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FORECASTING THE REALIZED VOLATILITY OF ISLAMIC EQUITIES USING MULTIVARIATE HAR-TYPE MODELS

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ABSTRACT

This study proposes nine multivariate intraday models using various realized variation measures with the aim to improve volatility forecasting in the Islamic stock market in Malaysia using a dataset from 1st April 2008 to 31st March 2018. The findings show that considering independently the jump-robust realized volatility, additional daily jump realized volatility, and continuous and discontinuous jump sample path variations improved the in-sample predictive regressions compared to using the standard realized volatility. For the out-of-sample volatility forecasts evaluation, it is observed that the volatility models that disentangled the realized volatility into its continuous and discontinuous jump components have outperformed the rest of the proposed models. This is because both the continuous and discontinuous variation of returns exhibit distinctive substantial information in yielding the final volatility dynamic and thus should be modeled disjointedly. However, the empirical results suggest that the simple autoregressive specification using the standard realized volatility is often performing better or as well as the new extension models. Lastly, this study may provide useful insight in portfolio management, risk assessment, and asset pricing, particularly in the *Shariah*-compliant equities.

Keywords: Realized volatility, volatility forecasting, multivariate heterogeneous autoregressive model, Islamic stock markets.

JEL Classification: C32, C58, G1.

INTRODUCTION

In the past decades, the literature of volatility modeling often analyses the return and volatility forecasting of the financial time series using the returns data sampled at daily or lower frequency. Even though one can discover much from the analysis using low-frequency data, it still fails to reveal both the news effect that incorporated exceedingly rapid as well as short-run dynamic effects and is unable to describe satisfactorily the stylized facts (Ferrari et al., 2021). Besides, using the squared daily return as a proxy of conditional volatility may also produce misleading findings due to its noisy nature, where the noise will mask the strong persistence in the volatility dynamics (Andersen et al., 2003; Jian Zhou, 2020). According to Degiannakis and Floros (2016), the noisier a volatility proxy, the less accurate the volatility forecast evaluation.

As a result, Andersen and Bollerslev (1998) introduced the concept of realized volatility (RV), which is computed from the high-frequency data, to curb this issue. RV is a non-parametric ex-post volatility measure of the return variation that can minimize noise accumulation due to market frictions. In principle, RV is treated as an 'observed' rather than a latent variable. Observable volatility builds entirely on a new opportunity in volatility modeling and forecasting because it requires a much simpler technique to be used, whereas if volatility is latent, a complex econometric model will be required (Andersen et al., 1999). In addition, RV is an unbiased, consistent, and highly efficient proxy (Barndorff-Nielsen & Shephard, 2002) for true volatility that is based on the theory of quadratic variation and arbitrage-free processes.

There are rising studies utilizing the theoretical and practical upsides of RV in volatility modeling (Chen et al., 2021; Degiannakis et al., 2022; Liu & Wang, 2021; Wen et al., 2021). The literature shows that the RV-based models generally outperform the traditional stochastic volatility (SV) models and the family of the generalized autoregressive conditional heteroscedastic (GARCH) models of Engle (1982) and Bollerslev (1986) that are based on squared daily returns in the volatility predictive performance (Bergsli et al., 2022; Wei, 2021; Tran & Tran, 2021). This is because high-frequency data contains more information about the real trading situations, and RV is considered an 'observable' volatility proxy. Nonetheless, the RV-based modeling is under-researched on the volatility transmission in a multivariate approach, specifically in the context of the Islamic stock market.

Over the past decades, the liberalization of capital movements and reformation of national financial systems have resulted in greater linkages between financial markets in countries around the globe. Moreover, the expansion of information technology has also allowed information to spread more freely than ever before. Thus, news and shocks initiated from the rest of the world are more likely to affect the market (Zhong et al., 2019). Numerous studies (Uludag & Khurshid, 2019; Tang et al., 2021) have shown that volatility changes are not only due to the dynamic evolution of its own market volatility but also changes of contagion effect and interdependency across markets. Consequently, the multivariate analysis has gained greater attention because it helps to control for possible endogeneity issues in volatility modeling and forecasting equations to overcome the weak assumption of market independence of the univariate analysis that neglected the dynamic linkages among the variance and covariance series. Financial institutions face a higher level of risk due to uncertainties in global and regional financial markets. Therefore, in the context of reducing portfolio risk, the subject of discovering the transmission of the financial return and volatility over time and across markets has become the central attraction among practitioners and academic researchers in the hope of formulating an effective financial risk management strategy.

Today, Islamic financial market runs in parallel with the conventional financial market. In addition, it provides investors with a distinct investment philosophy that is rapidly gaining acceptance worldwide. The rise in the number of Islamic equity indices and sub-indices available in the stock markets has created the opportunity for investors to allocate their portfolio in various types of sectors. Furthermore, with the availability of Islamic equities, Muslim investors no longer need to forgo profits to conform to their religious obligations. The global Islamic financial markets have experienced a remarkable growth of approximately 17 percent annually (IFDI, 2022), demonstrating their resilience and attracting investors. The recognition of the potential of diversification opportunities (Iftekhar et al., 2022) in the Shariah-compliant instruments and standardization of Shariah conformity procedures in the Islamic equities have precipitated the financialization of Islamic stock markets.

Considering the above-mentioned issues such as the noisy nature of squared daily return and endogeneity issues in volatility forecasting, hence, this study aims to analyze the forecasting accuracy of time-varying volatility spillover within the Islamic stock markets, deriving on the theory of realized volatility and the conceptual framework of heterogeneous market hypothesis (HMH). Motivated by the theoretical works of Barndorff-Nielsen and Shephard (2004b) and Andersen et al. (2012), who introduced the jump-robust realized measures and empirical evidence that supports the importance of the inclusion of daily jump regressors (Sévi, 2014) and jump-robust estimators (Chin et al., 2016) into the simple Heterogeneous Autoregressive (HAR) model of Corsi (2009), this study extends the univariate HAR that takes account for intraday jumps into a multivariate context. Also motivated theoretically (Andersen et al., 2007) and empirically (Sévi, 2014; Wen et al., 2016) on the significance of decomposing the realized volatility into its continuous and discontinuous jump sample path variations, this study extends it into the multivariate version as well. On the other hand, being motivated by the remarkable expansion of the Islamic finance over the past decade, the data selection in this study involves the Malaysian Islamic stock index and the Islamic sectoral stock indices of Dow Jones Islamic Market (DJIM). Under the role of high-frequency data, this study contributes a large-scale of empirical analysis in the volatility forecasting under the RV-based modeling derived from the Heterogeneous Autoregressive (HAR) model that considers various components of the realized volatility into a multivariate setting in the Islamic stock markets, particularly at the sectoral level. The findings of this study will have implications in risk management and portfolio selection, particularly to address the unique risks in Islamic investment.

The rest of the paper is organized as follows: Section 2 organizes the rest of the paper by reviewing relevant literature on the properties of popular jump-robust estimators and the development of HAR-type models. Section 3 presents the basic setup of jump-robust realized volatility and the framework of the multivariate HAR model. Section 4 discusses the results of the in-sample estimation performance and out-of-sample forecast evaluation. Section 5 concludes the paper.

LITERATURE REVIEW

Although the RV is broadly used in estimating the integrated variance of financial time series, the proxy encounters inconsistency issues under the occurrences of abrupt jumps in the asset prices (Andersen et al., 2003; Ewing & Malik 2016; Gao et al., 2022) due to economic shocks, political turmoil, natural disasters, institutional changes, and inter alia. Disregarding the presence of jumps may lead to false statistical conclusions such as inaccurate descriptive statistics, erroneous hypothesis inferences, and fallacious forecasts. Some past literature (Ma et al., 2018) show that the discontinuous jump component does not contain important information and fails to improve the volatility forecast. This assertion seems counter-intuitive since large increases in volatility are typically preceded by large jumps that change the investors' perception of the value of an underlying asset. A few potential explanations include the difficulty in achieving a precise measure of jump volatility (Garcia & Hill, 2017) and the potential for large sample bias in the realized estimators used in previous studies (Corsi et al., 2010). In contrast, several researchers, for example, Andersen et al. (2011), Bollerslev et al. (2009), Gao et al. (2022), Li et al. (2017), Maheu et al. (2013), and Qua et al. (2018), claimed that jump returns can provide significant information in exploring the risk relationships between assets. This is because the jump returns have a much higher signal to noise ratio and contain an independent measurable risk premium (Garcia & Hill, 2017).

To tackle this problem, several jump-robust realized variances are introduced in the literature, such as the realized bi-power variance (Barndorff-Nielsen & Shephard, 2004b), multi power variation (Andersen & Todorov, 2010), truncated realized variance (Andersen et al., 2012), and quantile realized variance (Christensen et al., 2010). As such, these estimators offer new and useful information on the dynamics of stock prices, for instance, it enables to evaluate the presence and significance of price jumps. The empirical evidence shows that an estimator that can well explain the realized variance jump process generates a significant impact on future volatility (Mamoona et al., 2022; Massiniliano, 2022) of the underlying assets whenever there is an arrival of new information (Yuan & Li, 2018) and provides substantial improvement in the accuracy of the volatility forecast (Fuertes & Olmo, 2012). Therefore, this study considers incorporating the intraday jump in the proposed volatility models since it has a strong impact on future volatility.

In addition, the theory of quadratic variation postulates that total variation of an asset return can be disentangled into continuous and discontinuous jump components. The empirical literature suggests that the continuous and discontinuous jump variation of returns exhibit distinct information about volatility dynamics. In other words, though both continuous and discontinuous jump components jointly produce the final volatility dynamics, their dynamics are different and thus should be modeled separately (Corsi et al., 2010). The jumps have a very short impact on future volatility, whereas continuous returns tend to have a persistent impact on future volatility. This is evidenced by Andersen et al. (2007) that they discovered the continuous component is noticeably more persistent than the volatility jump component, inferring the continuous component as a significant predictor of future volatility. The empirical studies also show that decomposition of the realized volatility into its continuous and discontinuous jump components has yielded improved out-of-sample volatility forecasts (Andersen et al., 2011). As such, these measures give useful and novel information on modeling the dynamics of stock prices, which leads this study towards decomposing the realized volatility into its continuous and discontinuous components in modeling the volatility measures.

The HAR model of Corsi (2009) has appeared as the preferred specification to further improve the RV-based modeling. It is because the HAR model can attain long memory in a parsimonious process without having to rely on fractional integration. In addition to this, this model does not necessitate restriction to the parameters, and it guarantees positive-definite estimates (Choi et al., 2010). Moreover, the HAR model is easily enhanced by external variables for improving the explanatory power of volatility dynamics.

Therefore, numerous researchers started to develop new volatility models based on the HAR-RV framework to further enhance volatility forecasting performance. For instance, the HAR-RV-J model of Andersen et al. (2007) with an additional daily jump component, the HAR-RV-CJ model of Andersen et al. (2007), in which the realized volatility is decomposed into its continuous and discontinuous jump components and the HAR-RSV model of Patton and Sheppard (2015) use the positive and negative realized semi variances (RSV), and among other things. Recent literature (Bergsli et al., 2022; Gong & Lin, 2019) exhibits evidence that HAR-type models provide superior volatility forecasting performance than the SV-type, GARCH-type, VAR-RV, and ARFIMA-RV models (Shin, 2018).

However, Bubák et al. (2011) use the VHAR model of logarithmic realized variances to look at how volatility was transmitted between Central European (CE) exchange rates and the EUR/USD foreign exchange from 2003 to 2009. A similar approach by Soucek and Todorova (2013), who extend the empirical model of Bubák et al. (2011) by using an orthogonalized version in studying the interrelationship of the equities (S&P 500, Nikkei 225, FTSE100) and energy (West Texas Intermediate, WTI) markets from 2002 to 2012. The study by Luo and Ji (2018) is also quite similar to Bubák et al. (2011); apart from using the realized volatility, they use the positive and negative realized semi-variance in examining the volatility connectedness between oil and agricultural markets. Based on the existing literature, there is a minimal study on modeling the volatility transmission using HAR-type models in the multivariate perspective, particularly in the context of Islamic equities. As a result, this paper extends the HAR model with the crucial intraday jump specifications into a multivariate setting to examine the in-sample and out-of-sample volatility forecasting of Islamic equities.

DATA AND METHODOLOGY

Data

The historical data used in this study is the 5-minutely intraday closing price of the Dow Jones Islamic Market Malaysia Titans 25 (DJMY25) index and ten sectoral indices of the Dow Jones Islamic Market (DJIM) World index from 1st April 2008 to 31st March 2018. The full sample of the intraday data is divided into the in-sample data, which spans from 1st April 2008 to 31st March 2017, and the out-of-sample data, which spans from 1st April 2017 to 31st March 2018. Dobrev and Szerszen (2010) found that two to five years of high-frequency data are sufficient to achieve a similar level of accuracy as 20 years of daily data. This study uses nine trading years of high-frequency data in modeling the market return and volatility, which has a more extended period than the required standard as aforementioned. On the other hand, the sample period includes several major financial crises, such as the global financial crisis period spiked in the early of 2008 and the European sovereign debt crisis period in the late of 2010, in order to ensure that the data is highly volatile with possible abrupt changes in the indices since this study examines the robustness of the jump-robust realized volatilities used in this study. The ten sectoral indices consist of the Dow Jones Islamic Market Basic Materials (DJIBSC), Dow Jones Islamic Market Consumer Services (DJICYC), Dow Jones Islamic Market Oil & Gas (DJIENE), Dow Jones Islamic Market Financials (DJIFIN), Dow Jones Islamic Market Healthcare (DJIHCR), Dow Jones Islamic Market Industrial (DJIIDU), Dow Jones Islamic Market Consumer Goods (DJINCY), Dow Jones Islamic Market Technology (DJITEC), Dow Jones Islamic Market Telecommunications (DJITLS), and Dow Jones Islamic Market Utilities (DJIUTI) indices. The DJMY25 index is paired with each of these global Islamic sectoral stock indices, forming ten bivariate markets.

Methodology

Using the theory of quadratic variation of semi-martingales, Andersen and Bollerslev (1998) introduced the first realized volatility (RV) as

$$RV_t = \sqrt{\sum_{i=1}^{M} r_{t,i}^2} \text{ for } \quad i = 1, 2, \ldots M \tag{1}$$

where r_t is the intraday return at t and M is the number of returns per day.

