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AN EMPIRICAL STUDY ON THE CONSTRUCTION OF A NON-CONVEX RISK PARITY PORTFOLIO USING A GENETIC ALGORITHM

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ABSTRACT

Risk-based portfolio optimization has become increasingly crucial due to the limitations and underperformance of traditional Mean-Variance (MV) portfolios. In the context of the Risk Parity (RP) portfolio, capital is allocated in such a way that each asset contributes equally to the overall risk of the portfolio. However, optimizing a non-convex RP portfolio presents significant challenges. While conventional numerical methods can be applied, they often struggle with inefficiency and fail to deliver optimal results. This study addresses these challenges by proposing the Genetic Algorithm (GA) and comparing its effectiveness against Successive Convex for RIsk Parity (SCRIP) to solve the non-convex RP portfolio. Using stock price data from companies listed on the Jakarta Islamic Index (JII), we demonstrate that the GA, although yielding a solution that slightly deviates from the ideal equal-risk-contribution, offers a more efficient and practical approach than SCRIP. Notably, the RP portfolio outperforms both the Equal Weight (EW) and Global Minimum Variance (GMV) portfolios in terms of lower Value at Risk (VaR) and Turnover (TO) ratios. This research contributes to the field by showcasing the GA as a viable tool for overcoming the limitations of traditional optimization methods, particularly in addressing the complexities of non-convex RP portfolios while considering practical investment constraints.

Keywords: genetic algorithm, non-convex optimization, risk parity portfolio, successive convex for risk parity

INTRODUCTION

In the complex world of financial investments, investors continually seek new methods to mitigate risks and maximize returns. One foundational approach to portfolio theory is the Markowitz Mean-Variance (MV) portfolio. This model aims to determine the optimal allocation of assets within a portfolio to either minimize risk for a given expected return or maximize expected returns for a specified level of risk (Markowitz, 1952). Notably, the MV portfolio relies on the estimation of the covariance matrix and expected returns. However, the model is susceptible to error estimations, meaning small changes in these predictions can lead to drastically different asset allocations (Chopra & Ziemba, 1993; Kolm et al., 2014; Michaud, 1989; Rubinstein, 2002). Additionally, MV portfolios often focus on low-variance assets, which contradicts the principles of diversification. While this might not be an issue under normal market conditions, it can present challenges during periods of crisis. For instance, the poor performance of MV portfolios resulted in substantial losses for many investors during the global financial crisis from mid-2007 to early 2009 (Roncalli, 2013). In response to these limitations, the Risk Parity (RP) model was developed. This risk-based optimization method allocates capital to ensure that each asset contributes equally to the overall risk of the portfolio (Bruder & Roncalli, 2012; Maillard et al., 2010).

The core concept of the RP portfolio is to create a portfolio where the risk contribution of each asset is equal. Various methods, such as Sequential Quadratic Programming (SQP), Interior Point Methods (IPM), and Successive Convex Optimization for RIsk Parity Portfolio (SCRIP), can be employed to solve the non-convex RP portfolio under constraints like $\mathbf{1}^\mathbf{T}\mathbf{w} = 1, \mathbf{w} \geq \mathbf{0}$ (Feng & Palomar, 2015; Maillard et al., 2010; Mausser & Romanko, 2014). However, these centralized techniques are computationally expensive and do not consistently perform well for high-dimensional problems (Maillard et al., 2010). For the convex RP portfolio optimization, classical methods such as the Newton method and Cyclical Coordinate Descent (CCD) can be employed (Chaves et al., 2012; Griveau-Billion et al., 2013; Spinu, 2013). Nevertheless, these approaches are unsuitable for more complex problems.

