JOURNAL OF COMPUTATIONAL INNOVATION AND ANALYTICS
https://e-journal.uum.edu.my/index.php/jcia

How to cite this article:
Moyi A.U. & Sani A. (2025). Magnetohydrodynamics inclination effects on couette fluctuating micro-gas flow. *Journal of Computational Innovation and Analytics*, *4*(1), 85-95. https://doi.org/10.32890/jcia2025.4.1.6

MAGNETOHYDRODYNAMICS INCLINATION EFFECTS ON COUETTE FLUCTUATING MICRO-GAS FLOW

[1]Aliyu Usman Moyi & [2]Ashafa Sani
Mathematical Sciences Department, Federal University Gusau, Nigeria

[1]Corresponding author: aliyumoyik@yahoo.com

Received: 23/06/2024 Revised: 29/11/2024 Accepted: 05/12/2024 Published: 30/01/2025

ABSTRACT

An analytical investigation is conducted in the laminar flow regime to examine the impacts of Magnetohydrodynamics (MHD) inclination on Couette fluctuating basic micro-gas-flow, in which the bottom plate remains fixed while the top plate swings in its plane with sinusoidal velocity. Using a transformation technique, the partial differential equations that govern the flow situation have been reduced to a system of analytically solvable ordinary differential equations. The impact of MHD inclination on the dimensionless velocity and skin friction's numerical values are displayed in a detailed analytical and graphical form. The analysis's conclusion reveals that velocity rises with increasing phase angle values but drops with increasing inclination angle, magnetism, and frequency of the fluctuating driving force.

Keywords: Couette flow, fluctuating flows, frequency of fluctuating driven force, Hartman number, MHD inclination

INTRODUCTION

Fluid flows are used by micro-scale devices for a variety of industrial and medical applications, advancing numerous facets of human endeavor. Inkjet printing, electronic cooling, and environmental testing all depend on micro-pumps. It has been discovered that micro-ducts have been crucial components of high-frequency fluidic control systems, diode lasers, and infrared detectors. Automobile airbags, accelerators, keyless entry devices, blood analysis tools, biological cell separation reactors, and thick micro-mirror arrays for high-definition optical screens are all examples of devices that use microdevices. In particular, small pumps are used in the medical profession to manufacture compounds in nanoliters, monitor and distribute tiny amounts of medication, and create an artificial pancreas. Notably, microfluidic flow is a popular field of study in hydrodynamics due to the wide range of applications for microfluidic systems.

Haddad et al.'s (2005) study is one of the most recent papers on variable micro-gaseous fluxes. Accordingly, four flow situations were examined in their study of the impact of changing driving force frequency on fundamental micro-flows of gases in the slip flow regime: the pulsing Poiseuille flow, the transient Couette flow, the Stokes second problem, and natural convection. According to the study's findings, the temperature jump and velocity slip increase as the frequency of driving forces and the Knudsen number rise. However, their impact is attenuated for sufficiently low driving force and Knudsen number frequencies. Meanwhile, Haddad et al. (2006) explored the result of regularly revolving driving forces on simple microflows in porous media. At the same time, Ashafa et al. (2017) investigated analytically the effects of Magnetohydrodynamics (MHD) inclination and unsteady heat transfer in a laminar, transitional, and turbulent flow of a simple gaseous micro-flow across a vertically moving oscillating plate. Similarly, Gosh (2021) studied Stoke's flow over a flat plate in oscillatory MHD with induced magnetic field effects, while Mahabaleshwar et al. (2022) examined the flow of a viscous, incompressible, laminar Casson fluid past a stretching/shrinking sheet. Moreover, Ashafa et al. (2017) researched the effects of suction and injection on transitory Couette basic gaseous fluctuating micro-flows. They discovered that both techniques slow down the velocity of the fluid in motion. In addition, an infinite inclined plate's MHD free convective flow has been studied by Osman et al. (2022). Sina et al. (2022) investigated mixed convection characteristics and MHD micropolar fluid flow using entropy production analysis of an inclined porous sheet.