Following the approach of Andersen et al. (2001), this study assumes the sample path of an asset price process belongs to the continuous-time semi-martingale jump-diffusion process as such

$$d \ln P_t = \mu_t dt + \sigma_t dW_t \ + \ J_t dN_t \ \ ; \ 0 \leq t \leq T \tag{2}$$

where J_t is the random jump size and N_t is a Poisson process that counts the number of jumps that takes the value of one in the case of a jump and zero otherwise. Under the finite jump-diffusion process, W_t, the one-period continuously compounded return is defined as

$$r_t \equiv \int_{t-1}^{t} \mu_s ds + \int_{t-1}^{t} \sigma_s dW_s + \sum_{t-1<s\leq t} J_s \quad ; 0 \leq t \leq T \tag{3}$$

where $J_t \neq 0$ only in the presence of a jump in the process. Again, using the theory of the quadratic variation, the daily realized variance converges in probability as the sampling frequency increases as such $RV_t \longrightarrow \int_{t-1}^{t} \sigma_s^2 ds_s + \sum_{t-1<s\leq t} J_s^2$. Hence, the RV_t for day t incorporates two components as the total variation of price process which are the integrated variance, $IV_t = \int_{t-1}^{t} \sigma_s^2 dW_s$ and the cumulative jumps component,$JV_t = \sum_{t-1<s\leq t} J_s^2$.

As a result, the RV is no longer a consistent estimator of integrated variance as it captures not only the return volatility at time t but also includes the jump variation (JV). To disentangle the continuous variation from the jump components, Barndorff-Nielsen and Shephard (2004b) have proposed the realized bi-power volatility (RBV) defined as

$$RBV_t = \sqrt{\xi^{-2} \sum_{i=2}^{M} |r_{t,i}||r_{t,i-1}|} \tag{4}$$

where $\xi = \frac{\Gamma(1)\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} = \sqrt{\frac{2}{\pi}}$ and $\Gamma(.)$ is the gamma function. Equation (4) is interpreted as the cumulative sum of products of the adjacent absolute returns. The RBV estimator ensures the jumps (under $M \rightarrow \infty$) will not impact the consistency of the volatility estimates. This is due to the return characterizing the jump being diminished by the multiplication of the adjacent diffusive (non-jump) intraday return. Asymptotically, as the sampling frequency increases, the jump impact becomes negligible. In other words, RBV purely measures the continuous component under the condition of the presence of jumps, hence, RBV is a consistent estimate of integrated variance ($RBV_t \longrightarrow IV_t$). The RV can be decomposed into its continuous (diffusive) and discontinuous jump (non-diffusive) components by the RBV ($RV_t - RBV_t \xrightarrow[M\rightarrow\infty]{} QV_t - IV_t = JV_t$).

The advantage of using RBV to estimate integrated variance is that it is robust to the finite and small magnitude of jumps. However, the RBV has some drawbacks in empirical applications. In practice, if the sampling frequency is not sufficiently high (finite sample), instead of having an adjacent diffusive intraday return, it might have an adjacent (large) jump intraday return that could lead to an upward bias in the RBV. Another drawback of RBV is that the presence of zero returns that are multiplied twice (with the previous and the following intraday return) will have caused a downward bias in the RBV.

To address the aforementioned issues, Andersen et al. (2012) proposed two estimators of integrated variance that are more robust to jumps in finite samples as the alternative to RBV. These two new jump-robust estimators are the median realized volatility,

$$medRV_t = \sqrt{\frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{M}{M-2}\right)\sum_{i=2}^{M-1} med\left(|r_{t,i-1}|,|r_{t,i}|,|r_{t,i+1}|\right)^2} \tag{5}$$

and minimum realized volatility,

$$min\,R\,V_t = \sqrt{\frac{\pi}{\pi-2}\left(\frac{M}{M-1}\right)\sum_{i=1}^{M-1} min\left(|r_{t,i}|,|r_{t,i+1}|\right)^2} \tag{6}$$

By applying scaling factors and manipulating short overlapping blocks, both estimators demonstrate better efficiency properties compared to RBV. Under the presence of a jump, minRV automatically discards a (large) jump return using the square of the minimum of a given block with two consecutive intraday absolute returns, and the computation will fully consider the adjacent non-jump (diffusive) return while medRV uses the square of the median of the block with three consecutive intraday absolute returns. This approach is referred to as one-sided and two-sided truncation neighboring returns for minRV and medRV, respectively. The nearest neighbor truncation approach serves as an endogenous control for the local level of volatility, improving the robustness of the estimators with the assumption that the returns within each block are I.I.D. Gaussian. Besides, it allows for the asymptotic distribution theory, which can shrink the effect of jumps at a quicker asymptotic rate than RBV. In the effort to obtain reliable results, all three jump-robust realized volatilities that consist of the RBV, medRV, and minRV are considered in this study.

Vector Heterogeneous Autoregressive (VHAR) Model

To analyze the potential volatility transmission patterns among the Islamic equities, the univariate HAR (Corsi, 2009) approach is extended to a multivariate version to model the joint behavior of the series as used by Bubák et al. (2011) and Soucek and Todorova (2013). In the multivariate version of the HAR (also known as vector HAR, VHAR), the volatility forecasts are of linear functions containing the daily, weekly, and monthly realized volatilities.

The general form of the VHAR specification, which models the vector of Cholesky factors using realized volatility, VHAR (RV) model, is given as follows

$$\begin{aligned} RV_{k,t}^{(d)} \quad = \quad & \alpha_{k,0} + \alpha_k^{(d)} RV_{k,t-1}^{(d)} + \alpha_k^{(w)} RV_{k,t-1}^{(w)} + \alpha_k^{(m)} RV_{k,t-1}^{(m)} \varepsilon_{k,t}; \\ & \varepsilon_t | \Omega_{t-1} \sim NIID(0,1) \end{aligned} \tag{7}$$

where the subscript $k = 1, 2$ represent the first and second markets respectively, $RV_{k,t-1}^{(d)}$, $RV_{k,t-1}^{(w)}$ and $RV_{k,t-1}^{(m)}$ are the one day lagged daily, weekly (5 days) and monthly (22 days) realized volatility vectors respectively, and ε_t are assumed to be Gaussian white noise α_0 is an $n \times 1$ vector of constants, $\alpha_k^{(\cdot)}$is an $n \times 1$ vector of parameters. The weekly and monthly realized volatility estimators are computed as $RV_{k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} RV_{k,t-i}^{(d)}$ and$RV_{k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} RV_{k,t-i}^{(d)}$, respectively.

VHAR-JR Model

The VHAR (RV) model in equation (7) using the realized volatility as the volatility proxy can be replaced with the jump-robust (JR) realized measures such as the realized bi-power volatility, median realized volatility, and minimum realized volatility estimators as follows.

VHAR-JR (RBV):

$$RV_{k,t}^{(d)} = \alpha_{k,0} + \alpha_k^{(d)} RBV_{k,t-1}^{(d)} + \alpha_k^{(w)} RBV_{k,t-1}^{(w)} + \alpha_k^{(m)} RBV_{k,t-1}^{(m)} + \varepsilon_{k,t} \quad (8)$$

VHAR-JR (medRV):

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)}\, m\, e\, dRV_{k,t-1}^{(d)} + \alpha_k^{(w)}\, m\, e\, dRV_{k,t-1}^{(w)} \\ & + \alpha_k^{(m)}\, m\, e\, dRV_{k,t-1}^{(m)} + \varepsilon_{k,t} \end{aligned} \quad (9)$$

VHAR-JR (minRV):

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)}\, min\, R\, V_{k,t-1}^{(d)} + \alpha_k^{(w)}\, min\, R\, V_{k,t-1}^{(w)} \\ & + \alpha_k^{(m)}\, min\, R\, V_{k,t-1}^{(m)} + \varepsilon_{k,t} \end{aligned} \quad (10)$$

where the weekly $RBV_{k,t-1}^{(w)}$, $medRV_{k,t-1}^{(w)}$ and $min\, R\, V_{k,t-1}^{(w)}$ and monthly $RBV_{k,t-1}^{(m)}$, $medRV_{k,t-1}^{(m)}$ and $min\, R\, V_{k,t-1}^{(m)}$ of the realized measures are computed as $RBV_{k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} RBV_{k,t-i}^{(d)}$, $medRV_{k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} medRV_{k,t-i}^{(d)}$, $min\, R\, V_{k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} min\, R\, V_{k,t-i}^{(d)}$, $RBV_{k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} RBV_{k,t-i}^{(d)}$, $medRV_{k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} medRV_{k,t-i}^{(d)}$ and $min\, R\, V_{k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} min\, R\, V_{k,t-i}^{(d)}$, respectively.

VHAR-CJ Model

Andersen et al. (2007) take the extension of the HAR-RV-J model a step further. They developed the HAR-CJ model by decomposing the realized volatility of the HAR-RV model into its continuous and discontinuous jump variation at different time horizons. In other words, the explanatory variables of the HAR-RV model, such as the daily, weekly, and monthly realized volatilities, are replaced by the daily, weekly, and monthly continuous and discontinuous jump components. Likewise, the HAR-CJ model is extended into the multivariate version. The general form of the extended model Vector HAR-CJ (VHAR-CJ) takes the following form.

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_0 + \alpha_k^{(d)} CV_{i,k,t-1}^{(d)} + \alpha_k^{(w)} CV_{i,k,t-1}^{(w)} + \alpha_k^{(m)} CV_{i,k,t-1}^{(m)} \\ & + \beta_k^{(d)} JV_{i,k,t-1}^{(d)} + \beta_k^{(w)} JV_{i,k,t-1}^{(w)} + \beta_k^{(m)} J\, \varepsilon_{k,t} \end{aligned} \quad (11)$$

where $CV_{i,k,t-1}^{(d)}$, $CV_{i,k,t-1}^{(w)}$ and $CV_{i,k,t-1}^{(wm)}$ are the daily, weekly, and monthly continuous realized variation, respectively; $JV_{i,k,t-1}^{(d)}$, $JV_{i,k,t-1}^{(w)}$and $JV_{i,k,t-1}^{(m)}$ are the daily, weekly, and monthly discontinuous jump variation, respectively with i =RBV, medRV or minRV. The weekly and monthly components of both continuous and discontinuous components are computed as follows $CV_{i,k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} CV_{i,k,t-i}^{(d)}$, $CV_{i,k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} CV_{i,k,t-i}^{(d)}$, $JV_{i,k,t-1}^{(w)} = \frac{1}{5}\sum_{i=1}^{5} JV_{i,k,t-i}^{(d)}$ and $JV_{i,k,t-1}^{(m)} = \frac{1}{22}\sum_{i=1}^{22} JV_{i,k,t-i}^{(d)}$.

Similarly, the equation (11) can be written specifically as follows:

VHAR-CJ (RBV):

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)} CV_{RBV,k,t-1}^{(d)} + \alpha_k^{(w)} CV_{RBV,k,t-1}^{(w)} + \alpha_k^{(m)} CV_{RBV,k,t-1}^{(m)} \\ & + \beta_k^{(d)} JV_{RBV,k,t-1}^{(d)} + \beta_k^{(w)} JV_{RBV,k,t-1}^{(w)} + \beta_k^{(m)} JV_{RBV,k,t-1}^{(m)} + \varepsilon_{k,t} \end{aligned} \quad (12)$$

VHAR-CJ (medRV):

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)} CV_{medRV,k,t-1}^{(d)} + \alpha_k^{(w)} CV_{medRV,k,t-1}^{(w)} \\ & + \alpha_k^{(m)} CV_{medRV,k,t-1}^{(m)} + \beta_k^{(d)} JV_{medRV,k,t-1}^{(d)} + \beta_k^{(w)} JV_{medRV,k,t-1}^{(w)} \\ & + \beta_k^{(m)} JV_{medRV,k,t-1}^{(m)} + \varepsilon_{k,t} \end{aligned} \quad (13)$$

VHAR-CJ (minRV):

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)} CV_{min\,RV,k,t-1}^{(d)} + \alpha_k^{(w)} CV_{min\,RV,k,t-1}^{(w)} \\ & + \alpha_k^{(m)} CV_{min\,RV,k,t-1}^{(m)} + \beta_k^{(d)} JV_{min\,RV,k,t-1}^{(d)} \\ & + \beta_k^{(w)} JV_{min\,RV,k,t-1}^{(w)} + \beta_k^{(m)} JV_{min\,RV,k,t-1}^{(m)} + \varepsilon_{k,t} \end{aligned} \quad (14)$$

VHAR-RV-J Model

The benchmark model (Equation (7)) assumes that the price process belongs to a continuous sample path variation. However, the actual price process consists of both continuous and discontinuous jump components. Therefore, both the continuous and discontinuous jump components contribute to the volatility of the price process. It has been shown earlier (in Section 3.2) that the realized variation estimators can be decomposed into their continuous and discontinuous jump parts. Knowing this, to evaluate whether the jump component can improve the forecast volatility, Andersen et al. (2007) proposed the HAR-RV-J by adding an explanatory variable, which is the daily discontinuous jump variation, into the HAR model (Equation (7)) of Corsi (2009). Motivated by Andersen et al. (2007), this study generalizes the HAR-RV-J model into the multivariate version as VHAR-RV-J presented.

$$\begin{aligned} RV_{k,t}^{(d)} = {} & \alpha_{k,0} + \alpha_k^{(d)} RV_{k,t-1}^{(d)} + \alpha_k^{(w)} RV_{k,t-1}^{(w)} \\ & + \alpha_k^{(m)} RV_{k,t-1}^{(m)} + \beta_k^{(d)} JV_{i,k,t-1}^{(d)} + \varepsilon_{k,t} \end{aligned} \quad (15)$$

where $JV_{i,k,t-1}^{(d)}$ is the daily discontinuous jump variation with i =RBV, medRV, or minRV. Therefore, another three new multivariate HAR models are proposed in this study and can be written as

VHAR-RV-J (RBV):

$$RV_{k,t}^{(d} = \alpha_{k,0} + \alpha_k^{(d)} RV_{k,t-1}^{(d)} + \alpha_k^{(w)} RV_{k,t-1}^{(w)} + \alpha_k^{(m)} RV_{k,t-1}^{(m)} + \beta_k^{(d)} JV_{RBV,k,t-1}^{(d)} + \varepsilon_{k,t} \quad (16)$$

VHAR-RV-J (medRV):

$$RV_{k,t}^{(d)} = \alpha_{k,0} + \alpha_k^{(d)} RV_{k,t-1}^{(d)} + \alpha_k^{(w)} RV_{k,t-1}^{(w)} + \alpha_k^{(m)} RV_{k,t-1}^{(m)} + \beta_k^{(d)} JV_{medRV,k,t-1}^{(d)} + \varepsilon_{k,t} \quad (17)$$

VHAR-RV-J (minRV):

$$RV_{k,t}^{(d)} = \alpha_{k,0} + \alpha_k^{(d)} RV_{k,t-1}^{(d)} + \alpha_k^{(w)} RV_{k,t-1}^{(w)} + \alpha_k^{(m)} RV_{k,t-1}^{(m)} + \beta_k^{(d)} JV_{min\,RV,k,t-1}^{(d)} + \varepsilon_{k,t} \quad (18)$$

RESULT AND DISCUSSION

Preliminary Analysis

Table 1 shows the summary of the descriptive statistics of the in-sample return and realized variation measures series. Most of the indices have a positive mean return, with values close to zero. The result shows DJIENE is the most volatile index relative to other indices because it has the largest standard deviation, 1.72, while the least volatile index is DJMY25 with a standard deviation of 0.744. The skewness of return in all markets is approximately symmetric, with the skewness value being less than 0.6 in modulus. However, all the realized variation measures across all markets exhibit a high degree of positive skewness, suggesting that the lower tail of the distribution is significantly longer at the right tail. The kurtosis coefficients exceeded three in the daily return and realized measures series across all markets, illustrating the presence of heavy-tailed behavior and a higher peak. This implies the series violated the normality properties, which is also supported by the results of the Jarque-Bera normality test at the 5 percent significance level. As a result, the estimation is based on the student's t-distribution.

