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THE MODIFIED MEAN ABSOLUTE DEVIATION INTEGER LINEAR PROGRAMMING MODEL FOR GLOBAL PORTFOLIO ASSET ALLOCATION

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ABSTRACT

This study develops a practical yet robust Mean Absolute Deviation (MAD) Mixed-Integer Linear Programming (MILP) model for a global investment portfolio tailored to the constraints faced by Malaysian retail investors, such as transactional fees and foreign currency exchange spreads, to provide more accurate estimations of portfolio returns. The MAD ILP model is modified to address these factors, aiming to enhance utility and efficiency. The study utilizes a Maybank 12-month fixed deposit cash account and ten selected Exchange-Traded Funds (ETFs) from iShares, Vanguard, and State Street Global Advisors for global portfolio construction. Forecasting techniques were utilized to generate expected returns for the ETFs and foreign currency exchange spreads. Based on the MAD model proposed by Konno and Wijayanayake (2001), the study introduces three modifications: incorporating transaction cost elements specific to Malaysian investors, including foreign currency exchange spreads in return calculations, and transforming the model from mixed-integer to pure integer model to accommodate minimum transaction lot constraint. Verification and validation exercises demonstrate the model's ability to replicate real-world portfolio construction and reliable simulations. The model offers a practical tool for self-ascribed investors aiming to optimize global investment portfolios while considering unique constraints and transaction costs.

Keywords: Linear programming, Integer programming, Optimization, Asset allocation, Global portfolio, ETFs.

INTRODUCTION

This study is motivated by three significant trends that have influenced the participation of retail investors in Malaysia over the past five years. Firstly, lower purchasing power among Malaysians, coupled with stagnant income levels, has prompted individuals to seek additional income streams and generate wealth independently. The gap between mean income between B40, M40, and T20 in Malaysia widened between 2016 and 2019 based on the compounded annual growth of 3.4%, 4.1%, and 4.7%, respectively, in line with the increase in GINI coefficient for Malaysia from 37.8% in 2016 to 40.7% in 2019 (Department of Statistics Malaysia, 2020). Secondly, there has been a heightened focus on financial literacy globally, including in Malaysia, with initiatives led by organizations like the Financial Education Network (FEN). Increased financial literacy has been shown to correlate with higher risk tolerance and a willingness to explore more complex investment options (Hsiao & Tsai, 2018; Kadoya et al., 2017; Panos et al., 2020). Lastly, advancements in technology, such as blockchain, artificial intelligence, and mobile computing, have democratized investing, granting retail investors access to a broader range of global assets.

The increased accessibility to various assets for retail investors in Malaysia has created opportunities for wealth creation but also poses challenges. Lack of knowledge and awareness and over-reliance on influencers or unqualified individuals for investment advice has led to poor investment decisions. At the same time, existing tools for portfolio construction and performance evaluation lack comprehensive real-life constraints, especially for cross-border trade activities.

To address these challenges, self-ascribed retail investors require a financial tool that considers real-life investment and operational constraints, risk tolerance levels, and realistic return expectations. Mean-Variance Optimization (MVO), introduced by Markowitz (1952), has been widely used but has become less robust and efficient with many constraints (Perold, 1984; Zhang et al., 2018). Researchers have explored improvements, such as using Mean Absolute Deviation (MAD) as a risk proxy, transforming MVO into a Linear Programming (LP) model, and increasing flexibility without compromising robustness (Kalayci et al., 2019).

Past studies have successfully incorporated real-life constraints into MAD Mixed-Integer Linear Programming (MILP) models. However, they have not considered transactional fee structures commonly faced by Malaysian retail investors or foreign currency exchange spread. These missing elements could lead to imprecise return estimations. Transactional fees in Malaysia, such as brokerage commissions and stamp duty, are usually a percentage of the transaction value or based on multiples of the value with minimum and maximum amounts.

Hence, this study aims to develop a practical and robust LP model that incorporates not only the fundamental constraints of a portfolio, such as risk level, investment policies, and foreign currency gain or loss. Nevertheless, some constraints are unique to Malaysian retail investors, i.e., the constant and linear-based transactional fees and foreign currency spread when deriving asset allocation decisions.

