzed. The risk function, derived from semi-variance and VaR, was approximated using quadratic terms, which are convex under standard conditions. However, the return function, influenced by non-linear interactions among assets, may exhibit partial non-convexity. Therefore, while the algorithm seeks to explore and exploit the entire solution space, the convexity assumption holds only for specific intervals, as validated through simulation and sensitivity analyses. Two methods can be used to solve MOO problems: scalarization and Pareto front (Gunantara, 2018). In the scalarization method, a single solution is generated for the multi-objective function, and the weights are determined before the optimization process.

Pareto Front

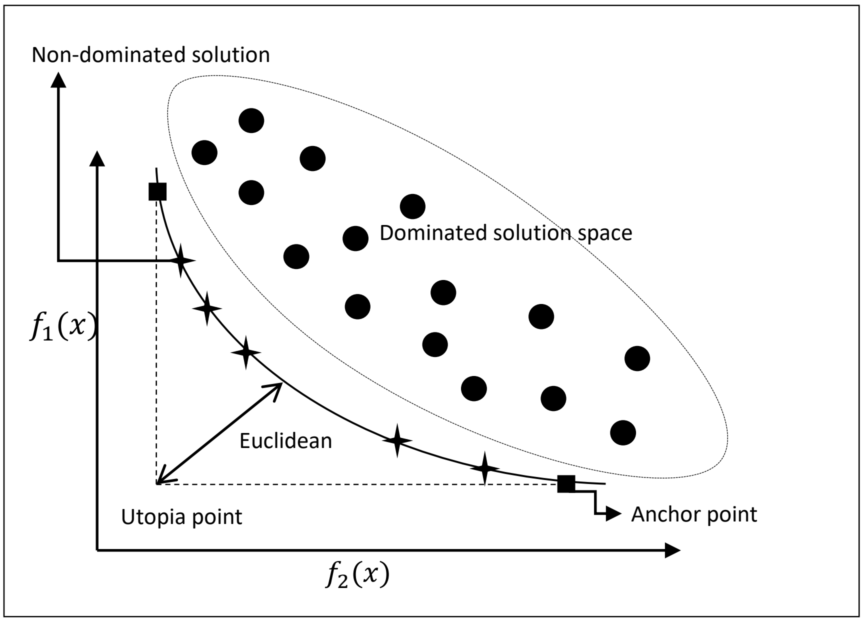
The Pareto front represents the trade-off between different objective functions and serves as a benchmark for evaluating the performance of MOO algorithms (Vaezi et al., 2020). This concept contributes to a deeper understanding of the fundamental concepts and principles required to solve MOO problems and provides a framework for more precise analysis and the selection of effective investment strategies. The Pareto method is used when the desired solutions and performance indicators are separate or independent and produce a trade-off solution that can be represented as a Pareto Optimal Front (POF). Mathematically, MOO can be expressed using the Pareto method as follows:

$$f_1.opt = \min f_1(x)$$
$$f_2.opt = \min f_2(x)$$
$$.$$
$$.$$
$$f_n.opt = \min f_n(x)$$

In the Pareto method, the elements of the solution vector remain independent throughout the optimization process, and the concept of dominance arises when it is possible to distinguish between dominated and non-dominated solutions. A dominant solution and an optimal value in MOO are achieved when one objective function can be improved without compromising another objective function. This condition is called Pareto optimality. Two terms are important on the Pareto front: anchor point and utopia point. The anchor point is derived from the best objective function, while the utopia point is derived from the intersection of the maximum/minimum value of one objective function and the maximum/minimum value of another objective function. Emmerich and Deutz (2018) claim that optimization with two objective functions and non-dominated solutions can be described on a two-dimensional surface within the Pareto front. Figure 3 illustrates this concept.

Figure 3

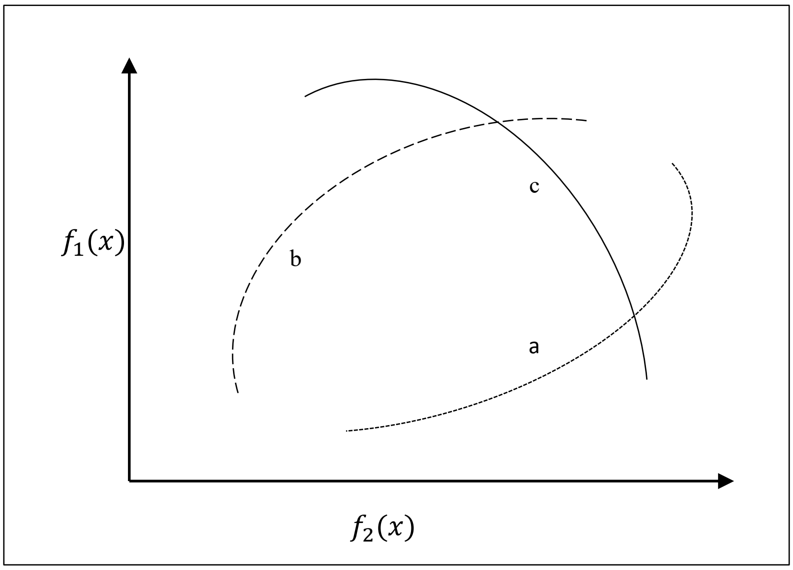
POF for Two Objective Functions



The optimal point on the Pareto Optimal Front (POF) is where the Euclidean distance is shortest (Bejarano et al., 2022). If there are two objective functions, f1(x) and f2(x), three combinations can be considered based on the POF. The minimization of the objective function f1(x) and the maximization of the objective function f2(x) can be seen in Figure 4 under curve (a). The maximization of the objective function f1(x) and the minimization of the objective function f2(x) are shown under curve (b). The maximization of both objective functions f1(x) and f2(x) is shown under curve (c).