For a two-layer fluid, Uddin and Murad (2022) examined analytical solutions to oscillatory Couette flow and Stoke's second problem. Sharma (2022) explored the impact of MHD on unsteady oscillatory Couette flow via porous media. Furthermore, the effects of radiation and rotation on MHD flow via an inclined plate with changing mass diffusion and wall temperature in the presence of Hall current have been studied by Rajput and Kumar (2018). In 2021, Ennayar et al. examined the numerical modeling of Stoke's second problem as it was impacted by the magnetic field. Chamuah and Ahmed (2024) investigated MHD flow in thermal diffusion and thermal radiation past an inclined plate that was begun impulsively and had a parabolic plate velocity. In line with this, an instability investigation of the MHD Couette flow of an electrically conducting plate was conducted by Husain et al. (2018). Moreover, Usman and Sani (2024) studied pulsating Poiseuille micro-gas flow with MHD inclination effects. Simultaneously, suction and injection effects on pulsing Poiseuille fundamental gaseous fluctuation micro-flows have been investigated by Sani and Jibril (2021). Nevertheless, to the best of the authors' knowledge, no literature has considered how MHD inclination affects Couette's fluctuating micro-gas flow.

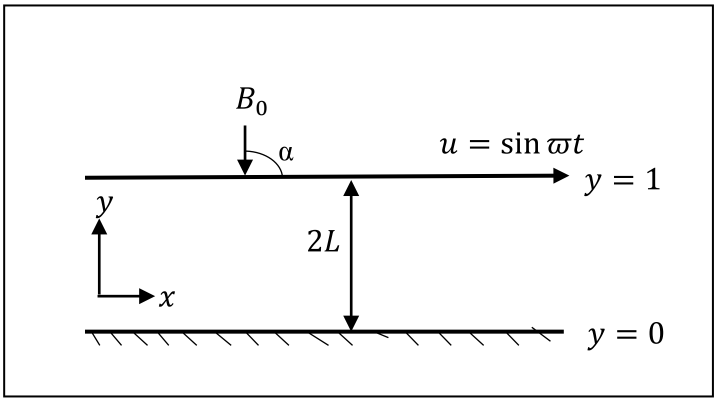
METHODOLOGY

MHD fluctuating Couette flow has applications in various human endeavors such as engineering, materials processing, biomedical, geophysics, and environmental sciences. In engineering, MHD fluctuating Couette flow helps optimize lubrication systems, improve the efficiency of heat exchangers and MHD power generators, design electromagnetic pumps for industries, and stabilize/maneuver spacecraft and aircraft. Meanwhile, in materials processing, MHD fluctuating Couette flow plays critical roles in continuous casting (controlling molten metal flow), regulates crystal growth processes, and helps synthesize advanced materials. Furthermore, in medical fields, MHD fluctuating flows help understand blood flow behavior in veins/arteries and the design of implantable devices such as the artificial heart/pancreas. Geophysical and environmental applications of MHD fluctuating Couette flow include modeling ocean currents, coastal dynamics, and Earth's core dynamics, as well as the study of atmospheric circulation patterns and geophysical phenomena. Accordingly, these applications demonstrate the significance of MHD fluctuating Couette flow in various fields, and research in this area continues to expand our understanding and explore new applications.

With an inclining magnetic field present, consider the hydrodynamically complete development of a basic micro-gas flow in the slip flow regime between two parallel plates of unlimited lengths separating a horizontal micro-channel. In contrast, the lower plate stays stationary, and the upper plate moves in its plane with a sinusoidal velocity ($u = \sin(\varpi t)$). The plates are separated by length L. As displayed in Figure 1, a coordinate system is utilized to ensure that the x-axis is parallel to the plates and the y-axis is orthogonal. Since it is assumed that the flow is hydrodynamically fully developed, there is no change in velocity with regard to x, i.e., $\partial u / \partial x = 0$.