The Genetic Algorithm (GA), a metaheuristic inspired by natural selection, is highly effective in solving non-convex optimization problems (Cavus & Allahham, 2024; Nalini et al., 2024; Qiao et al., 2024; Ranjha et al., 2024; Stefanoni et al., 2024; Zhang et al., 2024). While the convergence of the GA is not always guaranteed, this decentralized method is highly effective at exploring the entire solution space to find global optima or high-quality solutions, particularly for problems with complex objective functions. This paper will demonstrate how the GA can be utilized to address the challenges of optimizing non-convex RP portfolios. Accordingly, it will compare the effectiveness of GA with the SCRIP method, highlighting the limitations of conventional numerical approaches in achieving optimal results for non-convex problems. The structure of the paper is as follows: Section 2 provides a detailed explanation of the non-convex RP portfolio, GA, and SCRIP method: a Successive Convex Optimization (SCO)-based method that solves the non-convex problem by iteratively approximating it with a convex sub-problems. Section 3 outlines the methodology of the empirical study, followed by the presentation of the study's results and discussion in Section 4. Section 5 concludes the paper.

LITERATURE REVIEW

Risk Parity Portfolio

The RP portfolio is a specialized form of the Risk Budgeting (RB) portfolio, designed with the aim of allocating assets in such a way that each asset contributes equally to the overall risk of the portfolio (Maillard et al., 2010). Note that this methodology differs from traditional portfolio optimization strategies in that it focuses not just on maximizing returns or minimizing volatility but instead on balancing the risk contributions of each asset.

To better understand how this works mathematically, the total portfolio risk is expressed as the portfolio's volatility, denoted as $\sigma(w)$ or $R(w)$, which is computed using the portfolio's allocation vector w and the covariance matrix Σ. The expression for total risk is:

$$\sigma(w) = R(w) = \sqrt{w^T \Sigma w} = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1, j \neq \mathrm{i}}^{n} 2 w_i\, w_j \sigma_{ij}}. \tag{1}$$

Here, σ_i represents the volatility of asset i, and σ_{ij} represents the covariance between assets i and j. This equation reflects the total risk of the portfolio based on the individual asset volatilities and their pairwise covariances. Each asset's risk contribution to the portfolio is calculated using:

$$RC_i = \sigma_i(w) = w_i \frac{(\Sigma w)_i}{\sqrt{w^T \Sigma w}} = \frac{w_i^2 \sigma_i^2 + \sum_{j=1, j \neq \mathrm{i}}^{n} w_i w_j \sigma_{ij}}{\sqrt{w^T \Sigma w}}. \tag{2}$$

This equation quantifies the extent to which each asset's risk contributes to the overall portfolio risk, based on both its individual risk and how it correlates with other assets in the portfolio. The non-convex RP portfolio optimization seeks to allocate assets such that the risk contributions of all assets are equal. This objective can be represented as the following optimization problem:

$$\text{Minimize } \sum_{j=1, j \neq \mathrm{i}}^{n} \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2. \tag{3}$$

This minimizes the squared differences between the risk contributions of the different assets, ensuring a more even distribution of risk. Additionally, the constraint $\boldsymbol{w^T 1} = 1$, ensures that the sum of all asset weights equals 1, maintaining a fully invested portfolio. Alternatively, the optimization problem can also be written as:

$$\text{Minimize } \sum_{i=1}^{n} (w_i (\Sigma w)_i - \theta)^2, \tag{4}$$

subject to $\boldsymbol{w^T 1} = 1$, where $\theta = \frac{\sum_{j=1}^{n} w_j (\Sigma w)_j}{n}$ is the target risk contribution for each asset. Both equations (3) and (4) are non-convex, which indicates that the optimization problems are complex due to the potential presence of multiple local minima. This non-convexity arises from the complex relationships between asset weights, volatilities, and covariances. As a result, it is challenging to guarantee that the optimization process will find the global minimum solution, and the algorithm might converge to a suboptimal local solution depending on its starting point. Note that solving such non-convex problems typically requires sophisticated algorithms such as SCO or other meta-heuristics algorithms.