LITERATURE REVIEW

The goal of any investing approach is to optimize portfolio returns. One such approach is value investing, which constructs financial portfolios based on the dividend discount model initially proposed by Williams (1938) and popularized by Graham (1973) in his book The Intelligent Investor. Value investing focuses on capitalizing on the discrepancy between the market price and the intrinsic value of a security. In value investing, greater allocation is given to undervalued securities. In the early 1950s, Markowitz introduced a new approach

(Markowitz, 1952), which laid the foundation for Modern Portfolio Theory (MPT). Note that MPT approaches portfolio optimization differently by aiming to achieve the maximum return for a given level of risk or the minimum risk for a desired level of return attained through MVO.

Asset Allocation & Performance Attribution

Sharpe (1964, 1992) demonstrated the importance of asset allocation through performance attribution, which breaks down portfolio performance into multiple factors. Performance can be attributed to asset allocation policy, market timing, and instrument selection (Bacon, 2019). Other studies have identified additional factors, such as style-based (Chow et al., 2018; Froot & Teo, 2008) and machine-learning methods (Daul et al., 2022; Li et al., 2022). The intriguing aspect of asset allocation is how the allocation decision significantly contributes to portfolio performance by explaining variations in total return. Earlier research concluded that asset allocation policy, derived from optimization exercises, explained over 90% of return variations (Brinson et al., 1991; Ibbotson & Kaplan, 2000). More recent studies across different portfolio categories and periods found that asset allocation policy contributed between 35% and 75% of return variations while also recognizing the significant impact of market movements (Brown et al., 2010; Hensel et al., 1991; Ibbotson, 2010; Awaludin & Rahman, 2021; Xing et al., 2018).

Limitations and Enhancements of Optimization Models

The work of Ibbotson (2010) provides a foundation for ongoing interest and innovation in asset allocation techniques. Despite criticisms, Markowitz's MPT technique remains popular and widely adopted by pension funds, endowments, and mutual funds due to its alignment with the rational investor's perspective, acknowledging the positive relationship between risk and returns, risk tolerance awareness, and diversification for mitigating systematic risk. However, Markowitz's original model has faced practical criticisms (Kalayci et al., 2019; Zhang et al., 2018). The classic MVO quadratic programming optimization model is incompatible with certain types of constraints, such as non-convex and non-linear constraints, like buying thresholds and asset limits (Stein, 2007). These practical challenges and related works addressing them are summarized in Table 1.

Table 1

Practical Limitations of Markowitz's Classic Model

	Practical Limitations	Works Done to Address
1	Computational burden in solving large-scale quadratic program associated with a highly dense covariance matrix.	• Introduction of a covariance matrix using factor and scenario models of return (Perold, 1984). • Introduce assumptions that the returns of various securities are related to a common factor, such as market indices, reducing the required parametric input and computations (Sharpe, 1963).
2	An inaccurate assumption on investors' perception of risk and returns	• Introduce the downside-risk portfolio optimization model (Feiring et al., 1994; Jankova, 2019). • Introduce conditional value at risk as a measure of risk in quadratic mode (Rockafellar & Uryasev, 2000). • Introduce Prospect Theory through the usage of a dynamic sentiment-adjusted model to measure risk (Wei et al., 2021). • Introduce the principle of minimizing maximum loss over past observation periods as the objective of portfolio optimization in an LP model (Young, 1998).
3	Difficulty in incorporating real-life constraints of portfolio management.	• Introduce the mean-absolute deviation model in replacement of covariance as a proxy of risk. Transform the model into an LP model (Konno & Yamazaki, 1991). • Introduce transaction cost and minimum lot constraints in the LP model (Konno & Wijayanayake, 2001). • Inclusion of transaction limits and costs in large-scale quadratic programming (Perold, 1984).

Enhancements to Markowitz's theory have led to the development of the LP model using MAD as a risk measure by Konno and Yamazaki (1991). This model has addressed practical limitations while preserving the benefits of Markowitz's original model. Further research has focused on incorporating real-life constraints and transforming the MAD LP model into a MILP. Real-life constraints in portfolio management include transactional costs, namely brokerage commission, stamp duty, forex spread (Zarei et al., 2019), cardinality or number of investment instruments (Kalayci et al., 2019; Leung & Wang, 2022), minimum transaction lot (New York Stock Exchange, 2019), and investment policy (Securities Commission Malaysia, 2016).