Figure 4

POF for Two Objective Functions (Gunantara, 2018)



The modern portfolio theory of Markowitz serves as the primary basis and theoretical framework for this study. Accordingly, the Markowitz model, which is one of the most common models for portfolio selection, was developed as the main model in Equation 4:

$$\text{Minimize } G = \sum_{k=1}^{M} \sum_{l=1}^{n} \omega_k \omega_l \delta_{kl} \tag{4}$$
$$subject\ to:$$
$$\sum_{k=1}^{n} \omega_k \mu_k \geq R$$
$$\sum_{k=1}^{n} \omega_k = 1$$
$$\omega_k \geq 0 \quad for\ all\ k$$

Where, δ_{kl} denotes the covariance between assets k and l, μ_k represents the mean return on the asset and R indicates a certain level of efficiency. This model is solved for different values of R and the final results of the objective function based on risk are plotted together with the corresponding R values. Since R represents the investor's desired return, the resulting graph is referred to as the efficient frontier. In order to improve the realism and practicality of the model and provide investors with a reliable market, the scalability of the Markowitz model has been taken into account. By introducing λ as a weighting parameter, the objective function aims to strike a balance between risk and return by minimizing risk and maximizing return. More specifically, λ is a weighting parameter that varies within the range [0, 1].

However, one of the limitations of the Markowitz model is that it is not able to optimize the portfolio selection problem given the second constraint in Equation 5.

$$\sum_{i=1}^{n} Z_i = k \tag{5}$$

Based on this constraint, Z_i is equal to one if there is an investment in asset i, and zero otherwise. In this formulation, k represents the number of assets that the investor wishes to include in his portfolio. The extended portfolio selection and optimization model is therefore represented by Equation 6:

$$\max = \lambda \sum_{i=1}^{N} \omega_i \mu_i - (1-\lambda) \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \delta_{ij} \tag{6}$$
$$subject\ to:$$
$$\sum_{i=1}^{n} \omega_i = 1$$
$$\sum_{i=1}^{n} Z_i = k$$
$$\omega_i \geq 0, i = 1,2,3 \ldots, n$$
$$Z_i \in \{0.1\}$$

Metaheuristic algorithms operate over the objective function. In this study, based on Markowitz's modern portfolio theory, the developed mean semi-variance model, the above constraints and ratios such as Value at Risk (VaR), Conditional Value at Risk (CVaR) and Mean Absolute Deviation (MAD), the MOO problem SPEA-II is defined in Equation 7.

$$\min \quad Risk(\omega) \,.\, [1 + \beta \max\left(0,1 - \frac{\mu_i}{\mu_{i0}}\right)] \tag{7}$$
$$subject\ to:$$
$$\sum_{i=1}^{n} \omega_i = 1$$
$$\omega_i \geq 0, i = 1,2,3 \ldots, n$$

The SPEA-II Algorithm

The SPEA-II algorithm is a powerful metaheuristic method for solving MOO problems (Guarino et al., 2024). This algorithm uses evolutionary mechanisms such as selection, mutation and recombination to find the optimal Pareto front for the problem at hand (Rohman et al., 2023). In the context of portfolio optimization, SPEA-II can be used to find a trade-off between risk and return on investments (Zhang et al., 2023). The proposed framework for SPEA-II Optimization is shown in Figure 5.

Figure 5

SPEA Algorithm Process (Gharari et al., 2016)

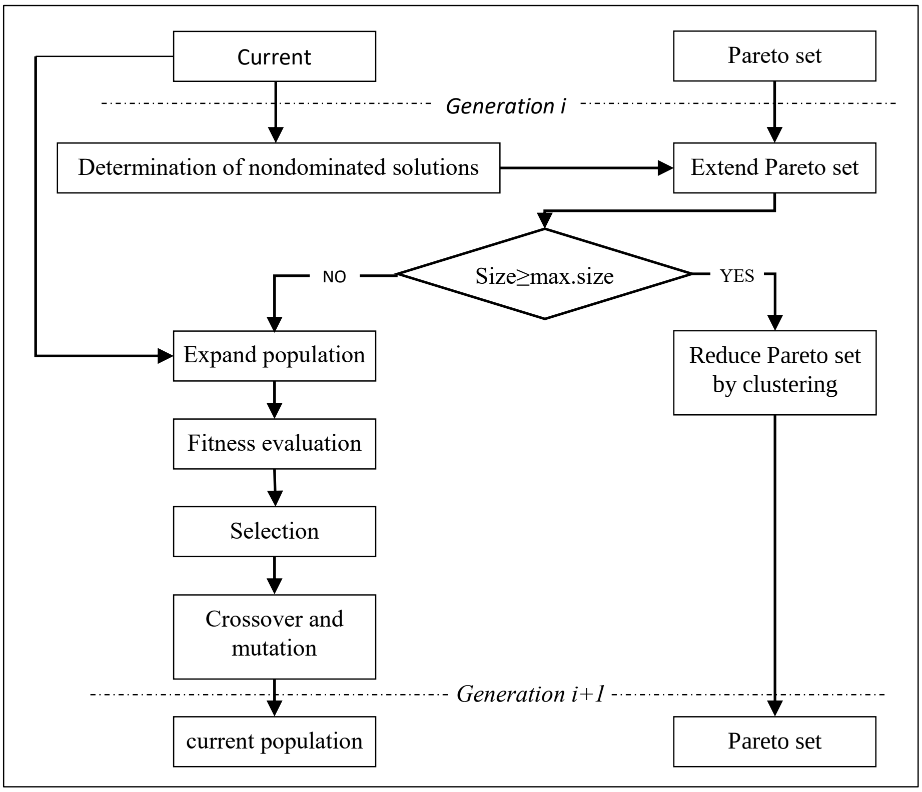


Figure 5 provides a schematic representation of the proposed SPEA-II algorithm. The framework integrates advanced fitness evaluation and archive management techniques to ensure optimal portfolio selection. The SPEA-II Optimization algorithm is presented in Algorithm 1.