Figure 1

Schematic diagram



Dimensional representations of the slip boundary conditions and the governing momentum equation are as follows:

Governing Equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2 \sin^2 \alpha}{\rho} u \ . \tag{1}$$

Boundary Conditions:

$$u(t,0) = \frac{2-\sigma_v}{\sigma_v} \lambda \left. \frac{\partial u}{\partial y} \right|_{y=0} , \tag{2}$$

$$u(t,L) - \sin(\varpi t) = -\frac{2-\sigma_v}{\sigma_v} \lambda \left. \frac{\partial u}{\partial y} \right|_{y=L} . \tag{3}$$

We now introduce the following dimensionless variables:

$$Y=\frac{y}{L},\tau=\frac{t}{t_r},U=\frac{u}{u_o},\ M^2=\frac{\sigma L^2 B_o^2}{\mu},\ \omega=\frac{\varpi}{\omega_r},\ Kn=\frac{\lambda}{L}. \tag{4}$$

Substituting Equation (4) into Equations (1-3), to obtain the:

Dimensionless Governing Equation:

$$\frac{\partial U}{\partial \tau}=\frac{\partial^2 U}{\partial Y^2}-M^2\sin^2\alpha U\ . \tag{5}$$

Dimensionless Boundary Conditions:

$$U(\tau,0)=\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=0}, \tag{6}$$

$$U(\tau,1)-\sin(\omega\tau)=-\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=1}, \tag{7}$$

where α is the angle between β_0 and $u\left(t,y\right)$ for $0\le\alpha\le\pi/2$, M is the magnetic force, and Kn is the Knudsen number.
An exact solution to this flow problem is obtained using the separation of variables used by Haddad *et al.* (2005):

$$\text{i.e. } U(\tau,Y)=\text{Im}\{\exp(i\omega\tau)V(Y)\}, \tag{8}$$

where Im stands for the complex solution's imaginary component and $i=\sqrt{-1}$. By differentiating Equation (8) and substituting into Equation (5), we obtained the following Ordinary Differential Equation (ODE):

$$V^{11}(Y)-\left(M^2\sin^2\alpha+i\,\omega\right)\ V(Y)=0\,. \tag{9}$$

Equation (8) is then solved analytically using the method of undetermined coefficient to obtain:

$$V(Y)=c_1\cosh\left(Y\sqrt{K}\right)+c_2\sinh\left(Y\sqrt{K}\right). \tag{10}$$

Substituting Equation (10) into Equation (8) to obtain:

$$U(\tau,Y) = \text{Im}\left\{\exp(i\,\omega\tau)\left(c_1\cosh\left(Y\sqrt{K}\right)+c_2\sinh\left(Y\sqrt{K}\right)\right)\right\}. \tag{10}$$

Using the boundary conditions in Equation (6) and (7) in Equation (10), the solution becomes:

$$U(\tau,Y) = \text{Im}\left\{\exp(i\,\omega\tau)\left(\frac{\sinh\left(Y\sqrt{K}\right)+a\cosh\left(Y\sqrt{K}\right)}{2a\cosh\left(\sqrt{K}\right)+\left(1+a^2\right)\sinh\left(\sqrt{K}\right)}\right)\right\}, \tag{11}$$

where

$$K = M^2\sin^2\alpha + i\,\omega \text{ and } a = \frac{2-\sigma_v}{\sigma_v}Kn\sqrt{K}. \tag{12}$$

The skin friction is obtained by differentiating Equation (11) with respect to Y.

$$\tau_Y = \frac{\partial U}{\partial Y} = \text{Im}\left\{\exp(i\,\omega\tau)\left[\frac{\sqrt{K}\left(\cosh\left(Y\sqrt{K}\right)+a\sinh\left(Y\sqrt{K}\right)\right)}{2a\cosh\left(\sqrt{K}\right)+\left(1+a^2\right)\sinh\left(\sqrt{K}\right)}\right]\right\}. \tag{13}$$