Ljung-Box Q-statistic in all the standardized and squared standardized residual series up to lag 12 are highly significant except for the $Q_{LB}^2(12)$ statistic in DJITEC. The significant sign of the standardized residual series, $Q_{LB}(12)$ indicates the evidence of serial correlation and dependency, implying the conditional mean is forecastable using historical data. Whereas, the significant sign of the squared standardized residual series, $Q_{LB}^2(12)$ reveals the presence of time-varying volatility effects in the series. The Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests reject the null hypothesis of a unit root in all series across markets at the 5 percent significance level. This implies that the return and realized measures series are stationary, suggesting the series could be modeled directly without any further transformation process. The Hurst exponent, H, measures the long-range dependence (LRD) behavior of a time series. It is observed that the realized measures show H values are between 0.641 and 0.991. As a result, the realized measures series are consistent across markets and exhibit LRD.

The Granger causality test and the unconditional correlation coefficient are conducted to capture an initial understanding of the causality and strength of the relationship, respectively, before exploring into forecasting the time-varying volatility spillover within the Islamic stock markets. Table 2 displays the results of the F-statistics obtained from the Granger causality test of the bivariate markets. The findings of the Granger causality test can be classified into three different outcomes. First, it is found that there is no significant Granger causality between DJMY25 and DJINCY, implying investors may gain potential benefits by diversifying in these stock markets. Second, the study uncovers a significant unidirectional Granger causality from DJMY25 to DJIFIN and DJITLS. This suggests that the price movement in DJMY25 may be useful in forecasting the future volatility of DJIFIN and DJITLS but not vice versa. Third, there is a significant bidirectional Granger causality found in DJMY25-DJIBSC, DJMY25-DJICYC, DJMY25-DJIENE, DJMY25-DJIHCR, DJMY25-DJIIDU, DJMY25-DJITEC, and DJMY25-DJIUTI. The significant causality reveals a correlation between the indices, suggesting that any changes in one stock market could potentially affect the other. On the other hand, Table 3 shows the unconditional correlation coefficient of the daily return and realized variation measures between DJMY25 and the DJIM sectoral stock markets. The measures of the unconditional correlation range from -0.0016 to 0.3996. These preliminary findings demonstrate that information is transmitted between the DJMY25 and DJIM sectoral indices. Therefore, it is worth further investigating these equities markets in terms of their volatility spillover effects meticulously to enhance the estimation and forecasting of volatility.

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Empirical Analysis

Table 4 presents the typology for ten models that consist of the benchmark model and nine proposed models derived from three main specifications estimated using the 5-minutely intraday data in this study. The three main specifications are the VHAR-JR, VHAR-RV-J, and VHAR-CJ specifications. The benchmark model is the VHAR (RV) (model A01), which uses the standard realized volatility as the volatility proxy.

Table 1

Summary of Descriptive Statistics of the Return and Realized Variation Measures

	Mean	S. D.	Skew	Kurt	JB	Q_{LB}	Q_{LB}^2	ADF	PP	Hurst	Mean	S. D.	Skew	Kurt	JB	Q_{LB}	Q_{LB}^2	ADF	PP	Hurst
	Panel A: DJMY25										Panel B: DJIBSC									
Ret	0.01	0.74	-0.29	9.2	**3451**	21	**811**	**-41**	**-41**	0.61	-0.01	1.63	-0.39	10.9	**5626**	**40**	**2145**	**-32**	**-37**	0.60
RVol	0.55	0.31	2.64	15.8	**1.7×10^{4}**	**317**	**501**	**-4**	**-28**	0.99	0.60	1.20	7.28	78.1	**5.2×10^{5}**	**373**	**738**	**-12**	**-34**	0.90
RBVol	0.50	0.28	2.33	11.1	**7824**	**356**	**422**	**-4**	**-26**	0.99	0.50	1.15	8.80	111.2	**1.1×10^{6}**	**314**	**683**	**-14**	**-30**	0.87
MedRVol	0.49	0.27	2.25	9.8	**5898**	**387**	**425**	**-4**	**-25**	0.99	0.52	1.27	8.14	96.5	**8.0×10^{5}**	**334**	**710**	**-14**	**-32**	0.87
MinRVol	0.49	0.28	2.45	12.4	**1.0×10^{4}**	**416**	**379**	**-4**	**-29**	0.99	0.52	1.29	8.44	102.3	**9.1×10^{5}**	**323**	**629**	**-14**	**-32**	0.86
ContRBVol	0.52	0.28	2.28	10.6	**7003**	**364**	**395**	**-4**	**-27**	0.99	0.51	1.16	8.09	93.9	**7.6×10^{5}**	**328**	**754**	**-10**	**-31**	0.88
ContMedRVol	0.51	0.28	2.24	10.1	**6287**	**398**	**384**	**-4**	**-27**	0.99	0.52	1.24	8.19	99.5	**8.6×10^{5}**	**316**	**816**	**-9**	**-31**	0.88
ContMinRVol	0.52	0.29	2.19	9.7	**5696**	**415**	**383**	**-4**	**-28**	0.99	0.52	1.25	8.41	104.1	**9.4×10^{5}**	**305**	**657**	**-9**	**-31**	0.87
JumpRBVol	0.09	0.20	4.65	50.9	**2.1×10^{5}**	**39**	**223**	**-18**	**-44**	0.72	0.19	0.43	6.10	54.6	**2.5×10^{5}**	**634**	**584**	**-5**	**-59**	0.88
JumpMedRVol	0.10	0.22	4.25	41.0	**1.4×10^{5}**	**44**	**348**	**-17**	**-44**	0.68	0.14	0.31	8.42	118.1	**1.2×10^{5}**	**289**	**134**	**-12**	**-51**	0.79
JumpMinRVol	0.08	0.21	5.22	55.1	**2.5×10^{5}**	**26**	**291**	**-43**	**-43**	0.64	0.13	0.32	8.13	105.7	**9.7×10^{5}**	**210**	**115**	**-14**	**-50**	0.77
	Panel C: DJICYC										Panel D: DJIENE									
Ret	0.03	1.04	-0.44	8.1	**2355**	15	**2430**	**-45**	**-45**	0.54	-0.02	1.72	-0.57	12.0	**7305**	**36**	**2449**	**-35**	**-46**	0.55
RVol	0.20	1.13	23.65	699.0	**4.3×10^{7}**	**170**	19	**-17**	**-39**	0.70	0.30	1.03	13.82	260.8	**6.0×10^{6}**	**405**	**352**	**-15**	**-32**	0.80
RBVol	0.15	0.90	19.12	427.1	**1.6×10^{7}**	**293**	**104**	**-15**	**-32**	0.69	0.23	1.00	16.34	343.7	**1.0×10^{7}**	**498**	**416**	**-15**	**-30**	0.74
MedRVol	0.14	0.89	18.68	413.1	**1.5×10^{7}**	**349**	**170**	**-16**	**-30**	0.68	0.22	1.05	16.60	357.1	**1.1×10^{7}**	**496**	**398**	**-15**	**-31**	0.72
MinRVol	0.15	0.93	19.39	453.3	**1.8×10^{7}**	**372**	**155**	**-16**	**-31**	0.68	0.22	1.06	16.70	358.5	**1.1×10^{7}**	**497**	**388**	**-15**	**-31**	0.72
ContRBVol	0.15	0.89	19.23	433.5	**1.7×10^{7}**	**266**	**82**	**-15**	**-32**	0.69	0.23	0.99	15.63	315.1	**8.8×10^{6}**	**396**	**340**	**-15**	**-31**	0.75
ContMedRVol	0.15	0.88	18.94	425.0	**1.6×10^{7}**	**346**	**170**	**-15**	**-30**	0.69	0.23	1.02	15.45	305.1	**8.2×10^{6}**	**412**	**456**	**-15**	**-31**	0.73
ContMinRVol	0.15	0.92	19.53	460.4	**1.9×10^{7}**	**330**	**129**	**-15**	**-31**	0.69	0.23	0.99	15.51	310.8	**8.5×10^{6}**	**387**	**328**	**-15**	**-31**	0.74
JumpRBVol	0.08	0.74	39.91	1740.2	**2.7×10^{8}**	12	0.01	**-45**	**-46**	0.67	0.11	0.34	9.63	121.5	**1.3×10^{6}**	**580**	**850**	**-7**	**-49**	0.85
JumpMedRVol	0.08	0.78	40.77	1794.2	**2.9×10^{8}**	8	0.01	**-15**	**-30**	0.66	0.10	0.33	10.11	135.2	**1.6×10^{6}**	**610**	**1023**	**-7**	**-48**	0.84
JumpMinRVol	0.07	0.78	41.15	1817.7	**3.0×10^{8}**	6	0.01	**-15**	**-31**	0.64	0.10	0.34	10.07	130.5	**1.5×10^{6}**	**533**	**78**	**-7**	**-48**	0.86
	Panel E: DJIFIN										Panel F: DJIHCR									
Ret	0.01	1.68	-0.48	26.3	**4.9×10^{4}**	**55**	**2105**	**-32**	**-55**	0.53	0.03	0.95	-0.32	12.2	**7541**	**30**	**1167**	**-34**	**-43**	0.55
RVol	0.39	0.91	11.11	161.3	**2.3×10^{6}**	**372**	**414**	**-8**	**-36**	0.90	0.25	0.97	24.04	715.1	**4.5×10^{7}**	**134**	**20**	**-18**	**-38**	0.74
RBVol	0.29	0.75	16.49	329.2	**9.6×10^{6}**	**456**	**575**	**-13**	**-23**	0.88	0.18	0.70	19.06	420.9	**1.6×10^{7}**	**208**	**82**	**-16**	**-27**	0.72
MedRVol	0.29	0.81	15.46	282.4	**7.1×10^{6}**	**272**	**414**	**-12**	**-26**	0.87	0.18	0.74	19.19	423.7	**1.6×10^{7}**	**223**	**103**	**-16**	**-27**	0.72
MinRVol	0.28	0.81	16.74	332.5	**9.8×10^{6}**	**419**	**483**	**-15**	**-23**	0.87	0.17	0.71	18.99	422.1	**1.6×10^{7}**	**205**	**79**	**-16**	**-26**	0.72
ContRBVol	0.30	0.79	15.40	279.5	**6.9×10^{6}**	**292**	**356**	**-12**	**-28**	0.88	0.19	0.70	19.03	417.2	**1.5×10^{7}**	**202**	**94**	**-16**	**-27**	0.72
ContMedRVol	0.30	0.79	15.00	265.1	**6.2×10^{6}**	**251**	**327**	**-12**	**-27**	0.88	0.18	0.71	18.80	406.7	**1.5×10^{7}**	**210**	**103**	**-16**	**-28**	0.72
ContMinRVol	0.30	0.79	15.83	301.6	**8.1×10^{6}**	**384**	**386**	**-13**	**-24**	0.87	0.18	0.69	18.84	413.1	**1.5×10^{7}**	**201**	**101**	**-16**	**-26**	0.72
JumpRBVol	0.15	0.52	11.36	214.2	**4.0×10^{6}**	**353**	17	**-6**	**-51**	0.89	0.11	0.69	39.01	1692.0	**2.6×10^{8}**	**51**	0.06	**-23**	**-47**	0.70
JumpMedRVol	0.14	0.52	11.34	210.0	**3.9×10^{6}**	**321**	17	**-7**	**-50**	0.87	0.11	0.68	38.97	1689.0	**2.5×10^{8}**	**35**	0.04	**-31**	**-46**	0.68
JumpMinRVol	0.14	0.59	13.76	275.3	**6.7×10^{6}**	**441**	**485**	**-7**	**-51**	0.90	0.11	0.70	39.31	1710.0	**2.6×10^{8}**	**32**	0.04	**-45**	**-46**	0.69

(continued)