Research and contributions that incorporate real-life constraints are not expansive and comprehensive, especially in the space of LP (Mansini et al., 2015; Zhang et al., 2018), as compared to quadratic programming. Table 2 lists the noticeable literature with contributions on additional real-life constraints used in linear and mixed-integer programming.

Table 2

Contributions to Real-life Constraints in Pure LP or Mixed-Integer LP Programming

	Descriptions of Real-life Constraints	Literature
1	Constant fixed charge, variable cost	(Young, 1998)
2	Minimum transaction lots	(Kellerer et al., 2000)
3	Constant transaction cost and minimum transaction lots	(Konno & Wijayanayake, 2001)
4	Piecewise linear and proportional cost for entering commission	(Chiodi & Mansini, 2003)
5	Non-convex fixed transaction cost, minimum transaction lots, and cardinality	(Konno & Yamamoto, 2005)
6	Piecewise constant transaction cost	(Le Thi et al., 2009)
7	Integral transaction units, a maximum weight of individual stocks, piecewise constant transaction costs, max number of stocks, dividend payments	(Baumann & Trautmann, 2013)
8	Piecewise linear and proportional transaction cost	(Beraldi et al., 2021)

Research Gap

Based on Table 2, existing studies have incorporated transaction costs in various forms, such as constant, piecewise linear, piecewise constant, or proportional functions. However, the combination of constant and linear transaction structures, which are commonly observed in cross-border transactions, has not been integrated into LP models. For example, brokerage commissions for US-listed securities charged by Rakuten Trade (2022) include a percentage of the investment amount with a minimum and maximum fee per transaction. Currency spread, which significantly impacts portfolio returns at the realization point, is another important constraint absent in the existing literature (Neely et al., 2009). Based on these observations, this study incorporates the constant-linear transaction cost and foreign currency exchange spread to increase the robustness and efficiency of the MAD MILP model.

MODEL DEVELOPMENT

Asset Selection & Data

For this study, a global portfolio was constructed using the Maybank 12-month fixed deposit cash account and ten selected Exchange-Traded Funds (ETFs) from iShares, Vanguard, and State Street Global Advisors. The chosen ETFs met specific criteria, including broader market exposure, large fund size for stability (Akhigbe et al., 2020; Bebchuk et al., 2019; Sherrill & Stark, 2018), high liquidity for efficient price discovery (Agrrawal & Clark, 2009), and a minimum operating history of three years for low closure risk (Akhigbe et al., 2020). The Maybank 12-month fixed deposit account was selected as the risk-free asset due to its AAA financial institution's national credit rating and large deposit size. Suitable forecasting techniques were applied using actual historical data to estimate the 12-month forward parametric input for the proposed model.

Reference Model

The proposed MAD LP model is developed based on the past MAD MILP model and real-life constraints and operating conditions of Malaysian retail investors. The proposed model was derived from the MAD model proposed by Konno and Wijayanayake (2001) and expanded the original MAD by incorporating concave transaction costs as given by the following:

Let

$$x_j = X_j \Big/ \sum_{k=1}^{n} x_k,$$

be the proportion of the fund to be allocated to asset *j,* where the amount of money to be invested in securities *j, i.e.,* X_j, is divided by the total amount of money invested in securities.

Decision variables:

x_j : % of total amount to be invested in securities j, j=1, 2, …, n.

Objective function:

$$\text{maximize} \quad f(x) = \sum_{j=1}^{n} \{r_j \, x_j - c_j(x_j)\}. \tag{1}$$

Subject to:

$$\sum_{t=1}^{T} pb_t \left| \sum_{j=1}^{n} (r_{jt} - r_j) \, x_j \right| \leq w \tag{2}$$

$$\sum_{j=1}^{n} x_j = 1 \tag{3}$$

$$0 \leq x_j \leq \propto_j$$

$c_j(x_j)$: cost as a percentage of portfolio allocation percentage, *j=1, 2, …, n.*

pb_t : $\Pr\{(R_1, \ldots, R_n) = (r_{1t}, \ldots, r_{nt})\}, t = 1, \ldots, T$ the probability that the expected rate of return of asset j is equal to its return at time t

w : the specified level of risk desired by an investor

$\propto_j$: maximum percentage allocation into asset j

Proposed Model

The proposed model performed modifications to equation (4) alongside adaptation of equation (1) as a constraint. The modifications were made in two main areas within the objective function:

a. the expected rate of return computations of the assets, which will take into consideration the forex cost;
b. the localized functions of transactional costs, namely brokerage commission and stamp duty, and
c. the modeling of the minimum transaction value/ lots applicable for each of the selected ETFs.