Accordingly, the skin friction at the upper and lower plates is calculated as follows:

$$\tau_0 = \left.\frac{\partial U}{\partial Y}\right|_{Y=0} = \text{Im}\left\{\exp(i\,\omega\tau)\left[\frac{\sqrt{K}}{2a\cosh\left(\sqrt{K}\right)+\left(1+a^2\right)\sinh\left(\sqrt{K}\right)}\right]\right\}, \tag{14}$$

$$\tau_1 = \left.\frac{\partial U}{\partial Y}\right|_{Y=1} = \text{Im}\left\{\exp(i\,\omega\tau)\left[\frac{\sqrt{K}\left(a\sinh\left(\sqrt{K}\right)+\cosh\left(\sqrt{K}\right)\right)}{2a\cosh\left(\sqrt{K}\right)+\left(1+a^2\right)\sinh\left(\sqrt{K}\right)}\right]\right\}. \tag{15}$$

RESULTS

For the purposes of a physical understanding, the graphical discussions have been conducted using arbitrary values of α (angle of inclination), M (magnetic force), ω (frequency of the fluctuating driven force), $\omega\tau$ (phase angle), and Kn (Knudsen number) in Figure (2-6). Unless otherwise specified, the values $\omega = 1$, $Kn = 0.1$, $\omega\tau = \pi/4$, $\alpha = \pi/6$, $M = 2$, and $\sigma_\upsilon = 0.7$ are used throughout the discussion.

Figure 2

Velocity profile when varying α.

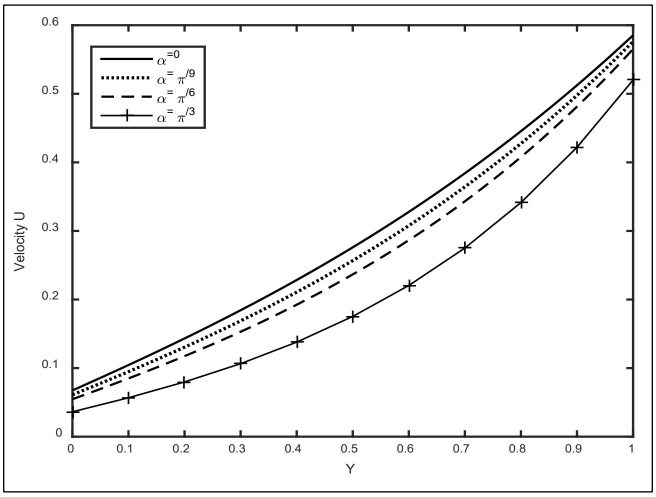


Figure 2 illustrates the effect of the magnetic field's angle of inclination. It is observed that the velocity drops with an increase in the angle of magnetic inclination. This is attributed to the fact that, as the angle of inclination increases, the component of the magnetic field perpendicular to the flow direction increases, resulting in a stronger force and decreased velocity.

Figure 3 illustrates the effect of magnetism on the flow. Notably, an increase in magnetism results in a decrease in velocity, while an increase in magnetism leads to enhanced Lorentz force, magnetic damping, Joule dissipation, and the formation of magnetic boundary layers. These effects contribute to a decrease in the fluid velocity.

Figure 4 displays how the flow is affected when the Knudsen number increases. As the Knudsen number rises, it is discovered that the fluid's velocity falls. The velocity was observed to be higher on the moving plate and decreased downward as y approached the stationary plate. Increasing values of the Knudsen raise the velocity at $y = 0$ to $y = 0.3$, where there is crossover flow and retards the velocity from $y = 0.3$ to $y = 1$.