	Mean	S. D.	Skew	Kurt	JB	Q_{LB}	Q_{LB}^2	ADF	PP	Hurst
Panel G: DJIIDU										
Ret	0.02	1.24	-0.47	9.7	**4056**	**27**	**1879**	**-32**	**-39**	0.59
RVol	0.34	0.91	11.65	192.9	**3.3×10^6**	**399**	**1040**	**-14**	**-26**	0.82
RBVol	0.28	0.89	14.41	275.7	**6.7×10^6**	**565**	**995**	**-12**	**-25**	0.78
MedRVol	0.29	0.96	13.68	253.1	**5.7×10^6**	**483**	**935**	**-11**	**-26**	0.77
MinRVol	0.29	0.99	14.20	272.6	**6.6×10^6**	**530**	**933**	**-12**	**-26**	0.77
ContRBVol	0.28	0.88	13.91	259.5	**5.9×10^6**	**496**	**1013**	**-12**	**-25**	0.78
ContMedRVol	0.29	0.94	14.04	268.2	**6.4×10^6**	**495**	**950**	**-12**	**-26**	0.78
ContMinRVol	0.30	0.97	14.50	285.1	**7.2×10^6**	**518**	**918**	**-12**	**-26**	0.78
JumpRBVol	0.12	0.32	6.91	63.1	**3.4×10^5**	**261**	**420**	**-5**	**-47**	0.88
JumpMedRVol	0.09	0.27	9.22	110.1	**1.1×10^6**	**241**	**598**	**-8**	**-45**	0.75
JumpMinRVol	0.08	0.26	9.65	120.3	**1.3×10^6**	**144**	**219**	**-11**	**-45**	0.75
Panel H: DJINCY										
Ret	0.02	0.92	-0.44	9.1	**3436**	**41**	**2030**	**-34**	**-42**	0.58
RVol	0.23	0.65	19.07	456.9	**1.9×10^7**	**909**	**549**	**-12**	**-27**	0.78
RBVol	0.19	0.63	21.28	530.8	**2.5×10^7**	**1135**	**676**	**-17**	**-26**	0.76
MedRVol	0.18	0.68	21.70	551.4	**2.7×10^7**	**1157**	**554**	**-15**	**-26**	0.75
MinRVol	0.18	0.68	21.45	537.8	**2.6×10^7**	**1163**	**681**	**-15**	**-26**	0.75
ContRBVol	0.19	0.63	21.36	538.1	**2.6×10^7**	**1078**	**549**	**-17**	**-26**	0.76
ContMedRVol	0.18	0.63	21.12	527.2	**2.5×10^7**	**1096**	**555**	**-17**	**-26**	0.76
ContMinRVol	0.18	0.63	21.07	525.6	**2.5×10^7**	**1084**	**553**	**-17**	**-26**	0.76
JumpRBVol	0.09	0.21	8.36	97.9	**8.3×10^7**	**552**	**1332**	**-6**	**-48**	0.83
JumpMedRVol	0.09	0.21	8.20	95.2	**7.8×10^5**	**550**	**1365**	**-7**	**-48**	0.82
JumpMinRVol	0.08	0.21	8.14	94.0	**7.6×10^5**	**531**	**1357**	**-7**	**-48**	0.82
Panel I: DJITEC										
Ret	0.03	1.26	-0.32	8.3	**2540**	13	**2184**	**-44**	**-44**	0.55
RVol	0.17	0.60	24.75	812.6	**5.9×10^7**	**137**	0	**-13**	**-49**	0.75
RBVol	0.12	0.21	25.36	861.6	**6.6×10^7**	**224**	17	**-11**	**-50**	0.81
MedRVol	0.11	0.29	32.30	1263.0	**1.4×10^8**	**107**	3	**-13**	**-47**	0.78
MinRVol	0.12	0.27	25.72	864.7	**6.7×10^7**	**162**	19	**-12**	**-48**	0.78
ContRBVol	0.12	0.21	25.12	850.3	**6.4×10^7**	**235**	17	**-11**	**-50**	0.81
ContMedRVol	0.12	0.29	32.21	1259.0	**1.4×10^8**	**132**	5	**-12**	**-47**	0.78
ContMinRVol	0.12	0.49	40.75	1793.0	**2.9×10^8**	**49**	0	**-45**	**-46**	0.70
JumpRBVol	0.08	0.57	24.46	782.8	**5.5×10^7**	**102**	0	**-17**	**-48**	0.72
JumpMedRVol	0.07	0.53	22.71	679.8	**4.1×10^7**	**122**	0	**-16**	**-48**	0.71
JumpMinRVol	0.06	0.36	14.60	307.0	**8.3×10^6**	**392**	**28**	**-8**	**-53**	0.767
Panel J: DJITLS										
Ret	-0.01	1.07	0.26	17.3	**1.8×10^4**	**46**	**1289**	**-35**	**-45**	0.54
RVol	0.44	0.62	12.19	194.0	**3.3×10^6**	**209**	**623**	**-15**	**-24**	0.83
RBVol	0.38	0.60	14.51	249.0	**5.5×10^6**	**281**	**469**	**-15**	**-22**	0.78
MedRVol	0.37	0.62	14.80	258.0	**5.9×10^6**	**292**	**529**	**-15**	**-21**	0.76
MinRVol	0.37	0.64	14.73	255.2	**5.8×10^6**	**255**	**445**	**-15**	**-22**	0.77
ContRBVol	0.39	0.59	14.58	253.5	**5.7×10^6**	**280**	**455**	**-15**	**-22**	0.78
ContMedRVol	0.38	0.59	14.39	247.1	**5.4×10^6**	**288**	**552**	**-11**	**-21**	0.78
ContMinRVol	0.39	0.62	13.85	225.8	**4.5×10^6**	**250**	**392**	**-11**	**-23**	0.78
JumpRBVol	0.12	0.29	6.66	73.6	**4.6×10^6**	**418**	**167**	**-6**	**-50**	0.96
JumpMedRVol	0.12	0.29	6.51	70.9	**4.3×10^5**	**397**	**193**	**-7**	**-52**	0.96
JumpMinRVol	0.10	0.29	6.66	75.6	**4.9×10^5**	**440**	**195**	**-8**	**-50**	0.97
Panel K: DJIUTI										
Ret	-0.03	1.26	0.22	21.6	**3.1×10^4**	**139**	**1682**	**-11**	**-42**	0.59
RVol	0.47	0.80	13.46	256.0	**5.8×10^6**	**425**	**357**	**-15**	**-33**	0.85
RBVol	0.40	0.73	18.19	427.5	**1.6×10^7**	**665**	**432**	**-15**	**-28**	0.80
MedRVol	0.39	0.76	18.63	448.8	**1.8×10^7**	**629**	**321**	**-15**	**-27**	0.80
MinRVol	0.39	0.78	18.72	446.4	**1.8×10^7**	**689**	**424**	**-15**	**-29**	0.79
ContRBVol	0.40	0.70	17.79	413.6	**1.5×10^7**	**614**	**403**	**-15**	**-28**	0.81
ContMedRVol	0.40	0.75	18.53	452.3	**1.8×10^7**	**544**	**241**	**-15**	**-27**	0.80
ContMinRVol	0.41	0.75	16.77	365.6	**1.2×10^7**	**560**	**567**	**-15**	**-29**	0.80
JumpRBVol	0.13	0.43	10.41	158.9	**2.2×10^6**	**663**	**87**	**-7**	**-53**	0.88
JumpMedRVol	0.13	0.43	10.73	169.6	**2.5×10^6**	**648**	**81**	**-7**	**-55**	0.89
JumpMinRVol	0.11	0.40	11.50	198.7	**3.5×10^6**	**421**	**29**	**-9**	**-50**	0.89

Notes:
(1). Figures in bold denote the rejection of relevant H_o at 5% significance level. (2. S.D., Skew, Kurt, JB, Q_{LB}, Q_{LB}^2, ADF and PP denote the standard deviation, skewness, kurtosis, Jarque Bera normality test, Ljung-Box Q statistics, Ljung-Box Q^2 statistics, Augmented Dickey-Fuller test and Phillips-Perron test, respectively. Q_{LB} and Q_{LB}^2 tests are set at lag 12 which commonly the selection of number of lags is between 10 and 20. (2). The null hypothesis for the following tests is as: JB test, H_o: The series has a normal distribution; Q_{LB}, H_o: The series has no serial correlation; Q_{LB}^2, H_o: The series has no volatility clustering effect; ADF and PP tests, H_o: The series has no unit root (unit root is also known as non-stationary). (3). Hurst exponent, H is to measure the long-range dependency with H<0.5, H=0.5 and H>0.5 indicate the series is anti-persistent, follows a random walk and persistent, respectively. Skewness equals to zero indicates the series is normal distributed and symmetric, negative value indicates skewed left and positive value indicates skewed right. Kurtosis equals to three indicates standard normal distributed, more than three indicates heavy-tailed and less than three indicates light-tailed.

Table 2

Results of the Bivariate Granger Causality Test

Lag	Ret	RVol	RBVol	Med RVol	Min RVol	Cont RBVol	Cont Med RVol	Cont Min RVol	Jump RBVol	Jump Med RVol	JumpMin RVol
Panel A: DJMY25 - DJIBSC											
1	**49.41**	**87.35**	**61.79**	**82.33**	**67.36**	**74.60**	**79.70**	**66.25**	**4.079**	2.611	3.430
	272.6	**34.69**	**37.46**	**49.10**	**50.92**	**43.37**	**48.59**	**46.22**	**8.494**	**5.182**	**5.030**
5	**7.934**	**7.619**	**6.455**	**7.541**	**5.903**	**7.505**	**7.671**	**6.376**	**5.091**	**10.76**	**9.105**
	59.84	**7.364**	**4.213**	**4.470**	**4.697**	**4.179**	**2.990**	**3.461**	**3.175**	1.615	1.465
22	**3.641**	**3.168**	**2.964**	**3.248**	**2.856**	**3.319**	**3.305**	**2.893**	**2.537**	**6.132**	**5.811**
	15.33	**3.001**	**2.623**	**3.235**	**3.142**	**2.449**	**2.203**	**2.315**	**4.443**	**2.465**	1.215
Panel B: DJMY25 - DJICYC											
1	**7.147**	**69.65**	**23.52**	**18.19**	**13.51**	**23.12**	**15.94**	**13.12**	**116.9**	**102.5**	**124.7**
	221.9	3.352	**4.865**	**4.079**	3.626	**6.064**	**3.217**	**5.743**	0.027	**25.41**	0.012
5	1.390	**25.07**	**12.43**	**9.836**	**6.610**	**11.32**	**7.522**	**6.520**	**35.87**	**34.90**	**45.62**
	47.92	**13.75**	**3.537**	**4.038**	**3.384**	**4.037**	**2.999**	**4.250**	**52.25**	**52.27**	**54.87**
22	**1.729**	**6.939**	**4.008**	**3.313**	**2.464**	**3.722**	**2.656**	**2.754**	**11.40**	**11.38**	**12.98**
	12.54	**3.806**	1.184	1.425	1.325	1.241	1.108	1.293	**12.80**	**12.38**	**12.46**
Panel C: DJMY25 - DJIENE											
1	**15.45**	**25.29**	**14.91**	**15.79**	**11.21**	**20.79**	**19.71**	**13.60**	0.005	0.064	1.745
	267.9	2.267	**4.953**	**6.414**	**4.495**	**6.902**	**7.260**	**6.845**	**6.450**	**4.465**	3.318
5	**2.743**	**8.500**	**8.009**	**6.737**	**5.212**	**9.520**	**6.626**	**5.602**	0.479	0.448	0.792
	58.75	**2.667**	1.184	1.760	1.349	1.821	1.592	**2.632**	**3.801**	**3.311**	**4.231**
22	**2.827**	**2.251**	**2.221**	**2.335**	**1.829**	**2.462**	**2.186**	**2.051**	1.451	1.132	0.982
	14.45	**1.821**	1.057	1.080	0.970	1.207	1.010	1.215	**2.562**	**2.306**	**1.949**
Panel D: DJMY25 - DJIFIN											
1	2.213	**34.70**	2.507	**10.25**	0.763	**14.44**	**12.65**	1.207	1.437	1.325	**41.77**
	202.7	2.458	1.157	3.520	1.567	**4.239**	2.795	1.414	**6.137**	**5.919**	**3.996**
5	0.979	**10.24**	**5.961**	**7.165**	**3.544**	**10.22**	**8.001**	**4.446**	**3.144**	**5.672**	**9.251**
	47.99	**5.517**	1.167	**3.728**	1.319	**3.189**	**3.417**	1.649	1.115	**2.546**	**14.78**
22	**4.703**	**3.390**	**3.063**	**3.358**	**2.943**	**3.784**	**3.122**	**2.419**	**2.602**	**3.017**	**4.295**
	13.48	**4.395**	0.876	1.502	0.864	1.195	1.320	0.951	**7.514**	**6.908**	**8.680**
Panel E: DJMY25 - DJIHCR											
1	**10.96**	**70.22**	**15.41**	**15.96**	**9.6203**	**15.23**	**15.70**	**10.81**	**119.8**	**105.7**	**133.5**
	159.0	**5.156**	**2.565**	**2.366**	1.616	3.314	2.288	**4.011**	0.142	**26.58**	0.198
5	1.695	**23.19**	**9.489**	**8.693**	**5.608**	**8.880**	**7.146**	**6.077**	**37.93**	**35.28**	**46.07**
	33.75	**13.88**	**3.527**	**5.455**	**4.349**	**3.687**	**4.063**	**4.919**	**52.78**	**54.62**	**54.74**
22	**1.715**	**6.853**	**3.035**	**2.897**	**2.097**	**2.847**	**2.321**	**2.385**	**12.03**	**11.17**	**13.66**
	8.900	**3.790**	1.094	**1.646**	1.487	1.095	1.286	1.383	**12.94**	**12.70**	**12.49**
Panel F: DJMY25 - DJIIDU											
1	**23.74**	**34.79**	**17.69**	**24.15**	**16.00**	**21.61**	**21.39**	**15.94**	2.963	1.897	0.329
	273.6	**6.189**	**5.579**	**8.521**	**6.678**	**7.138**	**7.462**	**5.690**	**13.61**	3.804	**19.74**
5	**4.607**	**6.813**	**3.824**	**4.192**	**2.862**	**4.757**	**3.631**	**2.721**	0.988	1.325	1.061
	58.30	**3.564**	1.647	1.231	1.106	1.550	0.584	0.476	**8.917**	**6.813**	**7.737**
22	**2.301**	**2.335**	1.310	1.486	1.123	**1.549**	**1.662**	1.390	**2.063**	**2.326**	**3.023**
	15.24	**1.807**	**1.852**	**2.132**	**1.788**	**1.779**	**1.885**	1.462	**3.759**	**2.539**	**4.429**