Decision Variables

The decision variables for the proposed model are the total investment capital allocated in each of the selected ETFs. The description of decision variables is detailed in Table 3.

x_j = the amount of investment capital allocated to asset j, j=1, 2, …, 11

Table 3

Decision Variables

j	*Assets*	*Exposure*
	U.S. Stocks	
1	SPDR S&P 500 ETF	U.S. Large cap
2	iShares Core S&P Mid-Cap ETF	U.S. Midcap
3	iShares Core S&P Small-Cap ETF	U.S. Small cap
4	Vanguard Real Estate ETFs	U.S. REITs
	U.S. Bonds	
5	Vanguard Intermediate-Term Bond ETF	U.S. Aggregate long-term
6	iShares Core 1-5 Year USD Bond ETF	U.S. Aggregate short-term
	International Bonds	
7	iShares Core International Aggregate Bond ETF	Global non-USD investment grade bonds
	International Stocks	
8	iShares Core MSCI EAFE ETF	All cap Europe, Australia, Asia, Far East
9	iShares Core MSCI Emerging Markets ETF	All-cap Emerging Markets
	Commodity	
10	SPDR Gold Shares	Gold
	Fixed Deposit	
11	Maybank 12-month FD	Fixed Deposit

Objective Function

The objective function of the proposed model is to maximize the portfolio total return net of all the fees, i.e., transactional costs and foreign currency exchange spreads. The study adapted equation (1) and incorporated three transactional cost elements, namely brokerage commission, stamp duty, clearing fee, and foreign currency exchange spreads. This is expressed in the following general function, in absolute terms as follows.

maximize $f(x) = R_{11}x_{11} + \sum_{j=1}^{10}\{R'_j\ x_j - c(x_j) - s(x_j) - q(x_j)\}$. (4)

R_{11} : *the expected rate of return from Maybank's 12-month fixed deposit*
R'_j : *the expected rate of total return, net of forex cost for ETF asset j, j=1, 2, 3, ..., 10*
$c(x_j)$: *brokerage commission for ETF asset j*
$s(x_j)$: *stamp duty for ETF asset j*
$q(x_j)$: *clearing fee for ETF asset j*

To formulate, let the total rate of return for USD-denominated ETF *j* (including dividend yield), net of forex gains or loss, be defined as R'_j and the formulation is detailed as follows:

Expected rate of total return, R'_j = $\left(\frac{value\ of\ investment\ in\ RM\ at\ t=1}{value\ of\ investment\ in\ RM\ at\ t=0} 1\right) \times 100.$

Expected Net Total Rate of Return

Hence, the expected total rate of return, net of forex cost for ETF asset j, R'_j can be expressed as follows:

$$R'_j = \left(\frac{(1 + R_j + d_j) \times (f_1 - p)}{f_0} - 1\right) \times 100. \quad (5)$$

R_j : *the expected price rate of return for ETF asset j, j=1, 2, 3, ..., 10*
f_1 : *the expected USD: RM rate at t=1*
f_0 : *the expected USD: RM rate at t=0*
p : *the forex spread/cost incurred during conversion from USD to RM at year 1*
d_j : *the expected dividend yield for ETF asset j, j=1, 2, 3, ..., 10*

Transaction Cost

The study considered the cost structures of online share trading platforms available to Malaysian retail investors, including Rakuten Trade Sdn. Bhd, MIDF Invest, and iFAST Capital Sdn Bhd. The cost structure used is summarized in the following Table 4.

Table 4

Adopted Cost Structure for the Proposed Model

Cost	*Description*
Brokerage commission	0.1% of trading value, a minimum of USD1.88, and a maximum of USD25.00 per transaction
Stamp duty for U.S.-listed ETFs	RM1.00 for every RM1,000 contract value, maximum RM200.
SEC Clearing fee	0.00229% on sale contract value with a minimum fee of USD 0.01.