Successive Convex Optimization for Risk Parity Portfolio

The SCRIP algorithm utilizes SCO to solve the non-convex RP portfolio optimization problem. SCO is an optimization technique frequently applied to non-convex problems, where the objective is to find a solution to a function that is not convex. In essence, non-convex functions are challenging since they may have multiple local minima or maxima, making it difficult to identify the global optimum directly. Thus, to address this challenge, SCO works by iteratively approximating the non-convex problem using a series of convex sub-problems. In each iteration, the non-convex problem is replaced with a convex approximation that is easier to solve. The convex approximation is typically constructed using a Taylor series expansion, which provides a local linearization of the non-convex function around the current solution point (Scutari & Sun, 2018). This suggests that instead of directly solving the original non-convex problem, the algorithm solves simpler, convex problems that are easier to oversee. The iterative process in SCO continues until the solution converges to a stationary point, implying no further improvement can be made based on the local gradient (the direction of the steepest descent). However, it is essential to note that while SCO guarantees convergence to a stationary point, it does not guarantee that the solution is the global minimum. This is attributed to the fact that non-convex problems can have multiple stationary points, and SCO may converge to a local minimum rather than a global one.

In the context of RP portfolio optimization, the SCRIP algorithm applies SCO to iteratively adjust the portfolio allocations so that the portfolio's risk is distributed equally among all assets. At each step, the algorithm adjusts the asset allocations based on the convex approximations, moving closer to the desired risk parity distribution. This iterative process continues until the allocations converge, resulting in a portfolio that allocates risk evenly across its assets, which is the primary objective of RP portfolio optimization. The SCRIP algorithm, therefore, offers a practical approach to solving the RP portfolio optimization problem by efficiently managing non-convexity and providing a robust solution that balances the risk contributions of the portfolio's assets. However, further techniques or algorithms may be needed to enhance the solution's accuracy or to explore potential global optima in more complex cases. The general formulation of the RP portfolio optimization problem tackled by the SCRIP algorithm is as follows (Feng & Palomar, 2015):

$$\text{Minimize } R(\mathbf{w}) + \lambda F(\mathbf{w}), \tag{5}$$

subject to $\boldsymbol{w}^{\boldsymbol{T}}\mathbf{1} = 1$, $\boldsymbol{w} \in \mathrm{W}$, and λ is a trade-off parameter. $R(\mathbf{w})$ represents the risk concentration, which can take various forms. One example is $R(w) = \sum_{i=1}^{n}\left(w_i(\Sigma w)_i - \frac{\sum_{j=1}^{n} w_j(\Sigma w)_j}{n}\right)^2 = \sum_{i=1}^{n}\big(g_i(w)\big)^2$ which is equivalent to the equation (4). $F(\mathbf{w})$, on the other hand, is a convex function that captures preferences within the portfolio, such as the expected return, where $F(w) = -\boldsymbol{\mu}^{\boldsymbol{T}}\boldsymbol{w}$. To solve the optimization problem in equation (5), the SCRIP algorithm iteratively solves the following convex sub-problem:

$$\text{Minimize } \left\|\boldsymbol{g}(\boldsymbol{w}^{\boldsymbol{k}}) + \boldsymbol{A}^{\boldsymbol{k}}(\boldsymbol{w} - \boldsymbol{w}^{\boldsymbol{k}})\right\|_2^2 + \frac{\tau}{2}\left\|\boldsymbol{w} - \boldsymbol{w}^{\boldsymbol{k}}\right\|_2^2 + \lambda F(\boldsymbol{w}), \tag{6}$$

subject to $\boldsymbol{w}^{\boldsymbol{T}}\mathbf{1} = 1$, $\boldsymbol{w} \in W$, where $\boldsymbol{g}(\boldsymbol{w}^{\boldsymbol{k}}) = \left[g_1(\boldsymbol{w}^{\boldsymbol{k}}), g_2(\boldsymbol{w}^{\boldsymbol{k}}), \ldots, g_n(\boldsymbol{w}^{\boldsymbol{k}})\right]^T$, and $\boldsymbol{A}^{\boldsymbol{k}} = \left[\nabla g_1(\boldsymbol{w}^{\boldsymbol{k}}), \nabla g_2(\boldsymbol{w}^{\boldsymbol{k}}), \ldots, \nabla g_n(\boldsymbol{w}^{\boldsymbol{k}})\right]^T$, the optimization problem in equation (6) can be rewritten as:

$$\text{Minimize} \frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{Q}^{\boldsymbol{k}}\boldsymbol{w} + \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{q}^{\boldsymbol{k}} + \lambda F(\boldsymbol{w}), \tag{7}$$

subject to $\boldsymbol{w}^{\boldsymbol{T}}\mathbf{1} = 1$, $\boldsymbol{w} \in W$, where $\boldsymbol{Q}^{\boldsymbol{k}} = \mathbf{2}(\boldsymbol{A}^{\boldsymbol{k}})^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{k}} + \tau\boldsymbol{I}$, $\boldsymbol{q}^{\boldsymbol{k}} = \mathbf{2}(\boldsymbol{A}^{\boldsymbol{k}})^{\boldsymbol{T}}\boldsymbol{g}(\boldsymbol{w}^{\boldsymbol{k}}) - \boldsymbol{Q}^{\boldsymbol{k}}\boldsymbol{w}^{\boldsymbol{k}}$. The SCRIP algorithm solves these convex sub-problems sequentially, using them as first-order approximations to the general RP portfolio optimization problem in equation (5). For this study, the SCRIP algorithm was implemented using the `riskParityPortfolio` package in `R` (Cardoso & Palomar, 2021).

Genetic Algorithm

The GA is a metaheuristic optimization technique that simulates the process of biological evolution to iteratively search for optimal solutions (Abdel-Basset et al., 2018). Introduced by John Holland in the 1960s, GA starts with an initial population of candidate solutions (chromosomes) and uses operators like selection, crossover, and mutation to evolve the population (Holland, 1992). In each generation, solutions are evaluated using a fitness function, and the most suitable individuals are selected to create offspring through crossover and mutation, promoting diversity. Before running the algorithm, the problem's variables, encoding, fitness function, genetic operators (along with their probabilities), and convergence criteria must be defined. Then, the process repeats until a stopping condition, such as a maximum number of generations or a satisfactory fitness level, is reached, progressively refining the population toward an optimal or near-optimal solution.

GA uses various selection methods, including linear-rank selection, non-linear rank selection, proportional (roulette wheel) selection, tournament selection, fitness-proportional selection with linear scaling, and fitness-proportional selection with Goldberg's sigma truncation scaling (Rangel-Merino et al., 2005). One of the key operators, crossover, combines two parent chromosomes to generate a new offspring chromosome with a specified crossover probability, P_X references (Michalewicz, 1994). Several crossover techniques are employed, such as single-point crossover, multi-point crossover, linear crossover, uniform crossover, and others, including arithmetic, queen-bee, blend, and simulated binary crossovers (Filograsso & di Tollo, 2023). Meanwhile, mutation, another essential operator, prevents the population from becoming uniform and stagnating by randomly altering the genes of a selected chromosome with a mutation probability, P_Y.

The GA and other metaphor-based meta-heuristic algorithms, like Swarm Intelligence (SI) algorithms, were flexible tools capable of addressing various portfolio optimization challenges. This includes unconstrained problems with downside risk measures and those involving various risk metrics like MV, semi-variance, mean absolute deviation, and variance with skewness, as well as cardinality constraints (Abdurakhman et al., 2024; Arnone et al., 1992; Chang et al., 2009). GA was also adapted to more complex scenarios, such as incorporating constraints related to cardinality and market capitalization (Soleimani et al., 2009). When stock returns were modeled with fuzzy numbers to account for uncertainty, GA proved effective in portfolio optimization (Fatoni & Kusumawati, 2022; Kar et al., 2019). Furthermore, GA excelled in solving problems involving cardinality constraints and effectively managed outliers and deviations from normal distribution assumptions (Mishra et al., 2012; Rosadi et al., 2020). Over time, GA was enhanced to tackle multi-objective optimization problems, with the NSGA-II being a successful approach for such challenges (Anagnostopoulos & Mamanis, 2010; Lwin et al., 2017). Moreover, its performance was further improved through hybrid methods, such as combining GA with the ϵ-constraint method to optimize the Mean-Absolute-Semi-Deviation (MASD)-Skewness portfolio under uncertain conditions or pairing it with the Firefly Algorithm (FA) for optimizing the MV-Skewness portfolio (Chen et al., 2018; Mehlawat et al., 2021). Additionally, hybridizing GA with Local Search (LS) proved successful in finding solutions for long-only and short-term portfolio problems (Hochreiter, 2015).