(continued)

Lag	Ret	RVol	RBVol	Med RVol	Min RVol	Cont RBVol	Cont Med RVol	Cont Min RVol	Jump RBVol	Jump Med RVol	JumpMin RVol
Panel G: DJMY25 - DJINCY											
1	**14.45**	1.491	1.397	1.597	0.987	1.802	2.115	1.404	0.389	0.100	0.011
	175.4	0.005	0.257	0.494	0.393	0.286	0.684	0.370	**4.501**	3.431	**7.726**
5	1.805	**2.689**	2.012	1.542	1.523	2.121	1.273	1.937	0.197	0.507	0.295
	37.89	0.086	0.393	0.673	0.570	0.279	0.518	0.304	**5.095**	**5.402**	**4.150**
22	1.429	1.040	0.959	0.921	0.915	0.928	0.725	0.951	1.065	**1.553**	0.705
	10.56	0.638	0.613	0.704	0.695	0.567	0.642	0.514	**1.956**	**2.340**	**0.002**
Panel H: DJMY25 - DJITEC											
1	**11.10**	**181.0**	**144.2**	**119.8**	**175.1**	**136.9**	**95.15**	**42.14**	**82.70**	**71.59**	0.592
	241.5	**22.31**	**8.810**	**10.47**	**26.83**	**7.063**	**18.15**	0.022	0.288	**22.84**	1.430
5	2.080	**32.04**	**30.44**	**22.47**	**29.80**	**27.47**	**21.14**	**7.830**	**19.08**	**13.50**	0.748
	51.91	**3.168**	**5.948**	**4.628**	**4.441**	**5.064**	**9.872**	**81.38**	**36.50**	**31.08**	1.360
22	**2.211**	**6.265**	**7.176**	**4.653**	**5.613**	**6.043**	**5.719**	**4.491**	**7.666**	**6.379**	1.203
	14.20	1.364	**2.196**	**1.678**	**1.649**	**2.104**	**3.159**	**20.16**	**9.815**	**8.570**	1.279
Panel I: DJMY25 - DJITLS											
1	**29.03**	**150.6**	2.071	1.954	1.819	2.158	2.122	1.738	0.583	3.081	1.030
	93.81	**15.58**	1.618	3.000	2.619	1.109	3.066	2.421	0.000	0.291	0.352
5	**4.395**	**27.51**	**4.765**	**3.467**	**4.910**	**4.377**	**3.063**	**4.591**	1.871	1.303	0.797
	24.16	**16.38**	0.693	1.845	1.307	0.696	1.437	0.658	1.700	1.691	0.210
22	**2.186**	**7.659**	**2.626**	**2.387**	**2.463**	**2.100**	**1.792**	**2.374**	1.118	1.489	1.507
	7.563	**5.211**	**2.274**	**2.610**	**2.106**	**1.936**	**2.005**	1.477	**2.062**	**2.002**	**1.621**
Panel J: DJMY25 - DJIUTI											
1	**16.23**	**33.09**	**11.84**	**11.71**	7.134	**14.76**	**13.02**	**14.28**	**38.63**	32.06	0.590
	160.9	**9.826**	**7.277**	**8.724**	6.486	**8.412**	**9.453**	**10.29**	0.672	7.258	0.409
5	**2.752**	**6.584**	**5.971**	**4.859**	3.971	**6.608**	**4.439**	**0.000**	**6.192**	6.660	1.084
	35.78	1.511	**2.304**	**2.509**	1.901	1.999	2.173	**0.012**	**7.745**	8.462	0.722
22	1.532	**2.387**	**2.124**	**2.090**	**1.775**	**2.095**	**1.748**	**0.001**	**2.929**	3.073	**1.677**
	10.27	**2.803**	1.347	1.322	1.175	1.357	1.431	**0.044**	**5.019**	4.500	**4.627**

Notes:

1. The Granger analysis is performed on series that display stationarity such as the daily return and the realized measures series through estimation of bivariate vector autoregressive (VAR) model at the lag lengths of 1, 5 and 22. The selection of lags at 1, 5 and 22 days is to correspond to the daily, weekly and monthly volatilities proxies used in the model specifications under the empirical analysis of this study.

2. Figures in bold denote the rejection of relevant H_o at 5% significance level.

3. The reported F-statistics are the Wald statistics for the joint null hypothesis of a given lag (1, 5 and 22) on the first column. The null hypothesis of the F-statistics reported on the first row of each lag is H_o: DJMY25 does not Granger Cause Market-*i* (the respective sectoral market), while the second row is H_o: Market-*i* (the respective sectoral market) does not Granger Cause DJMY25.

Table 3

Unconditional Correlation Coefficient of Daily Return and Realised Variation Measures Series

	DJMY25 -DJIBSC	DJMY25 -DJICYC	DJMY25 -DJIENE	DJMY25 -DJIFIN	DJMY25 -DJIHCR	DJMY25 -DJIIDU	DJMY25 -DJINCY	DJMY25 -DJITEC	DJMY25 -DJITLS	DJMY25 -DJIUTI
Ret	0.3996	0.1914	0.2646	0.1389	0.2002	0.3574	0.3114	0.2418	0.3637	0.3922
RVol	0.3679	0.1092	0.1521	0.1454	0.1181	0.2201	0.102	0.1665	0.1652	0.2077
RBVol	0.2977	0.0539	0.069	0.0051	0.0545	0.1616	0.0409	0.2247	0.0472	0.0938
MedRVol	0.3268	0.044	0.0654	0.0192	0.0527	0.1798	0.0359	0.1808	0.0354	0.0797
MinRVol	0.1846	0.0231	0.0346	-0.0016	0.0262	0.1008	0.0202	0.1115	0.0139	0.0348
ContRBVol	0.3199	0.055	0.0788	0.0345	0.056	0.1688	0.0437	0.2298	0.0515	0.1012
ContMedRVol	0.3219	0.0468	0.0756	0.0282	0.0593	0.1805	0.0394	0.1859	0.0455	0.093
ContMinRVol	0.3132	0.0429	0.0779	-0.0001	0.052	0.175	0.038	0.1449	0.0441	0.1035
JumpRBVol	0.1264	0.0097	0.0484	0.0391	0.0278	0.0674	0.0482	0.0128	0.0866	0.0172
JumpMedRVol	0.0322	0.0051	0.0606	0.0508	0.0095	0.0449	0.0649	0.0301	0.0871	0.0208
JumpMinRVol	0.033	0.007	0.0865	0.0077	0.011	0.0409	0.0556	0.041	0.0885	0.0133

Note: The Pearson correlation coefficients range from −1 to +1 where +1 implies a strong positive relationship, -1 implies a strong negative relationship and 0 implies there is no relationship between the two markets.

Table 4

Typology of Model Specifications

Main Specification	Model Name	Model Code
Benchmark model	VHAR (RV)	A01
	VHAR-JR (RBV)	A02
VHAR-JR	VHAR-JR (medRV)	A03
	VHAR-JR (minRV)	A04
	VHAR-RV-J (RBV)	A05
VHAR-RV-J	VHAR-RV-J (medRV)	A06
	VHAR-RV-J (minRV)	A07
	VHAR-CJ (RBV)	A08
VHAR-CJ	VHAR-CJ (medRV)	A09
	VHAR-CJ (minRV)	A10

For the VHAR-JR specification, it consists of three models which are the VHAR-JR (RBV) (model A02), VHAR-JR (medRV) (model A03), and VHAR-JR (minRV) (model A04). The model A02, A03 and A04 are extended from the benchmark model by replacing the standard realized volatility using alternative jump-robust realized measures which are the realized bi-power volatility (RBV), median (medRV) and minimum (minRV) realized volatility, respectively.

For the VHAR-RV-J specification, it consists of three models, which are the VHAR-RV-J (RBV) (model A05), VHAR-RV-J (medRV) (model A06) and VHAR-RV-J (minRV) (model A07). The model A05, A06 and A07 are extended from the benchmark model by inclusion of a daily jump component into the benchmark model which are the daily RBV, medRV and minRV, respectively.

For the VHAR-CJ specification, it consists of three models, which are the VHAR-CJ (RBV) (model A08), VHAR-CJ (medRV) (model A09) and VHAR-CJ (minRV) (model A10). The models A08, A09 and A10 are extended from the benchmark model by decomposing the realized volatility into its continuous and discontinuous jump components. These models enable readers to comprehend additional information about the continuous and discontinuous jump components in capturing the return volatility dynamic of the stock markets.

Next, this study discusses the comparison of the relative performance of the various VHAR-type models for the in-sample estimation based on the goodness-of-fit (R^2 values) shown in Table 5. The results show that the DJMY25 values are greater than 64 percent across all the VHAR-type models. This signifies the overall goodness-of-fit of all models can capture well the volatility dynamic of the DJMY25 market across all pairwise. However, the R^2 for the sectoral indices exhibit lower R^2 values with a range within 30 percent to 50 percent (except DJICYC, DJIHCR, and DJITEC).

Table 6 shows the final ranking of the ten competing models. It is found that VHAR-CJ (medRV) (model A09) is the best performing model, followed by VHAR-JR (medRV) (model A03) and VHAR-JR (RBV) (model A02). On the other hand, the benchmark model (model A01) exhibits the worst performance, followed by VHAR-RV-J (medRV) (model A06) and VHAR-RV-J (minRV) (model A07). It is observed that the jump-robust realized volatilities in the VHAR-JR specification appear to enhance the explanatory power in the volatility dynamic relative to the benchmark model. This result is in line with the findings in Chin et al. (2016) and Andersen et al. (2012) in their univariate analysis on the conventional stock market, namely the DAX and Dow Jones 30, respectively. As for the VHAR-RV-J specification, the inclusion of daily lagged realized jump regressors is seen to have marginally higher R^2 values compared to the benchmark model, indicating the daily realized jump component has an added value to improve the predictive regression. Also, the VHAR-CJ specification seems to exhibit higher explanatory power of regression compared to the benchmark model and the VHAR-RV-J specification, but it is only closely as good as the models of the VHAR-JR specification.

Out-of-Sample Forecast Evaluation

Compared with the in-sample forecasting performance, this study concerns more about the out-of-sample forecasting performance of the models for the out-of-sample predictive power, which delivers more significant and practical values for investors. This is due to the market players being more apprehensive towards the ability of a model to enhance future performance than its ability to analyse the past patterns. In order to make an effective evaluation of the out-of-sample forecasting performance, this study employs six loss functions, which are the mean squared error (MSE), mean absolute error (MAE), quasi-likelihood (QLIKE), LE, heterogeneous mean squared error (HMSE), and heterogeneous mean absolute error (HMAE). The loss functions are used to assess the significant difference in the predictive performance of each volatility model as well as a post-forecasting diagnostic regression. The best model produces the lowest loss value among the competing models.

Table 7 shows all the six loss functions with the corresponding ranking for all volatility models across the pairwise, whereas Table 8 summarizes the overall ranking of all competing models across the pairwise based on the final score summing from the total score of all the six loss functions from Table 7. Based on Table 8, it is observed that the benchmark model (model A01) has the lowest loss values in the DJMY25-DJIBSC, DJMY25-DJIHCR, DJMY25-DJIIDU, and DJMY25-DJINCY pairwise, indicating the benchmark model provides the best prediction on their future volatility. As for the DJMY25-DJIFIN and DJMY25-DJITEC pairwise, the VHAR-JR (RBV) (model A02) exhibits to be the superior model for forecasting its volatility. On the other hand, VHAR-CJV (medRV) (model A09) is found to be the best performing model in the DJMY25-DJITLS and DJMY25-DJIUTI pairwise. Whereas DJMY25-DJICYC and DJMY25-DJIENE pairwise are best captured by VHAR-CJVol (minRV) (model A10). The empirical results exhibit that there is no single superior methodology for the in-sample forecasting performance of each sectoral market. The finding is parallel to the work of Dudek et al. (2023), where different models may perform better depending on the specific market data, choice of the quantile estimations, and different distributional assumptions selected.

As for the out-of-sample forecast evaluation, based on the overall ranking shown in the last column of Table 8, the benchmark model (model A01) is found to be the top-performing model, followed by the VHAR-CJ (RBV) model (model A08) and the VHAR-CJ (medRV) (model A09). It can be witnessed that the models using the standard realized volatility (model A01) and decomposition of the realized volatilities into its continuous and discontinuous components (models A08 and A09) as the volatility proxy are seen to be superior compared to the models using jump-robust realized volatilities and the addition of the daily jump component. This suggests a simpler volatility model such as the VHAR (RV) using the standard realized volatility can capture the volatility forecast superiorly compared to those complex and sophisticated models that involve various regressors of realized measures. This result is in line with Sévi (2014), who claims that the simple yet innovative HAR model provides significantly better forecast evaluation compared to sophisticated models using the decomposition of realized variance into its positive and negative semivariances component. This can be explained by the principle of parsimony (Sharmaa & Vipulb, 2016), where the more parameters and more complex a model is, the larger the penalty factor; hence, a more parsimonious model is more rewarded than a complicated model.

On the other hand, the VHAR-CJ specification, which decomposes the realized volatility into its continuous and discontinuous components, seems to exhibit higher explanatory power of regression compared to the VHAR-JR and VHAR-RV-J specifications. The empirical literature (Bollerslev et al, 2016) suggests that the continuous and discontinuous jump variation sample paths of returns exhibit distinct information about volatility dynamics. This is due to the two components displaying different time series properties where the long memory of volatility is largely coming from the continuous component while the jumps have short-lived effects that are only useful for short-term forecasting. Thus, incorporating the continuous sample path and jump component measures in the volatility forecasting model ensures that the continuous part has a relevant predictive power to improve financial risk measuring, asset pricing, and financial derivatives pricing. As a result, how to separately model the continuous and discontinuous jump of the price process turns to be indispensable in volatility forecasting.