Brokerage Commission

The study also notes that the brokerage commission will be charged during the purchase and sale of the assets. For this reason, the brokerage commission function will be multiplied by 2. The function, denoted in Ringgit Malaysia term based on USD:RM exchange rate of 4.7000, is as follows:

$$c(x_j) = 2 \times max(8.836, min(0.1\% x_j, 117.500)) \text{ in RM terms} \quad (6)$$

x_j : the amount of investment capital allocated to ETF asset j, j=1, 2, …, 10

a : minimum brokerage commission

b : cost as % of the transaction amount

c : maximum brokerage commission

Note that brokerage commission and other transactional costs of an ETF are only applicable if the ETF is selected during the optimal portfolio. Hence, to incorporate equation (6) into our proposed model, let be the 0-1 integer program as follows:

$$y_j (j = 1, 2, 3, \dots 10) = \begin{Bmatrix} 1 & if\ ETF\ j\ is\ invested \\ 0 & otherwise \end{Bmatrix}. \quad (7)$$

And to link the decision variables x_j andy_j, the following function is added.

$$x_j (= 1,2,3, \dots, 10) \leq M y_j. \quad (8)$$

M : The maximum amount that could be allocated to ETF asset j.

$$c(x_j) = 2 \sum_{j=1}^{10} y_j\, max(8.836, min(0.1\% x_j, 117.5)). \quad (9)$$

Stamp Duty

The stamp duty is also applicable both at the purchase and sale of an ETF. However, the stamp duty is calculated by first identifying the number of integer multiples for every RM1,000 transaction amount. This can be computed by rounding up the multiples to the nearest integer, notated by $\lceil d \rceil$ means to ceiling *d*.

$$s(x_j) \quad = \quad 2\sum_{j=1}^{10} y_j \, min(\left\lceil (\frac{x_j}{1000}) \right\rceil, 200). \tag{10}$$

Clearing Fee

The clearing fee is only applicable on sale transactions. This can be expressed in the following formula.

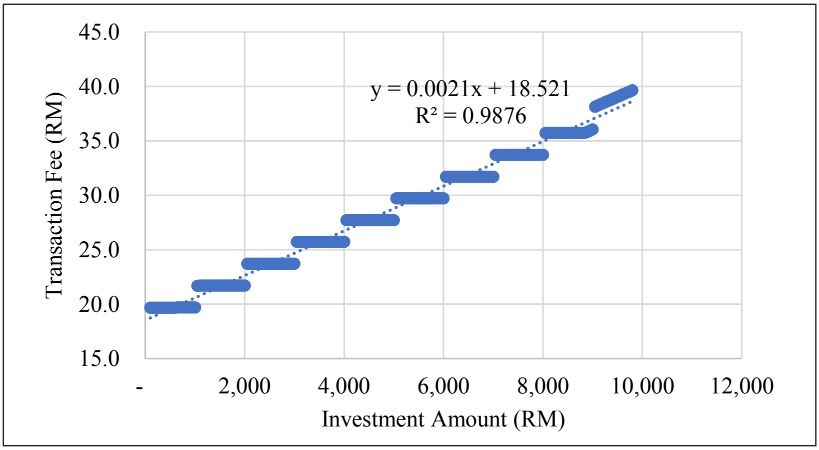
$$q(x_j) \; = \sum_{j=1}^{n} y_j \, min(0.05\,, 0.00229\% x_j) \quad \textit{in RM terms} \tag{11}$$

Modification to the formulation of transaction cost

Equations (9), (10), and (11) encompass non-linear functions individually. The aggregate configuration of these non-linear functions is visually depicted in the accompanying graph. To preserve linearity within the proposed model, linear regression analysis is employed to determine the combined function. The coefficient of determination (R square) yielded a value of 0.9876, which closely approaches unity. The mean absolute percentage error stood at 2.2%. In the context of a range of actual values under this study of between RM19.70 and RM39.70 per ETF, it means that, on average, the estimates or prediction deviates from the actual value by approximately between RM0.41 to RM0.87 of the actual value. The high R square and low mean absolute percentage error values substantiate the appropriateness of incorporating the regressed linear function into the proposed model. This is illustrated in the following Figure 1.