For non-convex RP portfolio optimization, GA aims to identify the optimal solution by selecting the best chromosome. Each chromosome in generation g^{th} is represented as $C_i^g = \{c_1^g, c_2^g, \dots, c_i^g, \dots, c_n^g\}$, where $c_i^g \in \mathbb{R}$ denotes the allocation of asset i, for $i = 1,2,\dots,n$. The objective function evaluates the performance of candidate solutions, and for non-convex portfolio optimization, the best chromosome minimizes the objective function (4). The fitness function is used to assess the quality of each chromosome, ranking them such that more optimal solutions are more likely to reproduce and generate superior offspring through selection, crossover, and mutation. The solution must also satisfy the equality constraint, which can be managed using techniques such as penalty functions, appropriate decoders, or specialized operators. In this case, a penalty term is introduced to penalize solutions that deviate from the equality constraint, encouraging the algorithm to find solutions that closely meet it. The fitness function, defined as the sum of the deviations of each risk contribution from the mean risk and the absolute difference between total allocations and 1, is minimized and represented as follows:

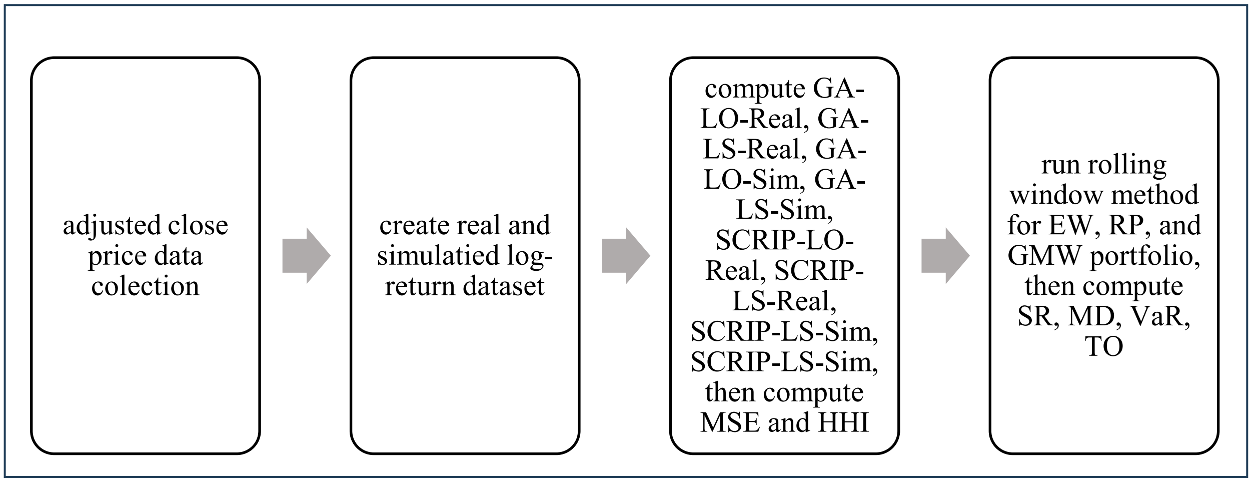
$$f(\text{w}) = \sum_{\text{i}=1}^{\text{n}} \left(w_i(\Sigma w)_i - \frac{\sum_{j=1}^{n} w_j(\Sigma w)_j}{n} \right)^2 + \left|\sum_{i=1}^{n} w_i - 1\right|. \tag{8}$$