Figure 3

Velocity profile when varying M

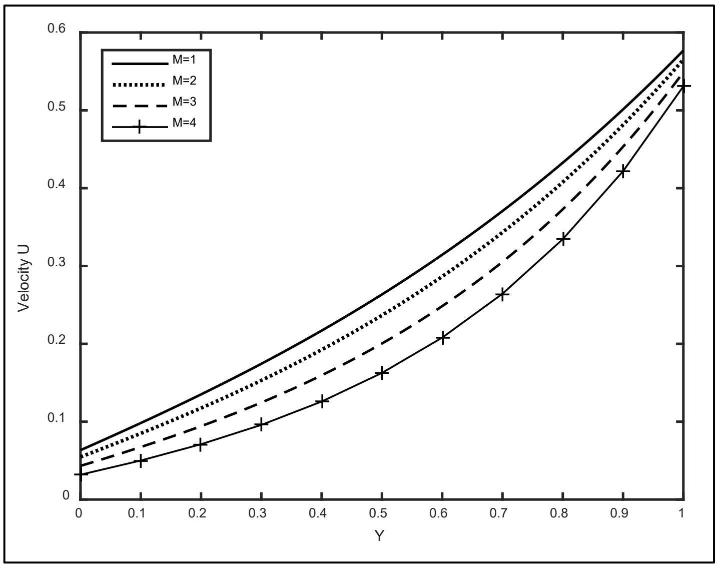


Figure 4

Velocity profile when varying Kn

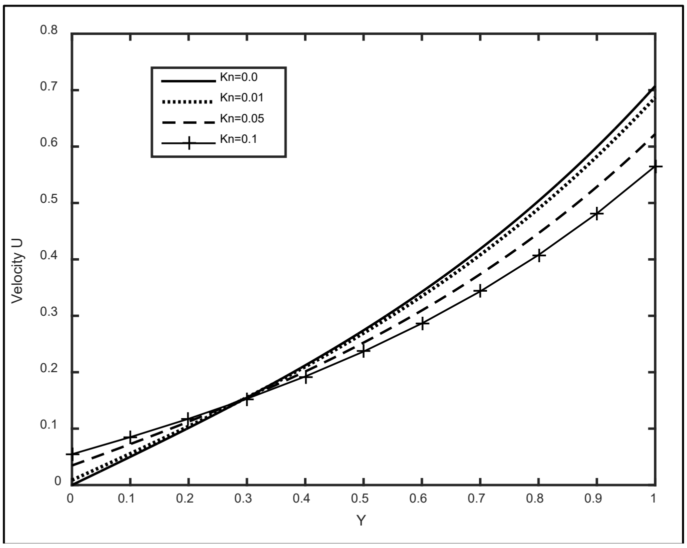


Figure 5 illustrates the velocity curve that results from varying driving force frequency values, ω. It is discovered that when the driving force's frequency increases, the fluid's local velocity decreases. This is due to an increase in energy losses from viscous dissipation, which is caused by the higher frequencies. In particular, the highest velocity is observed at $y = 1$ due to the sinusoidal movement of the plate, which decreases downward through the stationary plate at $y = 0$. At the same time, a crossover flow occurs around the channel's center for values of ω = 100 and 1000, at *y = 0.65* for ω = 10 and 100, at *y = 0.71* for ω = 10 and 1000, and at *y = 0.90* for the values of ω = 100 and 1000. It is essential to note that since velocity decreases as the frequency of the driven force increases, the fluid velocity approaches 0 more quickly at a higher frequency than at a lower one.

Figure 5

Velocity profile when varying ω.

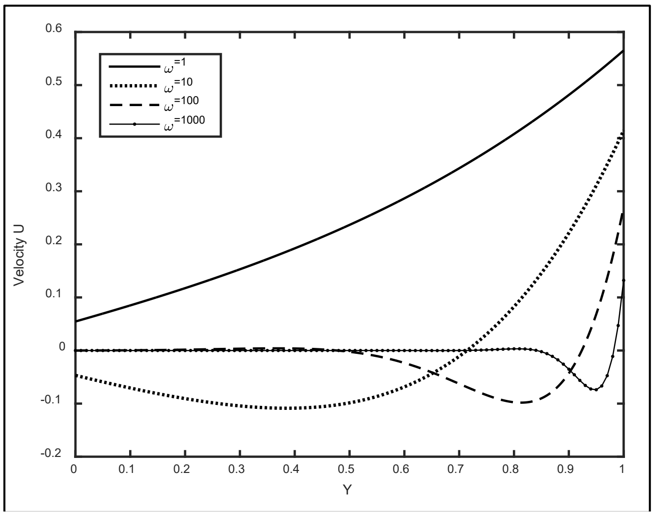


Figure 6 portrays velocity distribution when varying the phase angle. It is reported that increasing the values $\omega\tau$ increases the velocity of the fluid. Table 1 displays the skin friction values at the plates numerically. At *y = 0*, the skin friction decreases as α improves, whereas at *y = 1*, it increases as α increases.

Figure 6

Velocity profile when varying $\omega\tau$.