Algorithm 1. ***SPEA-II Algorithm.***

```
clc;
clear;
close all;
data=readmatrix("price.xlsx");
Symbols=readcell("Symbols.xlsx");
R=price2ret(data);
model.R=R;
model.method='cvar';
model.alpha=0.95;
CostFunction=@(x) PortMOC(x,model);
nVar=size(R,2);
VarSize=[nVar 1];
VarMin=0;
VarMax=1;
MaxIt=50;
nPop=50;
nArchive=100;
```

```
K=round(sqrt(nPop+nArchive));
pCrossover=0.7;
nCrossover=round(pCrossover*nPop/2)*2;
pMutation=1-pCrossover;
nMutation=nPop-nCrossover;
crossover_params.gamma=0.1;
crossover_params.VarMin=VarMin;
crossover_params.VarMax=VarMax;
mutation_params.h=0.2;
mutation_params.VarMin=VarMin;
mutation_params.VarMax=VarMax;
empty_individual.Position=[];
empty_individual.Cost=[];
empty_individual.Out=[];
empty_individual.S=[];
empty_individual.R=[];
empty_individual.sigma=[];
empty_individual.sigmaK=[];
empty_individual.D=[];
empty_individual.F=[];
pop=repmat(empty_individual,nPop,1);
for i=1:nPop
   pop(i).Position=unifrnd(VarMin,VarMax,VarSize);
   [pop(i).Cost, pop(i).Out]=CostFunction(pop(i).Position);
end
archive=[];
for it=1:MaxIt
   Q=[pop
     archive];
   nQ=numel(Q);
   dom=false(nQ,nQ);
   for i=1:nQ
      Q(i).S=0;
   end
   for i=1:nQ
      for j=i+1:nQ
         if Dominates(Q(i),Q(j))
            Q(i).S=Q(i).S+1;
            dom(i,j)=true;
        elseif Dominates(Q(j),Q(i))
            Q(j).S=Q(j).S+1;
            dom(j,i)=true;
            end
         end
   end
   S=[Q.S];
   for i=1:nQ
```

```
        Q(i).R=sum(S(dom(:,i)));
    end
    Z=[Q.Cost]';
    SIGMA=pdist2(Z,Z,'seuclidean');
    SIGMA=sort(SIGMA);
    for i=1:nQ
        Q(i).sigma=SIGMA(:,i);
        Q(i).sigmaK=Q(i).sigma(K);
        Q(i).D=1/(Q(i).sigmaK+2);
        Q(i).F=Q(i).R+Q(i).D;
    end
    nND=sum([Q.R]==0);
    if nND<=nArchive
        F=[Q.F];
        [F, SO]=sort(F);
        Q=Q(SO);
        archive=Q(1:min(nArchive,nQ));
        else
        SIGMA=SIGMA(:,[Q.R]==0);
        archive=Q([Q.R]==0);
        k=2;
        while numel(archive)>nArchive
            while min(SIGMA(k,:))==max(SIGMA(k,:)) && k<size(SIGMA,1)
                k=k+1;
            end
            [~, j]=min(SIGMA(k,:));
            archive(j)=[];
            SIGMA(:,j)=[];
        end
   end
    PF=archive([archive.R]==0);
    figure(1);
    PFC=[PF.Cost];
    plot(PFC(1,:),-PFC(2,:),'o');
    xlabel('Risk Assets');
    ylabel('Return Assets');
    grid on;
    disp(['Iteration ' num2str(it) ': Number of PF members = ' num2str(numel(PF))]);
    if it>=MaxIt
        break;
    end
    popc=repmat(empty_individual,nCrossover/2,2);
    for c=1:nCrossover/2
        p1=BinaryTournamentSelection(archive,[archive.F]);
        p2=BinaryTournamentSelection(archive,[archive.F]);
        [popc(c,1).Position,
popc(c,2).Position]=Crossover(p1.Position,p2.Position,crossover_params);
```

```
        [popc(c,1).Cost, popc(c,1).Out]=CostFunction(popc(c,1).Position);
        [popc(c,2).Cost, popc(c,2).Out]=CostFunction(popc(c,2).Position);
    end
    popc=popc(:);
    popm=repmat(empty_individual,nMutation,1);
    for m=1:nMutation
        p=BinaryTournamentSelection(archive,[archive.F]);
        popm(m).Position=Mutate(p.Position,mutation_params);
        [popm(m).Cost, popm(m).Out]=CostFunction(popm(m).Position);
  end
    pop=[popc
         popm];
end
disp(' ');
for j=1:size(PFC,1)
    disp(['Objective #' num2str(j) ':']);
    disp(['      Min = ' num2str(min(PFC(j,:)))]);
    disp(['      Max = ' num2str(max(PFC(j,:)))]);
    disp(['    Range = ' num2str(max(PFC(j,:))-min(PFC(j,:)))]);
    disp(['    St.D. = ' num2str(std(PFC(j,:)))]);
    disp(['     Mean = ' num2str(mean(PFC(j,:)))]);
    disp(['     RMSE = ' num2str(rmse(PFC(j,:)))]);
    disp(' ');
end
```

Table 2 presents the key parameters used in the implementation of the SPEA-II algorithm. These parameters, including population size, crossover probability, mutation probability, and selection method, were selected based on a combination of theoretical considerations and experimental validation. A population size of 50 was chosen to balance computational efficiency and solution diversity. Smaller populations may lead to premature convergence, while excessively large populations increase computational costs without significant improvements in results. A crossover probability of 0.7 aligns with standard practices in evolutionary algorithms, ensuring sufficient exploration of the solution space while maintaining the stability of convergence. The mutation probability was set at 0.2 to introduce diversity and prevent the algorithm from stagnating in local optima. This value was determined through sensitivity analysis, as excessively high mutation rates can disrupt convergence, while low rates limit diversity. Binary tournament selection is widely used for its simplicity and effectiveness in preserving high-quality solutions while maintaining diversity in the population.

γ-Crossover (0.1) and h-Mutation (0.2) were tuned to optimize the trade-off between exploration and exploitation in the algorithm. Lower values of γ-crossover and moderate h-mutation ensure gradual improvements without drastic deviations. The parameter values were refined through multiple test runs on historical data from the Tehran Stock Exchange (TSE). The stability and efficiency of the SPEA-II algorithm under these settings were confirmed by evaluating the Pareto front across different scenarios. The chosen parameters consistently produced solutions with high convergence and diversity metrics, indicating their near-optimality for the given dataset and objectives. To validate these parameter choices, additional experiments with varying population sizes, crossover/mutation probabilities, and selection methods were conducted. The results showed that the chosen parameters provided a consistent balance between computational cost and solution quality, demonstrating their suitability for this study.