METHODOLOGY

The empirical study will evaluate the performance of GA in optimizing both long-only and long-short non-convex RP portfolios using simulated and real-world datasets and compare its effectiveness with the SCRIP method. The study's methodology is illustrated in the diagrams in Figure 1. The real-world dataset includes daily returns of the adjusted closing prices for seven companies consistently listed on the Jakarta Islamic Index (JII) from May 1, 2020, to May 1, 2024, sourced from Yahoo! Finance via the `quantmod` package in `R` (Ryan et al., 2023). The companies considered—PT Adaro Energy Tbk (ADRO), PT AKR Corporindo Tbk (AKRA), PT Indofood CBP Sukses Makmur Tbk (ICBP), PT Kalbe Farma Tbk (KLBF), PT Telkom Indonesia (TLKM), PT United Tractors Tbk (UNTR), and PT Unilever Indonesia Tbk (UNVR)—have been part of the JII for the past decade (2010-2020). These companies are included in the Indonesian Sharia Stock Index (ISSI), meet high market capitalization and transaction volume criteria, and comply with Sharia principles, such as having a debt ratio below the maximum limit set by DSN-MUI and ensuring operational activities align with Sharia guidelines. For the simulation datasets, asset returns are generated using a multivariate normal distribution with a mean of 0, and the covariance matrix is based on the sample covariance of daily returns for the seven companies from May 1, 2020, to May 1, 2024. A noise factor is added to each asset, distributed as univariate normal with a variance of 0.1 times the variance of the respective asset.

Figure 1

Methodology Flowchart



The optimal asset allocations for the long-only RP portfolio are determined using a GA, which is configured with the parameters detailed in Table 1 and the `GA` package: linear scaling selection, arithmetic crossover, and uniform random mutation (Scrucca, 2013). In general, GA implementations tend to use a higher crossover probability (typically between 0.75 and 0.95) and a lower mutation probability (ranging from 0.005 to 0.01) to ensure an effective balance between exploring the solution space and refining the current solutions (Grefenstette, 1986; Patil & Pawar, 2015). In this study, a crossover probability of 0.9 is selected to promote genetic diversity, facilitating the search for a broad range of potential solutions. Meanwhile, a mutation probability of 0.01 is selected to focus on fine-tuning existing solutions, thereby enhancing the precision of the optimal allocations. These values were derived through trial and error, aiming to strike a balance between exploration (discovering new, potentially better solutions) and exploitation (refining the current promising solutions). In addition, to enforce the equality constraint that ensures the sum of the portfolio weights equals 1 (i.e., fully invested), a penalty term is introduced into the GA optimization. This penalty discourages solutions that deviate from the constraint, making the algorithm more likely to identify solutions that adhere to the required allocation condition as closely as possible.

Table 1

Parameters for GA

Parameter	Value
Number of populations	100
Crossover probability	0.9
Mutation probability	0.01
Elitism	10
Maximum iterations	500

When extending the analysis to the long-short RP portfolio, which allows both long and short positions, the risk profile changes. Short positions are inherently riskier than long positions, as the potential losses from short selling are unlimited if the price of the asset rises unexpectedly (Kim et al., 2016). Therefore, to mitigate such risks, financial advisors generally recommend limiting the proportion of short positions in a portfolio, typically to 0% to 5% of the total portfolio value or up to 10% for more risk-tolerant investors. For the long-short RP portfolio, GA is used to compute the optimal allocations, with the same GA parameters applied as for the long-only portfolio. However, for the long-short case, each stock is assigned a lower allocation bound of -0.3 (allowing up to 30% short position) and an upper allocation bound of 1 (for long positions). This ensures the flexibility to manage both long and short-term investments while maintaining a structured approach to portfolio risk. Accordingly, this dual application of GA to both long-only and long-short RP portfolios allows for a robust assessment of optimal asset allocations, accounting for varying levels of risk and constraints imposed by different market conditions.