Figure 1

Combined Transaction Cost



Hence, the transaction cost of the proposed model can be expressed as follows.

$$t(x_j) \quad = \quad \sum_{j=1}^{10} 0.0021 x_j + 18.521 \sum_{j=1}^{n} y_j. \tag{12}$$

With these changes, the study redefined the decision variables and rewritten equation (4) as follows.

$$\text{maximize } f(x) = R_{11}x_{11} + \sum_{j=1}^{10}\{R'_j\ x_j\ -\ t(x_j)\}. \tag{13}$$

R_{11} : *the expected rate of return from Maybank 12-month fixed deposit*

R'_j : *the expected rate of total return, net of forex cost for ETF asset j, j=1, 2, 3, ..., 10*

$t(x_j)$: transaction cost for ETF asset j

Constraints

Risk

Equation (2) was adapted to formulate the risk constraint for our proposed model, which is the formula for MAD. To find the composite MAD of the portfolio, the respective MAD is weighted with the percentage allocation of each asset.

Cardinality

In managing a portfolio, investors may want to limit the number of ETFs they would like to invest in due to cost and limited investment capital.

Investment policies formulation

The study considered total investment amount, minimum allocation into cash, minimum allocation into low-risk assets, and maximum allocation into high-risk ETFs as constraints for the proposed model. Flexibility is accorded to investment policy on the minimum or maximum allocation into an ETF or a category of ETF, given that the ETFs that have been selected for the study are already broadly diversified. Additionally, the risk control element through the MAD constraint is used as the tool to control the level of portfolio risk.

Total investment amount: The total available investment capital must be fully invested and able to cover the transaction costs during the purchase and sale of the ETFs.

Maximum allocation into cash: For the proposed model, the maximum allocation into the Maybank 12-month fixed deposit is set at 5% of the total available investment capital. Having some cash allocation in a portfolio provides instant access to liquidity in the event of unexpected costs that need to be incurred while managing the portfolio. Note that the low liquidity threshold is premised on the high liquidity of the ETFs selected.

Minimum allocation into low-risk assets: As a prudent measure to balance the risk of the portfolio, the proposed model will set a 25% minimum allocation into low-risk assets, namely the bond ETFs and the Maybank 12-month fixed deposits.

Maximum allocation into high-risk assets: To mitigate the systematic risk from the equity market, the maximum equity exposure through the equity-based ETFs is capped at 80%.

Minimum transaction value formulation*:* The minimum transaction unit for US-listed securities, including ETFs, is only one unit. Since this constraint could result in an inexact solution to the total investment amount constraint, we accord relaxation by introducing upper and lower buffers as represented by δ.

Modification to coefficients for Decision Variables

To enforce the minimum transaction value prescribed by equation (20) for the chosen ETFs in this study, modifications have been made to the proposed model, rendering it an integer model. This adaptation involves integrating the price of each ETF as a multiplier for both the objective function's coefficient and, where applicable, the coefficients of the constraints. As a result, the decision variables are converted into integer values, denoting the number of units allocated to each respective ETF. This simplification of the LP model enhances its robustness and overall effectiveness. Except for the cardinality constraints, the modifications have affected all other equations within the proposed model. Based on the formulations of the objective function and constraints and, consequently, the modifications of the equations as presented in equation (26), the proposed MAD MILP model is assembled and can be expressed as follows.

Decision variables:

x_j : the number of units to be invested in asset j, j=1, 2, …, 11.

Objective function:

maximize $f(x)$ =

$$R_{11}x_{11}p_{11} + \sum_{j=1}^{10}\left\{\left(\left(\frac{(1+R_j+d_j)\times(f_1-p)}{f_0} - 1\right)\times 100\right)p_j\, x_j \; - t(x_j)\right\}. \quad (14)$$

Subject to:

	Formulation	*Descriptions*
C1	$\frac{1}{7}\sum_{t=1}^{7}\left\lvert\sum_{j=1}^{11}(r_{jt} - r_j)\frac{x_j p_j}{A_0}\right\rvert \le w,$	*The maximum level of annual risk MAD (%)*
C2	$\sum_{j=1}^{10} y_j \le m$ $y_j(j = 1, 2, 3, \ldots 10) = \begin{Bmatrix} 1 & if\ ETF\ j\ is\ invested \\ 0 & otherwise \end{Bmatrix},$	*Cardinality/ Maximum number of ETFs*

(continued)