Table 2

Parameters Used in SPEA-II

Parameter	Tested Values	Optimal Value	Justification
Population size	10 – 90	50	Balanced diversity and computational cost.
Crossover Prob.	0.1 – 0.9	0.7	Ensures adequate exploration of solution space.
Mutation Prob.	0.1, 0.2 and 0.3	0.2	Prevents stagnation while maintaining stability.
Type of selection		Binary tournament selection	
γ-Crossover		0.1	
h-Mutation		0.2	

The parameter values were determined through experimental validation and sensitivity analysis, ensuring their suitability for the given dataset and objectives.

EVALUATION AND RESULTS

The dataset used in this study includes monthly stock prices of companies listed on the Tehran Stock Exchange (TSE) from 2011 to 2022. These data are publicly available and can be accessed through the TSE website at [https://tsetmc.ir/ and https://my.codal.ir/en/]. The aim of this study is to effectively use the SPEA-II algorithm for portfolio optimization. The data were collected through the TSE's official records, including historical monthly stock prices for all listed companies. After initial screening, 132 companies were selected based on criteria such as consistent trading activity, availability of complete data, and market capitalization. The dataset was then adjusted for stock splits and dividends to ensure accuracy. MATLAB was used for preprocessing and implementing the SPEA-II algorithm. When implementing this algorithm, the probabilities for crossover and mutation were set to 0.7 and 0.2 respectively. The gamma parameter (γ-crossover) was set to 0.1, the number of generations to 1,000 and the population size per generation to 50.

The following method was used for validation:

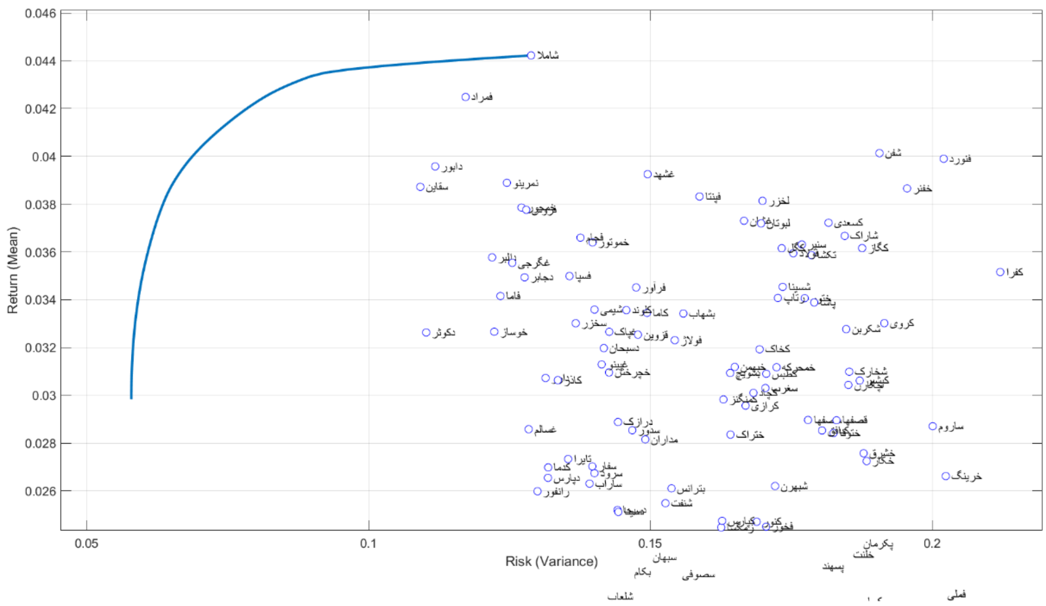
In traditional method, two main types of criteria are used to measure portfolio risk:

1. Dispersion criteria: These act as indicators of uncertainty and consider both positive and negative deviations from the mean as portfolio risk. The instruments used in this category include the standard deviation from the mean, the mean absolute deviation and the mean absolute deviation raised to the power of q.
2. Unfavorable portfolio risk criteria: This category includes semi-variance, moments of unfavorable deviations, Value at Risk (VaR), and Conditional Value at Risk (CVaR).

In this study, optimization is performed using classical methods based on the above criteria and the mean semi-variance, which has been repeatedly used in previous studies. Additionally, to leverage intelligent methods, the Markowitz model is developed and used as the objective function in the SPEA-II algorithm for minimization. To achieve MOO, traditional methods and criteria such as Value at Risk (VaR), Conditional Value at Risk (CVaR), and mean semi-variance were used to calculate the portfolio risk. The results of these calculations and the composition of the optimal portfolio are shown in Figures 6, 7 and 8:

Figure 6

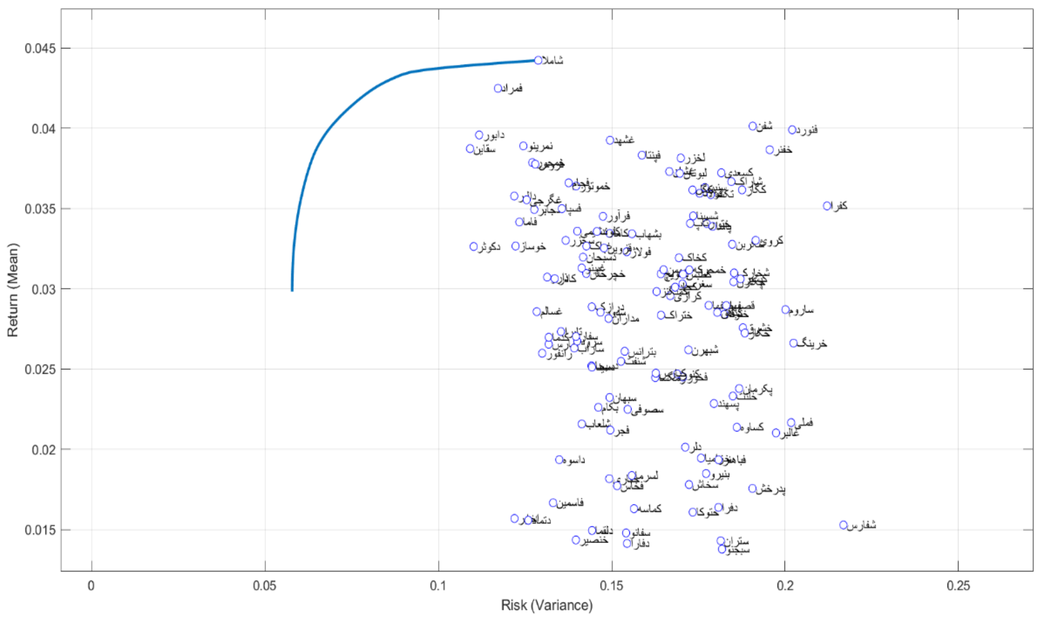
Optimal Portfolio Using the Traditional VaR Criterion



Note: The blue points represent company's names are shown in Persian.

Figure 7

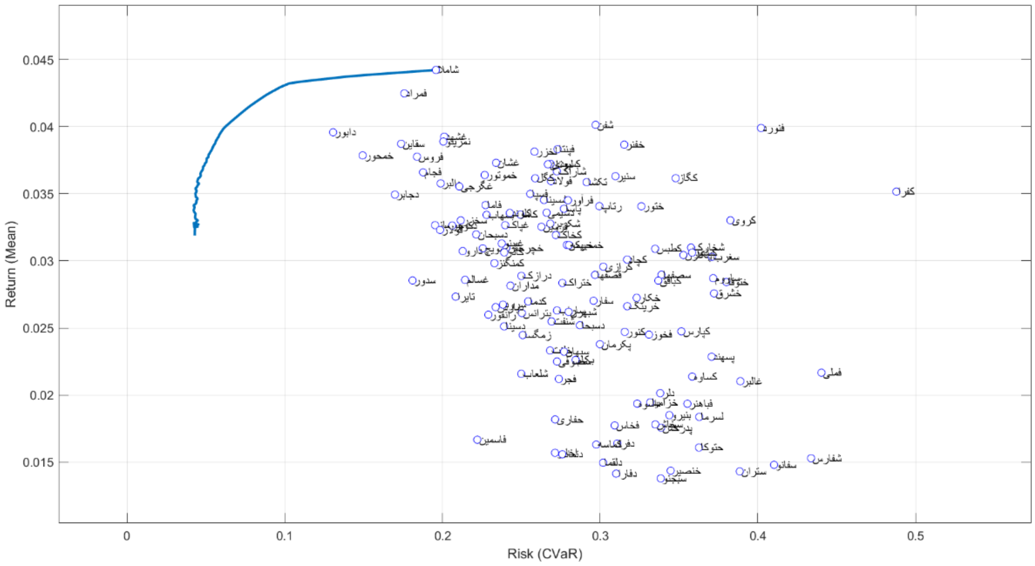
Optimal Portfolio Using the Traditional SemiVAR Criterion



Note: The blue points represent company's names are shown in Persian.

Figure 8

Optimal Portfolio Using the Traditional CVaR Criterion



Note: The blue points represent company's names are shown in Persian.

The portfolios optimized using traditional methods form the Pareto front in the diagrams corresponding to the VaR, semi-variance, and CVaR methods. These charts illustrate that the stock of a particular company, e.g., SHAMLA Company, lies on the efficient frontier and has an excellent performance. The Pareto frontier encompasses various criteria of company performance, including profitability, sales growth, return on investment, and rate of return on capital. Placing the company's stock on the efficiency frontier indicates above-average performance on these metrics, which can be a sign of successful management and the achievement of remarkable results compared to competitors or the industry as a whole. This status is of great importance to shareholders and investors as it indicates the company's successful strategy and can be perceived as a safe investment opportunity. Furthermore, this position can help to attract new investors and strengthen the confidence of current shareholders. Overall, the presence of a stock on the efficiency frontier is regarded as an indication of the company's above-average performance and can be seen as a motivator for future growth and progress. An overview of the risk and return of SHAMLA Company, which is on the Pareto optimal line, is given in Table 3.

Table 3

Overview of Risk and Return for SHAMLA Company

Method	Risk	Risk value
Value at Risk (VaR)	0.0298	0.0579
Value at Conditional Risk (CVaR)	0.0319	0.0429
Mean-semi-variance	0.0298	0.0579

Table 4

Analysis of AssetCovar of the Optimal Portfolio for the Top Eight Companies Using the VAR Method

AssetCoVar	Company1	Company2	Company3	Company4	Company5	Company6	Company7	Company8
Company1	0.014	0.0066	0.0063	0.0036	0.0073	0.0081	0.0064	0.0043
Company2	0.006	0.0236	0.0236	0.0066	0.0077	0.0129	0.0097	0.0054
Company3	0.0063	0.009	0.0094	0.0113	0.0094	0.0141	0.0109	0.0103
Company4	0.0036	0.0066	0.0066	0.0243	0.0063	0.0076	0.0085	0.0105
Company5	0.0073	0.007	0.0077	0.0063	0.0214	0.0114	0.0087	0.0081
Company6	0.0081	0.012	0.0129	0.0076	0.0114	0.0314	0.0099	0.0109
Company7	0.0064	0.009	0.0097	0.0085	0.0087	0.009	0.0183	0.0083
Company8	0.0043	0.0054	0.0054	0.0105	0.0081	0.0109	0.0083	0.0319

Table 4 illustrates the covariance between different assets in the optimal portfolio for the eight best companies based on the VaR method. The covariance indicates the degree of common variation between the returns of two assets and can be considered as an indicator of the relationship of common risk between them. Here the covariance values are analyzed as follows:

- High covariance (positive): Indicates a strong positive relationship between the returns of two assets. For example, Company-6 and Company-8 have the highest covariance with a covariance of 0.0314 and 0.0319, indicating a high correlation between their returns.
- Low covariance (positive): Indicates a weaker relationship between the returns of two assets. For example, Company-1 and Company-8 have the lowest covariance with a covariance of 0.0043, indicating a lower correlation between their returns.