Table 5

Model Ranking Based on R^2 Values for the In-sample Estimation of VHAR-type Models across the Pairwise Models

Model	DJMY25-DJIBSC Market 1 (Score)	DJMY25-DJIBSC Market 2 (Score)	DJMY25-DJIBSC Total Score (Rank)	DJMY25-DJICYC Market 1 (Score)	DJMY25-DJICYC Market 2 (Score)	DJMY25-DJICYC Total Score (Rank)	DJMY25-DJIENE Market 1 (Score)	DJMY25-DJIENE Market 2 (Score)	DJMY25-DJIENE Total Score (Rank)	DJMY25-DJIFIN Market 1 (Score)	DJMY25-DJIFIN Market 2 (Score)	DJMY25-DJIFIN Total Score (Rank)	DJMY25-DJIHCR Market 1 (Score)	DJMY25-DJIHCR Market 2 (Score)	DJMY25-DJIHCR Total Score (Rank)
A01	64.26 (10)	41.87 (9)	19 (10)	64.11 (10)	17.45 (10)	20 (10)	64.09 (10)	31.93 (7)	17 (10)	64.07 (10)	30.63 (10)	20 (10)	64.12 (10)	15.79 (10)	20 (10)
A02	68.35 (4)	42.54 (3)	7 (4)	68.28 (3)	28.98 (6)	9 (5)	68.27 (3)	32.4 (1)	4 (1)	68.24 (3)	45.12 (1)	4 (1)	68.26 (3)	33.51 (5)	8 (5)
A03	70.3 (1)	42.3 (5)	6 (3)	70.18 (1)	33 (4)	5 (2)	70.21 (1)	31.62 (8)	9 (2)	70.2 (1)	39.6 (4)	5 (2)	70.16 (1)	32.99 (6)	7 (4)
A04	66.36 (6)	40.52 (10)	16 (9)	66.21 (6)	31.38 (5)	11 (6)	66.21 (6)	31.22 (9)	15 (8)	66.21 (6)	44.28 (2)	8 (4)	66.19 (6)	35.98 (4)	10 (6)
A05	64.43 (7)	42.34 (4)	11 (5)	64.39 (7)	23.29 (8)	15 (7)	64.3 (7)	32.07 (2)	9 (2)	64.24 (8)	31.67 (8)	16 (8)	64.37 (7)	24.87 (7)	14 (7)
A06	64.37 (9)	42.13 (6)	15 (7)	64.32 (9)	22.93 (9)	18 (9)	64.21 (9)	31.95 (6)	15 (8)	64.16 (9)	31.59 (9)	18 (9)	64.31 (9)	23.69 (9)	18 (9)
A07	64.41 (8)	42.12 (7)	15 (7)	64.33 (8)	23.47 (7)	15 (7)	64.23 (8)	32.05 (3)	11 (6)	64.26 (7)	33.18 (7)	14 (7)	64.34 (8)	24.25 (8)	16 (8)
A08	68.44 (3)	42.89 (2)	5 (2)	68.17 (4)	33.03 (3)	7 (3)	68.2 (4)	31.99 (5)	9 (2)	68.13 (4)	37.42 (6)	10 (6)	68.18 (4)	39.3 (2)	6 (2)
A09	68.74 (2)	44.03 (1)	3 (1)	68.62 (2)	34.64 (1)	3 (1)	68.68 (2)	31.13 (10)	12 (7)	68.65 (2)	39.08 (5)	7 (3)	68.62 (2)	37.54 (3)	5 (1)
A10	67.95 (5)	41.9 (8)	13 (6)	67.89 (5)	34.3 (2)	7 (3)	67.85 (5)	32.04 (4)	9 (2)	67.79 (5)	43.4 (3)	8 (4)	67.9 (5)	39.32 (1)	6 (2)

Model	DJMY25-DJIIDU Market 1 (Score)	DJMY25-DJIIDU Market 2 (Score)	DJMY25-DJIIDU Total Score (Rank)	DJMY25-DJINCY Market 1 (Score)	DJMY25-DJINCY Market 2 (Score)	DJMY25-DJINCY Total Score (Rank)	DJMY25-DJITEC Market 1 (Score)	DJMY25-DJITEC Market 2 (Score)	DJMY25-DJITEC Total Score (Rank)	DJMY25-DJITLS Market 1 (Score)	DJMY25-DJITLS Market 2 (Score)	DJMY25-DJITLS Total Score (Rank)	DJMY25-DJIUTI Market 1 (Score)	DJMY25-DJIUTI Market 2 (Score)	DJMY25-DJIUTI Total Score (Rank)
A01	64.09 (10)	40.52 (6)	16 (9)	64.08 (10)	36.83 (10)	20 (10)	64.42 (10)	8.86 (8)	18 (10)	64.1 (10)	43.15 (10)	20 (10)	64.13 (10)	33.6 (10)	20 (10)
A02	68.23 (3)	40.71 (3)	6 (1)	68.21 (3)	37.67 (1)	4 (1)	68.32 (3)	10.36 (5)	8 (4)	68.24 (3)	47.13 (4)	7 (3)	68.29 (4)	35.92 (5)	9 (4)
A03	70.16 (1)	39.31 (8)	9 (4)	70.14 (1)	37.44 (3)	4 (1)	70.15 (1)	8.01 (9)	10 (5)	70.2 (1)	47.77 (2)	3 (1)	70.24 (1)	36.57 (3)	4 (2)
A04	66.2 (6)	38.02 (10)	16 (9)	66.16 (6)	37.58 (2)	8 (4)	66.2 (6)	6.86 (10)	16 (8)	66.22 (6)	47.22 (3)	9 (4)	66.26 (6)	34.14 (7)	13 (6)
A05	64.38 (7)	41.62 (1)	8 (3)	64.25 (7)	36.96 (7)	14 (7)	64.82 (7)	9.85 (6)	13 (7)	64.27 (7)	44.59 (7)	14 (7)	64.43 (7)	34.38 (6)	13 (6)
A06	64.2 (9)	40.54 (5)	14 (7)	64.17 (9)	36.93 (8)	17 (8)	64.68 (9)	9.48 (7)	16 (8)	64.19 (9)	44.17 (8)	17 (8)	64.32 (8)	34.11 (8)	16 (8)
A07	64.37 (8)	40.61 (4)	12 (6)	64.21 (8)	36.91 (9)	17 (8)	64.69 (8)	10.48 (4)	12 (6)	64.22 (8)	43.89 (9)	17 (8)	64.27 (9)	34.08 (9)	18 (9)
A08	68.22 (4)	40.73 (2)	6 (1)	68.17 (4)	36.99 (6)	10 (6)	68.19 (4)	13.81 (1)	5 (1)	68.1 (4)	47 (5)	9 (4)	68.36 (3)	36.93 (2)	5 (3)
A09	68.59 (2)	40.13 (7)	9 (4)	68.65 (2)	37.27 (5)	7 (3)	68.6 (2)	12.52 (3)	5 (1)	68.65 (2)	48.77 (1)	3 (1)	68.77 (2)	36.94 (1)	3 (1)
A10	67.94 (5)	39.3 (9)	14 (7)	67.86 (5)	37.31 (4)	9 (5)	67.97 (5)	12.99 (2)	7 (3)	67.84 (5)	45.55 (6)	11 (6)	67.98 (5)	36.37 (4)	9 (4)

Note: Each volatility model is given a score number with 1 up to 10 based on the R^2 values in ascending order for each market across the pairwise. The score number 1 denotes the volatility model with the highest R^2 value while the score number 10 denotes the volatility model with the lowest R^2 value.

Table 6

In-sample Final Ranking Across VHAR-type Models

Model	DJMY25-DJIBSC Total Score (Rank)	DJMY25-DJICYC Total Score (Rank)	DJMY25-DJIENE Total Score (Rank)	DJMY25-DJIFIN Total Score (Rank)	DJMY25-DJIHCR Total Score (Rank)	DJMY25-DJIIDU Total Score (Rank)	DJMY25-DJINCY Total Score (Rank)	DJMY25-DJITEC Total Score (Rank)	DJMY25-DJITLS Total Score (Rank)	DJMY25-DJIUTI Total Score (Rank)	Final Score (Final Ranking)
A01	19 (10)	20 (10)	17 (10)	20 (10)	20 (10)	16 (9)	20 (10)	18 (10)	20 (10)	20 (10)	190 (10)
A02	7 (4)	9 (5)	4 (1)	4 (1)	8 (5)	6 (1)	4 (1)	8 (4)	7 (3)	9 (4)	66 (3)
A03	6 (3)	5 (2)	9 (2)	5 (2)	7 (4)	9 (4)	4 (1)	10 (5)	3 (1)	4 (2)	62 (2)
A04	16 (9)	11 (6)	15 (8)	8 (4)	10 (6)	16 (9)	8 (4)	16 (8)	9 (4)	13 (6)	122 (6)
A05	11 (5)	15 (7)	9 (2)	16 (8)	14 (7)	8 (3)	14 (7)	13 (7)	14 (7)	13 (6)	127 (7)
A06	15 (7)	18 (9)	15 (8)	18 (9)	18 (9)	14 (7)	17 (8)	16 (8)	17 (8)	16 (8)	164 (9)
A07	15 (7)	15 (7)	11 (6)	14 (7)	16 (8)	12 (6)	17 (8)	12 (6)	17 (8)	18 (9)	147 (8)
A08	5 (2)	7 (3)	9 (2)	10 (6)	6 (2)	6 (1)	10 (6)	5 (1)	9 (4)	5 (3)	72 (4)
A09	3 (1)	3 (1)	12 (7)	7 (3)	5 (1)	9 (4)	7 (3)	5 (1)	3 (1)	3 (1)	57 (1)
A10	13 (6)	7 (3)	9 (2)	8 (4)	6 (2)	14 (7)	9 (5)	7 (3)	11 (6)	9 (4)	93 (5)

Note: The final ranking of the competing models is determined by summing up the Total Score columns (from Table 3) across all pairwise models. The lowest the sum of the final score, the highest the final ranking of the models.

Table 7

Out-of-Sample Forecast Evaluation Results where RV_t is the Volatility Proxy

Loss Function	MSE		MAE		QLIKE		LE		HSME		HMAE	
Model	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)
Panel A: DJMY25-DJIBSC												
A01	0.0026 (1)	0.0051 (1)	0.0338 (1)	0.0296 (1)	-0.9301 (1)	-2.2058 (1)	0.0708 (1)	0.5194 (1)	0.0846 (6)	2.1567 (7)	0.2284 (5)	0.9312 (7)
A02	0.0041 (8)	0.011 (7)	0.0383 (8)	0.0337 (2)	-0.9115 (8)	-2.0255 (6)	0.0926 (8)	0.5415 (2)	0.0748 (2)	0.9052 (1)	0.2227 (3)	0.6092 (3)
A03	0.0045 (9)	0.0114 (8)	0.0408 (9)	0.0355 (6)	-0.9004 (9)	-1.8631 (9)	0.108 (9)	0.6835 (6)	0.0795 (5)	0.9184 (3)	0.2317 (6)	0.6033 (2)
A04	0.0046 (10)	0.0116 (9)	0.0412 (10)	0.0351 (5)	-0.8987 (10)	-1.8829 (8)	0.1116 (10)	0.6511 (5)	0.0846 (7)	0.9113 (2)	0.2371 (10)	0.5927 (1)
A05	0.0028 (3)	0.0096 (5)	0.0343 (3)	0.0399 (9)	-0.9278 (4)	-2.0551 (5)	0.0748 (4)	0.8059 (9)	0.0894 (10)	4.4442 (9)	0.2337 (9)	1.2279 (9)
A06	0.0028 (4)	0.0081 (2)	0.0344 (4)	0.0402 (10)	-0.9278 (3)	-2.0811 (3)	0.0747 (3)	0.8222 (10)	0.089 (9)	4.9453 (10)	0.2334 (8)	1.3242 (10)
A07	0.0028 (2)	0.0085 (3)	0.0343 (2)	0.0379 (7)	-0.9284 (2)	-2.0996 (2)	0.0738 (2)	0.7375 (7)	0.0881 (8)	3.6743 (8)	0.2328 (7)	1.1696 (8)
A08	0.0037 (5)	0.0118 (10)	0.0366 (5)	0.0382 (8)	-0.9182 (5)	-1.7658 (10)	0.085 (5)	0.7753 (8)	0.0784 (4)	1.6203 (6)	0.2235 (4)	0.8512 (6)
A09	0.0039 (7)	0.0095 (4)	0.0374 (7)	0.0343 (3)	-0.9151 (7)	-1.9911 (7)	0.0873 (7)	0.6247 (4)	0.071 (1)	1.1428 (5)	0.2175 (1)	0.7343 (5)
A10	0.0038 (6)	0.01 (6)	0.037 (6)	0.0345 (4)	-0.9168 (6)	-2.0764 (4)	0.0865 (6)	0.5453 (3)	0.0769 (3)	1.0337 (4)	0.2224 (2)	0.7077 (4)
Panel B: DJMY25-DJICYC												
A01	0.0026 (1)	0.0012 (7)	0.0338 (1)	0.011 (10)	-0.93 (1)	-3.385 (3)	0.0711 (1)	4.2509 (7)	0.0855 (7)	7.6714 (10)	0.2293 (5)	2.9058 (10)
A02	0.0041 (8)	0.0012 (3)	0.0382 (8)	0.0077 (6)	-0.9121 (8)	-3.4891 (2)	0.0918 (8)	1.9599 (3)	0.0749 (2)	6.0806 (7)	0.2225 (4)	1.6743 (6)
A03	0.0045 (9)	0.0012 (6)	0.0406 (9)	0.0073 (3)	-0.901 (9)	-3.6032 (1)	0.1072 (9)	1.8925 (2)	0.0794 (5)	4.9622 (5)	0.231 (6)	1.4333 (3)
A04	0.0046 (10)	0.0012 (5)	0.041 (10)	0.007 (1)	-0.8994 (10)	-3.2039 (4)	0.1107 (10)	1.2681 (1)	0.0848 (6)	4.2119 (4)	0.2367 (10)	1.4122 (1)
A05	0.0028 (3)	0.0014 (8)	0.0344 (3)	0.0098 (8)	-0.9277 (3)	-1.6903 (5)	0.0754 (3)	3.9431 (5)	0.0911 (10)	6.0956 (8)	0.2355 (9)	2.3767 (8)
A06	0.0028 (4)	0.0014 (10)	0.0345 (4)	0.0103 (9)	-0.9277 (4)	-0.4565 (9)	0.0754 (4)	5.1273 (8)	0.0907 (9)	6.586 (9)	0.2353 (8)	2.6776 (9)
A07	0.0028 (2)	0.0014 (9)	0.0343 (2)	0.0097 (7)	-0.9283 (2)	-1.6378 (7)	0.0741 (2)	3.9865 (6)	0.0891 (8)	5.5937 (6)	0.2338 (7)	2.2777 (7)
A08	0.0036 (5)	0.0012 (4)	0.0363 (5)	0.0076 (5)	-0.9194 (5)	-1.4898 (8)	0.0829 (5)	5.8766 (10)	0.0768 (4)	3.7634 (1)	0.2216 (2)	1.5064 (4)
A09	0.0038 (7)	0.0011 (2)	0.0371 (7)	0.0071 (2)	-0.9163 (7)	-0.3812 (10)	0.0856 (6)	5.3668 (9)	0.0703 (1)	3.7757 (2)	0.2157 (1)	1.4135 (2)
A10	0.0038 (6)	0.0011 (1)	0.0369 (6)	0.0073 (4)	-0.9173 (6)	-1.6646 (6)	0.0857 (7)	3.5326 (4)	0.0766 (3)	4.1826 (3)	0.2221 (3)	1.5078 (5)