	Formulation	Descriptions
C3	$A_o - \delta \leq R_{11}x_{11}p_{11} + \sum_{j=1}^{n} x_j p_j + t(x_j) \leq A_0 + \delta,$	*Total investment amount (RM)*
C4	$x_{11}p_{11} \leq 5\% \times A_0,$	*Maximum allocation in cash (RM)*
C5	$x_5p_5 + x_6p_6 + x_7p_7 + x_{11}p_{11} \geq 25\% \times A_0,$	*Minimum allocation into low-risk assets (RM)*
C6	$x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_8p_8 + x_9p_9 \leq 80\% \times A_0,$	*Maximum allocation in high-risk assets (RM)*
C7	$x_jp_j(= 1,2,3,\ldots,10) \leq My_j,$	*Link constraint*
C8	$x_j \geq 0,$	*Non-negative constraint*

where

r_j : the historical average rate of return of assets j across the observation period

r_{jt} : the historical rate of return of assets j over period *t*, *j*=1, 2, …, *11* and *t*=1, …, *7*.

w : the specified % level of risk desired by an investor

m : Maximum number of ETFs to be invested

A_0 : Total available investment capital.

δ : Buffer to investment capital

RESULTS

Parametric Input & Complete Proposed Model

The author conducted a study using specific data sources and statistical techniques on monthly data points up until May 2023 to create the input for their model. The model was designed to work with a portfolio that could include up to five ETFs. This choice assumed that the initial investment capital, was RM10,000, which allowed self-ascribed investors to acquire up to five ETFs, with the highest value possible. The author assigned a value of 11.02% to parameter representing the annualized risk level associated with a balanced portfolio. This value was derived from the average risk level observed in the balanced equity portfolio category, reported by Morningstar for a 5-year period ending on April 30, 2023 (*Morningstar*, n.d.). To calculate the total returns for each ETF, the author combined the forward dividend yield (12-month forward dividend estimates) for each ETF obtained from Market Chameleon (n.d.) with the annualized price returns of each ETF. The outcomes of this analysis have been compiled and organized in Tables 4 and 5.

Table 4

Parametric Input for the Proposed Model – R'_j, MAD, p_j

x_j	R'_j	$MAD = \frac{1}{7}\sum_{t=1}^{7}\sum_{j=1}^{11} r_{jt} - r_j$	p_j
	The expected rate of total return, net of forex cost for ETF asset j	*The mean absolute deviation for ETF asset j*	*The closing price per unit of ETF j (RM)*
x_1	7.9%	9.3%	1,948.85
x_2	11.0%	16.2%	1,135.05
x_3	6.7%	22.8%	437.38
x_4	3.6%	11.1%	368.48
	0.7%	4.9%	353.11
	1.6%	2.5%	219.63
	1.3%	2.9%	228.70
	3.0%	12.8%	313.63
	4.6%	19.6%	224.28
	5.4%	7.9%	846.94
	3.1%	0.0%	Not applicable

The parameter price, , which were computed in RM term, will be the multiplier to the coefficients of the constraints after modifying the mixed integer form of the LP model to an integer LP model.

Table 5

Parametric Input for the Proposed Model –

m	A_0	M
The maximum number of ETFs to be invested	The total investment capital	The maximum amount that could be allocated to ETF asset j
5	RM10,000	RM9,800
	21	
w	δ	
The maximum level of annual risk	δ *is a small positive value*	
11.02%	$5\% \times A_o$ $= 5\% \times 10{,}000$ $= 500$	

Modeling & Model Verification

The proposed model was executed in stages using simplex LP to solve a series of batches of constraints. In the first batch, constraints related to maximum risk level, portfolio size, total investment amount, link constraint, and non-negativity were selected due to inter-dependencies amongst these constraints. In the second batch, a constraint on the maximum allocation in cash was introduced to assess the reallocation of the investment amount from risk-free assets to higher-risk assets. In the third batch, constraints on the minimum allocation into low-risk assets and the maximum allocation into high-risk assets were introduced. These constraints could be introduced independently as they did not have dependencies among them. The analysis presented outcomes in Table 6, showing the range of returns and corresponding Sharpe ratios. Correspondingly, the results demonstrated that the model successfully generated plausible outcomes and optimized investment portfolios within the specified constraints. The inclusion of the constraint on maximum allocation into fixed deposits in the second stage had minimal impact on the results because of the low weightage assigned to fixed deposits in the overall portfolio. However, the introduction of constraints on allocation to low-risk and high-risk assets in the third stage significantly influenced the optimal solution. These constraints forced the model to allocate the investment capital into low-risk assets even though the total returns were low and to stop the allocation into high-risk assets even though their potential total returns were high. This resulted in changes to both basic and non-basic variables and the optimal value at this stage. The enforcement of diversification and control over exposure to risky assets through constraints 5 and 6 led to a reduction in the optimal portfolio risk level in the complete model, as demonstrated by the outcomes in the analysis table.