This analysis can help investors better understand the risk associated with the combination of different assets in their portfolio and to make more informed decisions regarding the diversification of their investments. The aim of this diversification is to reduce the overall risk of the portfolio, in particular by investing in assets that are less correlated with each other.
The stability test of the algorithm is one of the fundamental pillars in the portfolio optimization process, which is essential to confirm the performance accuracy of the algorithm. This test is designed to ensure that the algorithm does not produce inconsistent results with each run and that the consistency of the optimal solutions is guaranteed. To evaluate the stability of the SPEA-II algorithm, the optimization process was repeated 10 times, and the objective function values across runs were analyzed (Table 5). The low variance observed across these runs indicates high stability. Specifically, the variance of the fitness values was approximately 0.000004, demonstrating the algorithm's ability to produce consistent solutions under varying initial conditions. Stability is further supported by the convergence behavior observed in Figure 6, where the fitness values stabilize over iterations. This consistency is critical for ensuring reliable decision-making in real-world applications, as it confirms that the algorithm does not exhibit erratic behavior due to stochastic elements. The results are shown in Table 5.

Table 5

The Stability of the SPEA-II Algorithm Over 10 Algorithm Runs

	5th Run & Objective Function Value	4th Run & Objective Function Value	3th Run & Objective Function Value	2th Run & Objective Function Value	1th Run & Objective Function Value
Mean	0.0812	0.0807	0.0817	0.0807	0.0804
Std.	0.0982	0.0906	0.0970	0.0913	0.0922
	10th Run & Objective Function Value	9th Run & Objective Function Value	8th Run & Objective Function Value	7th Run & Objective Function Value	6th Run & Objective Function Value
Mean	0.0806	0.0806	0.0811	0.0816	0.0813
Std.	0.0945	0.0930	0.0936	0.0899	0.0891

The results in Table 5 show that there is a small difference between the results of the replicates. The very low variance of 0.000004 indicates that the algorithm is highly stable over 1000 runs. While the risk function, based on semi-variance, generally exhibits convexity, the return function introduces partial non-convexity due to non-linear interactions among portfolio assets. Figure 9 illustrates an example of the convex and non-convex regions observed in the objective functions. The SPEA-II algorithm effectively handles these non-convex regions by maintaining solution diversity and exploring the Pareto front comprehensively. This adaptability ensures that even in cases where convexity is not guaranteed, the algorithm produces reliable and efficient solutions.

Figure 9

Convex and Non-convex Regions

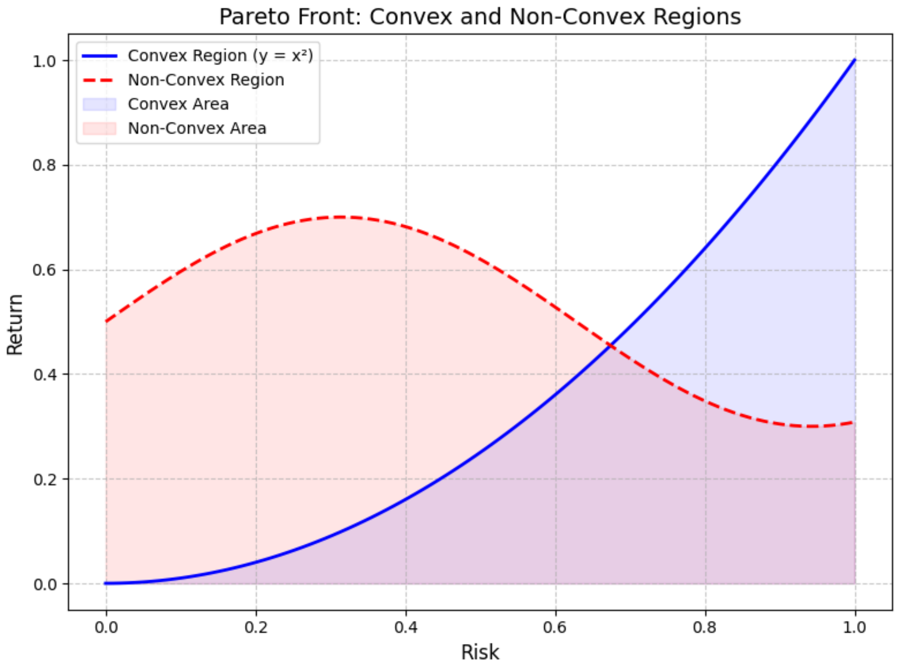


Table 6 illustrates the efficiency of the SPEA-II algorithm for stock portfolios with different optimization sizes. It can be observed that both risk and efficiency increase proportionally with increasing portfolio size, while they decrease with decreasing size. The price of a portfolio of 50 stocks over 144 months is depicted in Figure 10. The horizontal axis represents time in months, and the vertical axis represents price.