(continued)

Loss Function	MSE		MAE		QLIKE		LE		HSME		HMAE	
Model	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)
Panel C: DJMY25-DJIENE												
A01	0.0026 (1)	0.0019 (8)	0.0337 (1)	0.0109 (7)	-0.9303 (1)	-4.3282 (2)	0.0703 (1)	2.6238 (6)	0.0834 (6)	18.916 (8)	0.2271 (5)	4.4676 (7)
A02	0.0041 (8)	0.0018 (2)	0.0382 (8)	0.0094 (4)	-0.9117 (8)	2.2378 (7)	0.0922 (8)	2.4073 (3)	0.0746 (2)	12.6721 (2)	0.2223 (4)	3.6661 (5)
A03	0.0045 (9)	0.0018 (1)	0.0407 (9)	0.0093 (3)	-0.9005 (9)	27.2917 (10)	0.1079 (9)	2.5434 (4)	0.0793 (5)	12.7872 (3)	0.2313 (8)	3.6635 (4)
A04	0.0046 (10)	0.0019 (6)	0.0412 (10)	0.0091 (1)	-0.899 (10)	4.3811 (8)	0.1112 (10)	2.3788 (2)	0.0845 (7)	13.1196 (4)	0.2368 (10)	3.5516 (2)
A05	0.0028 (3)	0.0018 (3)	0.0343 (3)	0.0118 (10)	-0.9281 (4)	-4.2604 (4)	0.0741 (4)	2.9015 (10)	0.0876 (10)	21.5806 (10)	0.2318 (9)	4.9714 (10)
A06	0.0028 (4)	0.0019 (7)	0.0343 (4)	0.0115 (9)	-0.9281 (3)	-4.289 (3)	0.0739 (3)	2.7914 (9)	0.0869 (9)	20.4057 (9)	0.2311 (7)	4.7649 (9)
A07	0.0028 (2)	0.0019 (9)	0.0342 (2)	0.011 (8)	-0.9286 (2)	-4.3321 (1)	0.0729 (2)	2.6382 (7)	0.086 (8)	15.4561 (7)	0.2306 (6)	4.5967 (8)
A08	0.0037 (5)	0.0018 (4)	0.0365 (5)	0.0095 (5)	-0.9186 (5)	-0.2839 (6)	0.0837 (5)	2.5555 (5)	0.0753 (3)	13.4756 (5)	0.2204 (2)	3.5526 (3)
A09	0.0039 (7)	0.002 (10)	0.0375 (7)	0.0101 (6)	-0.9148 (7)	9.3517 (9)	0.0875 (7)	2.7507 (8)	0.0697 (1)	14.1725 (6)	0.216 (1)	3.8959 (6)
A10	0.0038 (6)	0.0019 (5)	0.0371 (6)	0.0092 (2)	-0.9165 (6)	-4.1813 (5)	0.0865 (6)	2.204 (1)	0.0756 (4)	11.3855 (1)	0.2215 (3)	3.5059 (1)
Panel D: DJMY25-DJIFIN												
A01	0.0026 (1)	0.0013 (1)	0.0338 (1)	0.0133 (10)	-0.9301 (1)	-3.7834 (4)	0.0708 (1)	1.5012 (10)	0.0845 (6)	12.1327 (9)	0.2281 (5)	2.4955 (10)
A02	0.0041 (8)	0.0016 (4)	0.0382 (8)	0.0105 (1)	-0.9119 (8)	-3.806 (1)	0.0922 (8)	1.1263 (1)	0.075 (2)	6.443 (1)	0.2225 (4)	1.8657 (1)
A03	0.0045 (9)	0.0017 (8)	0.0406 (9)	0.0115 (4)	-0.9008 (9)	-3.6661 (10)	0.1075 (9)	1.275 (6)	0.0795 (5)	7.8935 (4)	0.2309 (6)	2.0311 (5)
A04	0.0046 (10)	0.0016 (7)	0.0411 (10)	0.0108 (2)	-0.899 (10)	-3.7207 (8)	0.1113 (10)	1.1983 (2)	0.0848 (7)	7.119 (2)	0.2368 (10)	1.9518 (2)
A05	0.0028 (3)	0.0016 (3)	0.0344 (3)	0.0127 (9)	-0.9278 (3)	-3.7807 (5)	0.0748 (4)	1.4479 (9)	0.0893 (10)	12.363 (10)	0.2337 (9)	2.3929 (9)
A06	0.0028 (4)	0.0015 (2)	0.0344 (4)	0.0125 (8)	-0.9278 (4)	-3.7902 (3)	0.0747 (3)	1.4254 (8)	0.0888 (9)	12.0607 (8)	0.2333 (7)	2.3635 (8)
A07	0.0028 (2)	0.0016 (5)	0.0343 (2)	0.0109 (3)	-0.9282 (2)	-3.8018 (2)	0.0741 (2)	1.2162 (4)	0.0888 (8)	9.2988 (5)	0.2336 (8)	1.9867 (4)
A08	0.0036 (5)	0.0017 (9)	0.0363 (5)	0.0115 (5)	-0.9194 (5)	-3.7541 (7)	0.0827 (5)	1.2161 (3)	0.0764 (3)	9.6201 (6)	0.2208 (2)	1.9862 (3)
A09	0.0038 (7)	0.0017 (10)	0.0373 (7)	0.0125 (7)	-0.9159 (7)	-3.6819 (9)	0.086 (6)	1.3246 (7)	0.07 (1)	10.1858 (7)	0.2161 (1)	2.134 (7)
A10	0.0038 (6)	0.0016 (6)	0.037 (6)	0.0115 (6)	-0.917 (6)	-3.7617 (6)	0.0861 (7)	1.2446 (5)	0.0765 (4)	7.4381 (3)	0.2222 (3)	2.0401 (6)
Panel E: DJMY25-DJIHCR												
A01	0.0026 (1)	0.0024 (1)	0.0338 (1)	0.0184 (7)	-0.9301 (1)	-2.862 (2)	0.0709 (1)	1.0932 (4)	0.085 (7)	4.2439 (8)	0.2287 (5)	1.472 (8)
A02	0.0041 (8)	0.0046 (7)	0.0382 (8)	0.0147 (1)	-0.9119 (8)	-2.97 (1)	0.0921 (8)	0.5738 (1)	0.0749 (2)	0.9139 (3)	0.2225 (3)	0.6599 (3)
A03	0.0045 (9)	0.0048 (10)	0.0406 (9)	0.0155 (3)	-0.9008 (9)	-2.7876 (4)	0.1075 (9)	0.7921 (3)	0.0795 (5)	0.8925 (2)	0.2311 (6)	0.6548 (2)
A04	0.0046 (10)	0.0048 (9)	0.0411 (10)	0.0149 (2)	-0.8992 (10)	-2.8019 (3)	0.111 (10)	0.6223 (2)	0.0848 (6)	0.6393 (1)	0.2367 (10)	0.5804 (1)
A05	0.0028 (3)	0.0043 (4)	0.0344 (4)	0.021 (9)	-0.9278 (3)	-0.5937 (10)	0.0752 (4)	2.8369 (9)	0.0906 (10)	4.2608 (9)	0.2351 (9)	1.4885 (9)
A06	0.0028 (4)	0.0041 (3)	0.0344 (3)	0.0217 (10)	-0.9278 (4)	-1.1339 (9)	0.075 (3)	3.0759 (10)	0.0898 (9)	4.9935 (10)	0.2342 (8)	1.6394 (10)
A07	0.0028 (2)	0.004 (2)	0.0343 (2)	0.0198 (8)	-0.9283 (2)	-1.8771 (6)	0.074 (2)	2.7478 (8)	0.0887 (8)	3.2515 (7)	0.2337 (7)	1.465 (7)
A08	0.0036 (5)	0.0047 (8)	0.0364 (5)	0.016 (5)	-0.9193 (5)	-2.1411 (5)	0.0831 (5)	1.1952 (5)	0.0769 (3)	1.2233 (5)	0.2218 (2)	0.7274 (5)
A09	0.0038 (7)	0.0045 (6)	0.0372 (7)	0.0165 (6)	-0.9161 (7)	-1.4556 (8)	0.0859 (6)	1.3509 (6)	0.0702 (1)	1.2725 (6)	0.2161 (1)	0.7513 (6)
A10	0.0038 (6)	0.0044 (5)	0.0369 (6)	0.0156 (4)	-0.9171 (6)	-1.4689 (7)	0.0861 (7)	1.7224 (7)	0.077 (4)	1.1979 (4)	0.2225 (4)	0.7094 (4)

(continued)

Loss Function	MSE		MAE		QLIKE		LE		HSME		HMAE	
Model	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)
Panel F: DJMY25-DJIIDU												
A01	0.0026 (1)	0.001 (1)	0.0337 (1)	0.0104 (1)	-0.9303 (1)	-3.1523 (2)	0.0704 (1)	0.3814 (3)	0.084 (6)	1.2668 (7)	0.2276 (5)	0.7494 (7)
A02	0.0041 (8)	0.0022 (8)	0.0383 (8)	0.0122 (5)	-0.9117 (8)	-3.1093 (4)	0.0923 (8)	0.3548 (2)	0.0749 (2)	0.7666 (3)	0.2227 (3)	0.5842 (3)
A03	0.0045 (9)	0.0022 (9)	0.0407 (9)	0.013 (8)	-0.9006 (9)	-3.0512 (9)	0.1079 (9)	0.4169 (6)	0.0795 (5)	0.784 (5)	0.2315 (7)	0.5886 (4)
A04	0.0046 (10)	0.0024 (10)	0.0412 (10)	0.0128 (7)	-0.8989 (10)	-3.0613 (8)	0.1114 (10)	0.3953 (4)	0.0849 (7)	0.753 (1)	0.2371 (10)	0.5776 (2)
A05	0.0028 (3)	0.0015 (6)	0.0343 (4)	0.014 (10)	-0.9281 (3)	-3.0698 (6)	0.0743 (4)	0.5522 (10)	0.0886 (10)	2.2249 (10)	0.2329 (9)	0.9393 (10)
A06	0.0028 (4)	0.0011 (3)	0.0343 (3)	0.0127 (6)	-0.9281 (4)	-3.0876 (5)	0.0741 (3)	0.5178 (9)	0.0878 (9)	2.0438 (9)	0.232 (8)	0.9124 (9)
A07	0.0028 (2)	0.0011 (2)	0.0342 (2)	0.0116 (4)	-0.9287 (2)	-3.1364 (3)	0.0728 (2)	0.4171 (7)	0.0861 (8)	1.47 (8)	0.2308 (6)	0.7855 (8)
A08	0.0037 (5)	0.0021 (7)	0.0366 (5)	0.0137 (9)	-0.9186 (5)	-2.9972 (10)	0.0842 (5)	0.5061 (8)	0.0773 (4)	1.1949 (6)	0.2227 (2)	0.7085 (6)
A09	0.0038 (7)	0.0014 (5)	0.0373 (7)	0.0111 (3)	-0.9153 (7)	-3.0648 (7)	0.0869 (6)	0.4038 (5)	0.0705 (1)	0.7688 (4)	0.2166 (1)	0.6051 (5)
A10	0.0038 (6)	0.0013 (4)	0.0372 (6)	0.0106 (2)	-0.9162 (6)	-3.1622 (1)	0.0873 (7)	0.2949 (1)	0.077 (3)	0.7617 (2)	0.2231 (4)	0.537 (1)
Panel G: DJMY25-DJINCY												
A01	0.0026 (1)	0.002 (1)	0.0338 (1)	0.0115 (1)	-0.9302 (1)	-3.0258 (1)	0.0706 (1)	0.3292 (2)	0.0846 (6)	1.0143 (7)	0.2282 (5)	0.6857 (7)
A02	0.0041 (8)	0.0044 (8)	0.0382 (8)	0.0137 (7)	-0.912 (8)	-2.8721 (8)	0.0919 (8)	0.3624 (4)	0.0749 (2)	0.4833 (6)	0.2224 (4)	0.4718 (6)
A03	0.0045 (9)	0.0045 (9)	0.0406 (9)	0.0142 (9)	-0.9009 (9)	-2.789 (9)	0.1074 (9)	0.4265 (9)	0.0795 (5)	0.431 (2)	0.2311 (6)	0.4521 (4)
A04	0.0046 (10)	0.0045 (10)	0.0411 (10)	0.0143 (10)	-0.8993 (10)	-2.7724 (10)	0.1108 (10)	0.4351 (10)	0.0848 (7)	0.4495 (3)	0.2367 (10)	0.4602 (5)
A05	0.0028 (3)	0.0028 (4)	0.0344 (3)	0.0137 (8)	-0.9279 (3)	-2.9871 (4)	0.0749 (3)	0.4102 (8)	0.0897 (10)	1.3244 (9)	0.2342 (9)	0.7803 (10)
A06	0.0028 (4)	0.0027 (2)	0.0345 (4)	0.0136 (6)	-0.9278 (4)	-2.9913 (3)	0.0749 (4)	0.4046 (7)	0.0894 (9)	1.3401 (10)	0.2341 (8)	0.7777 (9)
A07	0.0028 (2)	0.0027 (3)	0.0344 (2)	0.0133 (4)	-0.9283 (2)	-2.9957 (2)	0.074 (2)	0.3932 (6)	0.0884 (8)	1.2692 (8)	0.2335 (7)	0.7589 (8)
A08	0.0036 (5)	0.004 (5)	0.0365 (5)	0.0131 (2)	-0.9191 (5)	-2.9431 (5)	0.0834 (5)	0.3238 (1)	0.0773 (4)	0.4735 (4)	0.2222 (3)	0.4372 (2)
A09	0.0038 (7)	0.0041 (6)	0.0372 (7)	0.0132 (3)	-0.9159 (7)	-2.9099 (6)	0.0862 (7)	0.3393 (3)	0.071 (1)	0.3309 (1)	0.2168 (1)	0.415 (1)
A10	0.0038 (6)	0.0041 (7)	0.0368 (6)	0.0135 (5)	-0.9174 (6)	-2.9036 (7)	0.0856 (6)	0.3635 (5)	0.0765 (3)	0.4767 (5)	0.2214 (2)	0.4436 (3)
Panel H: DJMY25-DJITEC												
A01	0.0027 (1)	0.0103 (4)	0.034 (1)	0.0125 (6)	-0.9296 (1)	2.0928 (10)	0.072 (1)	1.8394 (6)	0.0868 (7)	2.377 (6)	0.2309 (5)	0.9128 (6)
A02	0.0041 (8)	0.0102 (2)	0.0381 (8)	0.0108 (1)	-0.912 (8)	-2.3493 (1)	0.0921 (8)	0.5092 (1)	0.0753 (2)	0.4976 (1)	0.2228 (4)	0.5147 (1)
A03	0.0045 (9)	0.0103 (6)	0.0405 (9)	0.0113 (4)	-0.9011 (9)	-0.937 (8)	0.1072 (9)	0.7279 (4)	0.0798 (5)	0.6576 (3)	0.2313 (6)	0.5798 (3)
A04	0.0046 (10)	0.0103 (3)	0.0411 (10)	0.0109 (2)	-0.8994 (10)	-1.9471 (2)	0.1109 (10)	0.5501 (2)	0.0851 (6)	0.5647 (2)	0.2371 (10)	0.5406 (2)
A05	0.0029 (3)	0.0103 (9)	0.0347 (3)	0.0128 (8)	-0.9274 (3)	-1.193 (5)	0.0758 (3)	4.7719 (9)	0.0908 (10)	2.926 (7)	0.2359 (8)	0.9492 (8)
A06	0.0029 (4)	0.0103 (8)	0.0348 (4)	0.013 (9)	-0.9272 (4)	-1.856 (3)	0.076 (4)	2.9042 (7)	0.0905 (9)	3.9456 (9)	0.2362 (9)	1.0175 (9)
A07	0.0028 (2)	0.0102 (1)	0.0346 (2)	0.0127 (7)	-0.9279 (2)	-1.1436 (6)	0.075 (2)	4.4105 (8)	0.0902 (8)	2.9998 (8)	0.2355 (7)	0.916 (7)
A08	0.0036 (5)	0.0103 (5)	0.0362 (5)	0.0112 (3)	-0.9197 (5)	-1.0725 (7)	0.0825 (5)	0.6125 (3)	0.0767 (4)	0.7258 (4)	0.2212 (3)	0.5874 (4)
A09	0.0038 (7)	0.0103 (7)	0.037 (7)	0.0118 (5)	-0.9167 (7)	1.5591 (9)	0.0849 (6)	1.1781 (5)	0.0699 (1)	1.423 (5)	0.2152 (1)	0.7233 (5)
A10	0.0037 (6)	0.0104 (10)	0.0368 (6)	0.0135 (10)	-0.9176 (6)	-1.2898 (4)	0.0851 (7)	5.1656 (10)	0.0759 (3)	4.5447 (10)	0.2207 (2)	1.0361 (10)