Table 6

The Outcome of the Modelling Exercise on Microsoft Excel Solver

Stages	Model Component	Non-basic variables	Risk (%)	Return (%)	Sharpe Ratio
1	Objective function, C1, C2, C3, C7, C8	x_1, x_2, x_{11}	10.95%	8.0%	0.45
2	Objective function, C1, C2, C3, C7, C8, C4	x_1, x_2, x_{11}	10.95%	8.0%	0.45
3	Objective function, C1, C2, C3, C7, C8, C4, C5, C6 – complete model	$x_2, x_{6,}, x_{10,} x_{11}$	10.68%	6.7%	0.33

The complete model, which was established under Stage 3, generated the following optimal solution.

Optimal Solution by Excel Solver
Z = 672, RM672 or 6.7% maximum net total returns on investment.

Non-Basic Variables
x_2 = 5, RM5,675 into iShares Core S&P Mid-Cap ETF
x_6 = 15, RM3,295 into iShares Core 1-5 Year USD Bond ETF
x_{10}= 1, RM847 into SPDR Gold Shares
x_{11}= 178, RM178 into Maybank 12-month FD
y_2, y_6, y_{10}, y_{11}= 1, the corresponding 0-1 variables for selected ETFs

Based on the optimal solution, out of the total investment capital of RM10,700, the amount that goes into ETFs totaled RM9,995, while the balance is to cover the transaction cost.

Basic Variables

$x_1, x_3, x_4, x_5, x_7, x_8, x_9 = 0$, no allocation into SPDR S&P 500 ETF, iShares Core S&P Small-Cap ETF, Vanguard Real Estate ETFs, Vanguard Intermediate-Term Bond ETF, iShares Core International Aggregate Bond ETF, iShares Core MSCI EAFE ETF and iShares Core MSCI Emerging Markets ETF. Variables $y_1, y_3, y_4, y_5, y_7, y_8, y_9$ are the corresponding 0-1 variables for ETFs that are not selected.

Binding and Non-Binding Solutions

The binding solutions of the proposed model are all for the constraints of the 0-1 variables for ETFs that are not selected where the coefficients for y_j are zero, namely $y_1, y_3, y_4, y_5, y_7, y_8$ *and* y_9 . The rest of the variables are non-binding, which means none of these variables satisfies equality to the right-hand side values of the constraints.

CONCLUSION

The study presents a comprehensive methodology and techniques for developing a practical and robust MAD ILP model to optimize a global investment portfolio specifically designed for Malaysian retail investors. The model incorporates constant and piecewise linear transactional fees and foreign currency exchange spreads. It uses a portfolio of ten US-listed ETFs and a Maybank 12-month fixed deposit account. To handle minimum transaction value constraints, the proposed model is transformed into an integer LP model. The transaction cost structure is converted from a non-linear to a linear model component using linear regression. The study verifies the model's effectiveness in optimizing returns within identified constraints through a staged verification process using Microsoft Excel Solver. As constraints increase, the maximum potential return and Sharpe Ratio decrease. The complete model generates an optimal solution with a maximum return of 6.7%, MAD of 10.68%, and a Sharpe Ratio of 0.33, achieved through investments in two ETFs and fixed deposits. The model can easily accommodate revisions in ETF expected returns for portfolio rebalancing, providing refreshed solutions in line with the market outlook. It contributes significantly to investment portfolio optimization for Malaysian retail investors and offers a reliable tool to optimize global portfolios considering unique constraints and transaction costs. Further research can build upon this foundation to explore additional factors and incorporate dynamic elements, enhancing the model's effectiveness in real-world investment scenarios.

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