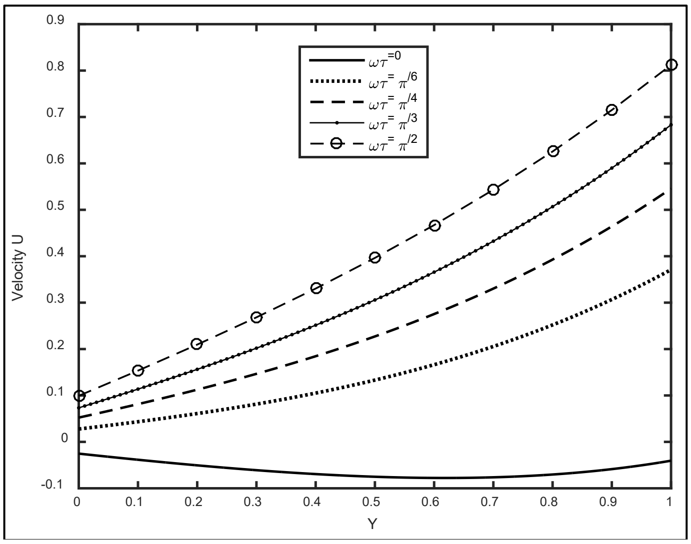


Table 1

Numerical values for skin friction

α	τ_0	τ_1
0	0.25966571	0.68672898
π/18	0.25090567	0.70595580
π/9	0.22685707	0.74869656
π/6	0.19222664	0.78791418
π/3	0.10222894	0.83028603
π/2	0.07618158	0.83055564
M	τ_0	τ_1
0	0.25966571	0.68672898
1	0.24180963	0.72392390
2	0.19222664	0.78791418
3	0.12878874	0.82403656

CONCLUSION

MHD inclination effect on Couette fluctuating micro-gas flow has been investigated in this research. It has been discovered that velocity decreases with an increase in the angle of inclination (α), magnetism (M), and frequency of the fluctuating driven force (ω). It also increases with an increase in the values of phase angle $(\omega\tau)$. Nevertheless, the outcome of the research is applicable to optimizing lubrication systems and improving the efficiency of heat exchangers and MHD power generators.

ACKNOWLEDGEMENT

The authors are grateful to Federal University Gusau and the Tetfund for the grant support and assistance under the Institutional Base Research (IBR) with number TETFUND/DR&D/CE/UNI/GUSAU/IBR/B8/2023/VOL. 1. We also appreciate the anonymous reviewers' thoughtful criticism and recommendations, which helped to raise the quality and clarity of the paper.

Nomenclature:

Kn	Knudsen number $(= \lambda / L)$
L	Reference length
t	Time
t_r	Reference time $(= L^2 / \upsilon)$
U	Dimensionless axial velocity $(= u / u_o)$
u	Axial velocity (in x-direction)
u_o	Velocity of moving wall
V	Complex solution function for velocity

X Dimensionless axial coordinate $(= x/L)$

x Axial coordinate

Y Dimensionless transverse coordinate $(= y/L)$

y Transverse coordinate

Greek symbols:

λ Mean-free-path length

ρ Density

μ Dynamic viscosity

υ Kinematic viscosity

σ_υ Tangential momentum accommodation coefficient $(= 0.7)$

ϖ Frequency

ω Dimensionless frequency $(= \varpi / \omega_r)$

ω_r Reference frequency $(= \upsilon / L^2)$

τ Dimensionless time $(= t / t_r)$

REFERENCES

Ashafa, S.; Abiodun, O.A; Haruna, M.J;. (2017). Effects of Suction/Injection on Transient Couette Basic Gaseous Fluctuating-Micro-Flow. *IOSR Journal of Mechanical and Civil Engineering, 14*(5), 56-62.

Ashafa, S.; Ahmad, A.A.: Sakir, A.A;. (2017). Analytical Solution of the Effect of MHD Inclination and Unsteady Heat Transfer in a Laminar, Transition and Turbulent Flow of a Basic Gaseous Micro-Flow past a Vertically Moving Oscillating Plate. *American Journal of Engineering & Natural Sciences (AJENS), vol. 1*(2), pp. 29-35.