Table 6

Validates the Objective Function Value, Mean, Portfolio Efficiency, and Variance Using the SPEA-II Algorithm

Portfolio Size	30 Stock	40 Stock	50 Stock
Objective function value	0.150	0.091	0.098
Portfolio risk	0.089	0.094	0.091
Portfolio return	0.021	0.026	0.031

Figure 10

The Price of a 50-stock Portfolio Over 144 Months

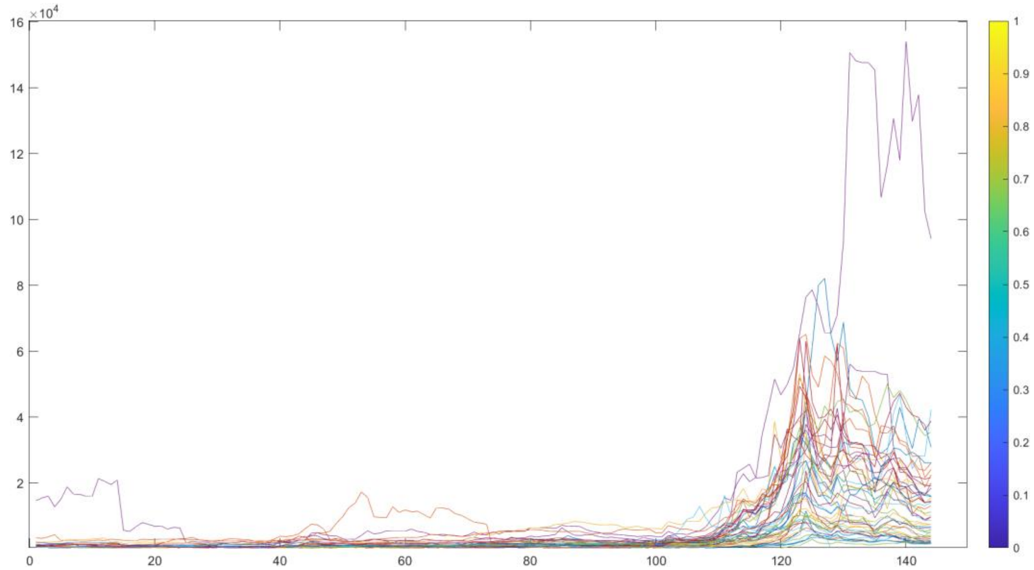


Figure 11 illustrates how the SPEA-II algorithm works over 1000 iterations. The descending trend in minimizing the objective function value while exposed to risk shows the effectiveness of this algorithm. Figure 12 shows the result and the optimal efficiency frontier of MOO using the SPEA-II algorithm. In this method, the optimized portfolio is multi-objective and includes investments in 132 companies with a weighting determined by the algorithm. The result of the algorithm and the investment amount with defined weightings can be seen in the analyzed companies with known risk and return values. This portfolio is created taking into account the complex and non-linear conditions of the capital market. The Pareto front - efficient frontier for the portfolio optimized with the SPEA-II algorithm for more than 1000 runs of the algorithm is shown in Figure 13.

Figure 11

The Process of Error Reduction and Optimization of the SPEA-II Algorithm Over 1000 Algorithm Runs

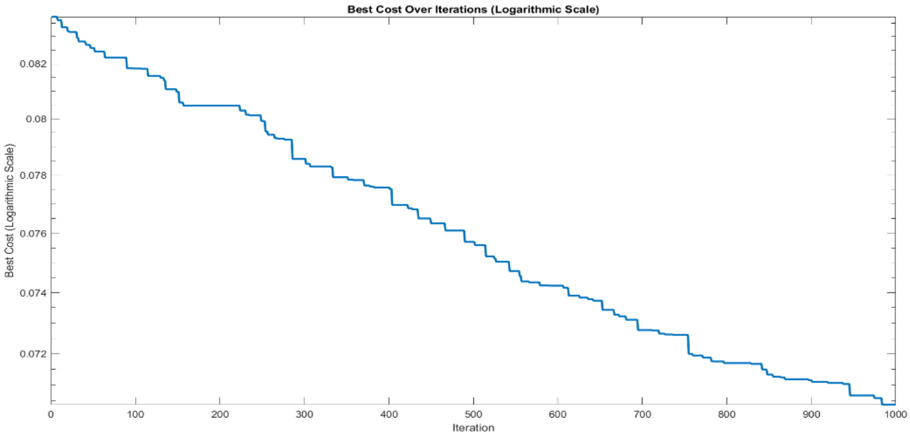


Figure 12

The Pareto Front - the Efficiency Frontier with the NSGA-II Algorithm During the Initial 50 Iterations of Algorithm Execution

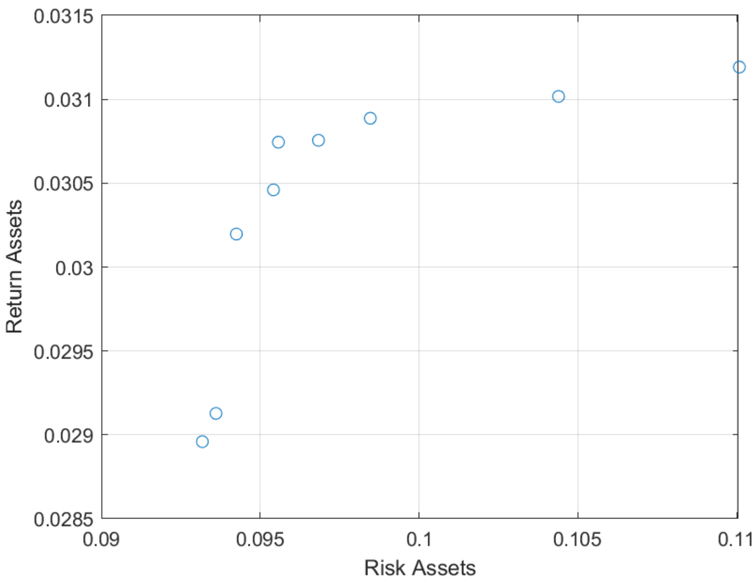
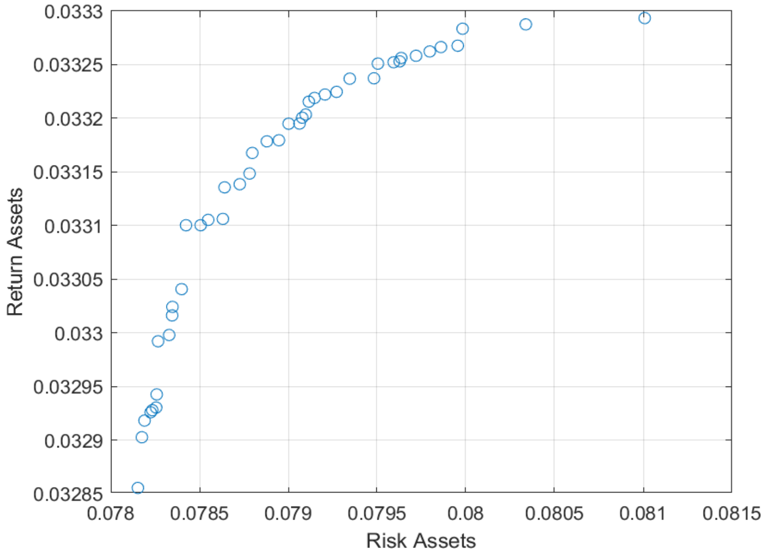