(continued)

Loss Function	MSE		MAE		QLIKE		LE		HSME		HMAE	
Model	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)	Market 1 (Score)	Market 2 (Score)
Panel I: DJMY25-DJITLS												
A01	0.0026 (1)	0.0018 (1)	0.0338 (1)	0.0174 (6)	-0.9302 (1)	-1.368 (2)	0.0705 (1)	0.046 (5)	0.0843 (6)	0.0627 (7)	0.2278 (5)	0.1927 (7)
A02	0.0041 (8)	0.0032 (6)	0.0382 (7)	0.0163 (3)	-0.9119 (8)	-1.3672 (3)	0.0921 (8)	0.0393 (2)	0.075 (4)	0.0401 (2)	0.2222 (4)	0.1538 (4)
A03	0.0045 (9)	0.0038 (9)	0.0406 (9)	0.0168 (4)	-0.9008 (9)	-1.3594 (6)	0.1076 (9)	0.0469 (6)	0.0796 (5)	0.0428 (4)	0.2312 (6)	0.1531 (3)
A04	0.0046 (10)	0.004 (10)	0.0411 (10)	0.0196 (7)	-0.8993 (10)	-1.3538 (7)	0.1109 (10)	0.0569 (7)	0.0849 (7)	0.0546 (6)	0.2367 (10)	0.1828 (6)
A05	0.0028 (3)	0.0036 (8)	0.0343 (3)	0.0286 (10)	-0.9281 (4)	-1.3447 (10)	0.0744 (4)	0.0927 (10)	0.089 (10)	0.1271 (10)	0.2333 (9)	0.3044 (10)
A06	0.0028 (4)	0.0034 (7)	0.0343 (4)	0.028 (9)	-0.9281 (3)	-1.3467 (9)	0.0741 (3)	0.0892 (9)	0.0878 (9)	0.1221 (9)	0.2323 (8)	0.2981 (9)
A07	0.0028 (2)	0.0032 (4)	0.0342 (2)	0.0248 (8)	-0.9287 (2)	-1.3526 (8)	0.073 (2)	0.0762 (8)	0.0865 (8)	0.1016 (8)	0.2314 (7)	0.2657 (8)
A08	0.0038 (5)	0.0032 (5)	0.0369 (5)	0.0161 (2)	-0.9172 (5)	-1.3671 (4)	0.085 (5)	0.04 (3)	0.0739 (2)	0.0416 (3)	0.2189 (2)	0.1524 (2)
A09	0.0041 (7)	0.0031 (3)	0.0383 (8)	0.0155 (1)	-0.9122 (7)	-1.3695 (1)	0.0908 (7)	0.036 (1)	0.0696 (1)	0.0373 (1)	0.2169 (1)	0.1451 (1)
A10	0.004 (6)	0.0031 (2)	0.0376 (6)	0.017 (5)	-0.9143 (6)	-1.3659 (5)	0.089 (6)	0.0448 (4)	0.0741 (3)	0.0529 (5)	0.2198 (3)	0.165 (5)
Panel J: DJMY25-DJIUTI												
A01	0.0026 (1)	0.0009 (1)	0.0337 (1)	0.0236 (8)	-0.9303 (1)	-1.8598 (6)	0.0698 (1)	0.2177 (9)	0.0819 (6)	0.8343 (10)	0.2254 (5)	0.5309 (9)
A02	0.0041 (8)	0.0012 (6)	0.0383 (8)	0.0209 (4)	-0.9115 (8)	-1.8644 (3)	0.0925 (8)	0.186 (4)	0.0741 (2)	0.6019 (4)	0.2214 (2)	0.4407 (4)
A03	0.0045 (9)	0.0013 (9)	0.0407 (9)	0.0195 (1)	-0.9002 (9)	-1.8613 (4)	0.1082 (9)	0.1739 (2)	0.0791 (5)	0.4962 (1)	0.2308 (8)	0.3931 (1)
A04	0.0046 (10)	0.0014 (10)	0.0412 (10)	0.0218 (6)	-0.8987 (10)	-1.851 (10)	0.1115 (10)	0.2042 (6)	0.0841 (7)	0.6587 (6)	0.2362 (10)	0.4568 (5)
A05	0.0028 (3)	0.0012 (5)	0.0342 (4)	0.0236 (7)	-0.9282 (4)	-1.8571 (8)	0.0739 (4)	0.2152 (7)	0.0873 (10)	0.7264 (7)	0.2311 (9)	0.5061 (7)
A06	0.0028 (4)	0.0011 (3)	0.0341 (2)	0.0238 (9)	-0.9283 (3)	-1.8581 (7)	0.0735 (3)	0.216 (8)	0.0861 (9)	0.7657 (8)	0.2297 (7)	0.518 (8)
A07	0.0028 (2)	0.0012 (8)	0.0341 (3)	0.0244 (10)	-0.9287 (2)	-1.8539 (9)	0.0725 (2)	0.2274 (10)	0.0846 (8)	0.8174 (9)	0.229 (6)	0.533 (10)
A08	0.0037 (5)	0.0011 (4)	0.0365 (5)	0.0207 (3)	-0.9186 (5)	-1.8657 (2)	0.0843 (5)	0.1831 (3)	0.0787 (4)	0.5952 (3)	0.223 (4)	0.4365 (3)
A09	0.0039 (7)	0.0011 (2)	0.0375 (7)	0.0197 (2)	-0.9151 (7)	-1.873 (1)	0.0873 (7)	0.1658 (1)	0.0715 (1)	0.5076 (2)	0.2177 (1)	0.4119 (2)
A10	0.0038 (6)	0.0012 (7)	0.0372 (6)	0.0215 (5)	-0.9162 (6)	-1.8609 (5)	0.0872 (6)	0.1967 (5)	0.0773 (3)	0.6552 (5)	0.2227 (3)	0.4594 (6)

Notes: 1. Each volatility model is given a score number of 1 up to 10 based on the average loss value in ascending order for each market across the pairwise. The score number 1 denotes the volatility model with the lowest average loss value while the score number 10 denotes the volatility model with the highest average loss.

2. Certain loss values within the market seem to have similar numerical value but given different rankings is due to the issue of accuracy of decimal places shown in this table. In actual, their loss values are dissimilar with very small difference in magnitude, thus, they are given different ranking.

Table 8

Final Ranking across Pairwise and Overall Ranking of the Out-of-Sample Forecast Evaluation

Pairwise	DJMY25-DJIBSC	DJMY25-DJICYC	DJMY25-DJIENE	DJMY25-DJIFIN	DJMY25-DJIHCR	DJMY25-DJIIDU	DJMY25-DJINCY	DJMY25-DJITEC	DJMY25-DJITLS	DJMY25-DJIUTI	Overall Score (Overall Rank)
Model	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	Final Score (Final Rank)	
A01	33 (1)	63 (4)	53 (2)	59 (4)	46 (1)	36 (1)	34 (1)	54 (3)	43 (2)	58 (3)	479 (1)
A02	58 (3)	65 (5)	61 (4)	47 (1)	53 (2)	62 (5)	77 (8)	45 (1)	59 (5)	61 (4)	588 (4)
A03	81 (9)	67 (7)	74 (6)	84 (10)	71 (7)	89 (9)	89 (9)	75 (7)	79 (7)	67 (6)	776 (8)
A04	87 (10)	72 (9)	80 (9)	80 (9)	74 (8)	89 (9)	105 (10)	69 (6)	100 (10)	100 (10)	856 (10)
A05	79 (8)	73 (8)	80 (9)	77 (8)	83 (9)	85 (8)	74 (7)	76 (8)	91 (9)	75 (8)	793 (9)
A06	76 (6)	87 (10)	76 (8)	68 (6)	83 (9)	72 (6)	70 (6)	79 (9)	83 (8)	71 (7)	765 (7)
A07	58 (3)	65 (5)	62 (5)	47 (1)	61 (4)	54 (3)	54 (4)	60 (4)	67 (6)	79 (9)	607 (6)
A08	76 (6)	58 (3)	53 (2)	58 (3)	58 (3)	72 (6)	46 (2)	53 (2)	43 (2)	46 (2)	563 (2)
A09	58 (3)	56 (2)	75 (7)	76 (7)	67 (6)	58 (4)	50 (3)	65 (5)	39 (1)	40 (1)	584 (3)
A10	54 (2)	54 (1)	46 (1)	64 (5)	64 (5)	43 (2)	61 (5)	84 (10)	56 (4)	63 (5)	589 (5)

Note: The scores for the 6 loss functions in Table 7 are summed and the final ranking of the models are based on these total scores. The model with the lowest total score is identified as the best model.

CONCLUSION

The main aim of this study is to investigate a methodology for improving the dynamic of stock return volatility based on the theory of realized volatility (RV) and the theoretical framework of heterogeneous market hypothesis (HMH). The RV is known to be a consistent, efficient, and unbiased proxy of the unobservable return volatility, while HMH captures the heterogeneity in the market and long memory in the time series. This study offers a comprehensive comparison of in-sample estimation and out-of-sample volatility performance, utilizing ten multivariate RV-based models that effectively capture the long memory property, which significantly impacts market efficiency and predictability. The estimation of the parameters is performed through a simple ordinary least squares (OLS) regression, while the computation of the realized variation measures is computed from the 5-minutely data of the Malaysian Islamic stock index and global Islamic sectoral stock indices. The findings of this study exhibit that all nine proposed models have outperformed the benchmark model, VHAR (RV), in the in-sample estimation. On the other hand, for out-of-sample volatility forecasting, the benchmark model is found to be the best performing model. More than that, this study shows that using a volatility proxy of standard realized volatility and breaking down realized volatility into its continuous and discontinuous jump components gives us more information than just using jump-robust realized volatilities and adding the daily jump regressor to the benchmark model when predicting volatility in Islamic stock markets. Overall, the findings shows that a simple autoregressive specification mimicking long memory and using standard realized volatility as volatility proxy does not perform significantly worse than more complicated models in including the various realized variation measures. To provide ~~a~~ more generalized and reliable inferences of a superior volatility forecasting model, the methodology used in this study can be extended to account for different timespans of a more recent dataset, different distributional assumptions, different market data, different sampling frequencies and the incorporation of other possible methodologies, such as adding the multivariate conditional volatility modeling and its various extensions. The findings of this study may provide useful insights for policymakers, academic researchers, and market investors that have practical applications in market risk regulation, portfolio management, option strategy formulation, and pricing.

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