Chamuah, K., & Ahmed, N. (2024). MHD Flow Past an Impulsively Started Inclined Plate with Parabolic Plate Velocity in the Presence of Thermal Diffusion and Thermal Radiation. *Biointerface Research in Applied Chemistry, 13*(3), 1 - 18. https://doi.org/https://doi.org/10.33263/BRIAC143.060

Ennayar, H., Boeck, T., & Karcher, C. (2021). Numerical simulation of Stokes' second problem affected by magnetic field. *10th International Conference on Electromagnetic Processing of Materials.* Riga, Latvia.

Ghosh, S. (2021). Oscillatory magnetohydrodynamic (MHD) Stoke's flow. *African Journal of Engineering Research, 9*(1), 1-12.

Haddad, O.M.; Al-Nirm, M.A.; Abuzaid, M.M. (2005). The effect of frequency of fluctuating driven force on basic gaseous micro-flows. *Acta-Mechanica*(179), 249-259.

Haddad, O.M.; Al-Nirm, M.A.; Abuzaid, M.M.;. (2006). Effect of periodically oscillating driving force on basic microflows in porous media. *Journal of Porous Media, 9*(7), 695-707.

Hussain, Z., Hussain, S., & Kong, T. (2018). Instability of MHD Couette flow of an electrically conducting fluid. *AIP Advances, 8*, 105209. https://doi.org/https://doi.org/10.1063/1.5051624

Mahabaleshwar, U. S., Maranna, T., & Sofos, F. (2022). Analytical investigation of an incompressible viscous laminar Casson fluid flow past a stretching/shrinking sheet. *Scientific Reports*, 18404. https://doi.org/https://doi.org/10.1038/s41598-022-23295-6

Osman, H. I., Omar, N. F., Vieru, D., & Ismail, Z. (2022). A Study of MHD Free Convection Flow Past an Infinite Inclined Plate. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, 92*(1), 18 - 27. https://doi.org/https://doi.org/10.37934/arfmts.92.1.1827

Rajput, U. S., & Kumar, G. (2018). Rotation and radiation effects on MHD flow past an inclined plate with variable wall temperature and mass diffusion in the presence of Hall current. *Applications and Applied Mathematics: An International Journal (AAM), 13*(1), 484 - 495. Retrieved from https://digitalcommons.pvamu.edu/aam/vol13/iss1/31

Sani, A., & Jibril, H. M. (2021). Suction/Injection Effects on Pulsating Poiseuille Basic Gaseous Fluctuating-Micro-Flows. *International Journal of Science and Global Sustainability, 7*(4), 31 - 38. https://doi.org/https://doi.org/10.57233/ijsgs.v7i4.311

Schaaf, S., & Chambre, P. (1961). *Flow of rarefied gases.* Pinceto, NJ: Princeton University Press.

Sharma, B. K. (2022). Effect of MHD on Unsteady Oscillatory Couette Flow through Porous Media. *Int. J. of Applied Mechanics and Engineering, 27*(1), 188 - 202. https://doi.org/DOI: 10.2478/ijame-2022-0012

Sina, S., Hossein, A., Mohsen, J., & Hossein, A. (2022). An analytical approach to entropy production in MHD mixed convection micropolar fluid flow over an inclined porous stretching sheet. *Frontiers in Mechanical Engineering*.

Uddin, M., & Murad, A. (2022). Stokes' Second Problem and Oscillatory Couette Flow for a Two-Layer Fluid: Analytical Solutions. *Alexandria Engineering Journal, 61*(12), 10197 - 10218. https://doi.org/https://doi.org/10.1016/j.aej.2022.03.023

Usman, A. M., & Sani, A. (2024). Pulsating Poiseuille micro-gas flow with MHD inclination effect. *Journal of Mathematical Problems, Equations and Statistics, 5*(1), 19 - 23. https://doi.org/https://doi.org/10.22271/math.2024.v5.i2a.144

Zohar, Y. (2003). *Heat Convection in Micro Ducts.* Springer Science & Business Media.