Figure 13

The Pareto Front - the Effective Frontier for the Optimized Portfolio with the SPEA-II Algorithm Over 1000 Algorithm Runs



The optimization results for each objective of the Pareto front can be found in Tables 7 and 8. For the first objective, the portfolio return, the minimum return is 0.09318, the maximum return is 0.11008 and the average return is 0.0979777. The situation similar, for the second objective, the minimum risk is 0.031192, the maximum risk is 0.028959 and the average risk is -0.030371. The RMSE value of 0.05702 indicates the high accuracy and robustness of the proposed algorithm in approximating the true Pareto front. In the algorithm mentioned above, the objectives are calculated directly in the cost function. This cost function receives a vector (portfolio) as input and calculates the values of both objectives simultaneously. The first objective represents the return (Table 7) and the second objective represents the risk (Table 8). Therefore, both objective values (return and risk) are calculated in the cost function for each portfolio and sent as output. The results are shown in Tables 7 and 8.

Table 7

Optimization Results for the First Objective (Portfolio Return) with SPEA-II

Objective 1	
Min	0.09318
Max	0.11008
Range	0.01690
St.D	0.00567
Mean	0.09797

Table 8

Optimization Results for the Second Objective (Portfolio Return) With SPEA-II

Objective 2	
Min	-0.031192
Max	-0.028959
Range	0.0022323
St. Dev	0.00080782
Mean	-0.030371
RMSE	0.05702

Tables 7 and 8 show that the portfolio returns optimized by the SPEA-II algorithm are within a relatively limited range, indicating the stability and reliability of the algorithm in finding portfolios with desirable returns. The low standard deviation also indicates the stability of the results across different algorithm runs. This observation aligns with prior studies that highlight the ability of SPEA-II to maintain consistent performance across multiple runs (Deb et al., 2002; Zitzler et al., 2001). The low standard deviation further supports the stability of the results, confirming findings from studies that emphasize the robustness of SPEA-II in handling MOO problems under varying conditions (Van Veldhuizen & Lamont, 2000). The negative values for risk result from the definition of risk in the model used. The narrow risk range and the low standard deviation show that the algorithm is able to find portfolios with low and similar risks across different runs. The average risk also indicates the average risk level that the algorithm predicts for the optimized portfolios. Overall, the results show the efficiency of the SPEA-II algorithm in finding balanced and stable portfolios with both desirable returns and risks that can be valuable for data-driven investment decisions.

CONCLUSION

In this study, we focused on the optimization of stock portfolios using innovative algorithms in addition to traditional and intelligent methods. First, well-known criteria and models from existing research were used to minimize the minimum expected return in classical optimization. Then, taking into account the needs and complexity of the financial markets, the Markowitz mean-variance model was modified by replacing the variance with the semi-variance and defining the risk minimization function as the objective function of the problem. By applying certain restrictions, the developed mean-semi-variance model was chosen as the base model for the study. Subsequently, the SPEA-II algorithm with the proposed objective function was used to optimize stock portfolio. Finally, an empirical example was presented to demonstrate the experimental results of the SPEA-II algorithm. These results show that SPEA-II outperforms classical methods in the selection and optimization stock portfolios. In addition, the algorithm converges quickly and provides all necessary information in a single pass. Overall, SPEA-II has proven to be a suitable tool for optimizing stock portfolios. In the classical method, criteria such as Value at Risk (VaR), Conditional Value at Risk (CVaR), and mean semi-variance were used for optimization. These criteria are used to assess the risk and return of companies.

With this classical approach, the company with the highest potential return and the lowest risk based on these criteria is selected as the best company. Tables 5 and 6 show the optimal distribution of investments across 132 companies with specific investment weights using the SPEA-II algorithm. For each given investment weight, the amount invested in each company is determined. To analyze this distribution, one can examine how the investments in companies change with different investment weights. When the weights change, the companies that receive the most investment at higher weights may vary. By observing the changes in investments in the different weight distributions, you can analyze the strengths and weaknesses of each distribution. In some distributions, a higher weighting investment in companies with high returns can also increase the investment risk. Conversely, in other distributions, increasing the investment weighting in companies with lower-risk can reduce return. By analyzing the Pareto optimum points of the SPEA-II optimization, it is also possible to identify companies that consistently hold superior positions in all optimal distributions. These companies can be considered as suitable candidates for investment in an optimized portfolio. Based on the research results, it is recommended to develop advanced multi-objective models. Future studies should explore further multi-objective models that, incorporate other elements besides risk and return, such as diversification, sustainability, and financial constraints in portfolio optimization. The use of parallel optimization techniques is also proposed. Exploring the potential to improve the performance of SPEA-II through parallel optimization techniques, such as population distributions and divide-and-conquer strategies, may be beneficial.

The innovative aspect of this research lies in the integration of the SPEA-II algorithm with a tailored multi-objective optimization framework specifically designed for the context of portfolio selection in Iran. Unlike conventional optimization methods, the proposed framework addresses both convex and non-convex regions of the objective space, providing a more comprehensive and accurate representation of Pareto fronts. This ensures stability, reliability, and improved accuracy in the optimization process. Furthermore, the novelty of this study is highlighted in the inclusion of a region-specific parameter tuning approach, which ensures that the algorithm is well-suited to the characteristics of the Iranian financial market. This tailored approach, combined with the extended analysis of convexity and parameter selection, offers a significant contribution to the field of multi-objective optimization, particularly in the context of portfolio management.

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