To assess the performance of GA and SCRIP in optimizing the non-convex RP portfolio, two key metrics are utilized: Mean Squared Errors (MSE) and Herfindahl-Hirschman Index (HHI). MSE is a statistical measure that evaluates the discrepancy between the actual and target relative risk contributions of each asset in the portfolio. The target relative risk contribution for each asset is set to $1/n$, where n is the total number of assets in the portfolio. This metric quantifies how closely each asset's risk contribution aligns with the ideal allocation of risk, with smaller MSE values indicating better alignment between the actual and desired risk distribution. Thus, by minimizing MSE, the optimization process aims to achieve a portfolio where all assets contribute proportionally to the total risk, as defined by the RP framework. HHI is used to measure the concentration of risk within the portfolio. A higher HHI indicates that the portfolio's risk is concentrated in a few assets, suggesting a less diversified portfolio with higher exposure to specific risks. On the other hand, a lower HHI value suggests a more diversified portfolio with a more equal risk distribution across all assets (Kaucic, 2019). In this context, HHI is an essential metric for evaluating the effectiveness of the optimization algorithm in spreading risk evenly among the portfolio assets.

To compare the performance of the optimized RP portfolios against standard portfolio allocation strategies such as the Equal Weight (EW) and Global Minimum Variance (GMV) portfolios, the rolling window method with two different look-back rolling windows, namely (a) 12*20 days and (b) 24*20 days are employed. This method helps account for changes in market conditions by frequently updating the covariance matrix, which is crucial for accurately estimating the risk and correlations between assets over time. In particular, the rolling-window approach allows the portfolio's performance to be re-evaluated periodically, ensuring that the optimization adapts to shifts in asset returns and risk dynamics. Additionally, walk-forward analysis is used to evaluate the robustness and out-of-sample performance of the optimized portfolios. In this approach, portfolio allocations are calculated using a training dataset, and the resulting allocations are then tested on a separate testing subset. This ensures that the evaluation periods do not overlap, reducing the risk of overfitting and providing a more reliable measure of performance. The walk-forward method is applied sequentially across various datasets to simulate the portfolio's performance over time, helping to assess its ability to adapt to evolving market conditions. The analysis is conducted using the `portfolioBacktest` library (Palomar & Zhou, 2022).

Various performance metrics are used to evaluate the portfolios during this process, including the Sharpe Ratio (SR), which measures risk-adjusted returns, helping to determine how well the portfolio compensates for the risk taken. Meanwhile, Maximum Drawdown (MD) quantifies the largest peak-to-trough loss in the portfolio, providing insights into the downside risk. In addition, Value at Risk (VaR) estimates the potential loss in portfolio value under normal market conditions at a given confidence level. At the same time, Turnover (TO) measures the frequency of portfolio rebalancing, indicating the portfolio's trading activity and potential transaction costs.

EMPIRICAL STUDY

Result

The descriptive statistics for the simulated and real datasets are summarized in Table 2. With the exception of UNVR, which has a negative average return, most of the stocks in the real datasets, which comprise returns from seven stocks, demonstrate positive returns on average, according to the analysis. Although ADRO has the highest average return of any company, its huge return variance indicates that it also carries the highest risk. This implies that ADRO is the riskiest asset in the dataset since its performance is more volatile than that of the other stocks. As observed by their kurtosis values of more than 3, several equities, such as ICBP and KLBF, display traits that depart from the assumption of a normal distribution. Kurtosis quantifies a distribution's "tailedness" in statistical terms. A kurtosis value greater than 3 indicates that the stock returns exhibit higher peaks and heavier tails than a normal distribution, indicating that these stocks are more prone to extreme outcomes (either positive or negative) than would be expected from a normally distributed variable.

In the simulation dataset, which was generated under controlled conditions, several stocks reveal negative average returns, including ADRO, AKRA, TLKM, and UNTR. This contrasts with the real dataset, where most stocks have positive returns. Among the simulated stocks, ICBP has the highest average return, much like in the real dataset. However, ADRO remains the riskiest stock in this simulated set as well, with the highest variance in its returns.

The main distinction between the simulation and the real data is that all simulated stock returns are made to follow a normal distribution, meaning that their kurtosis values are smaller than 3. This suggests a decreased chance of extreme deviations in the simulated returns. To comprehend how the assets perform under various assumptions and market conditions, it is crucial to distinguish between the real and simulated datasets. With its controlled features, simulated data offers a helpful foundation for evaluating portfolio optimization techniques away from the noise and unpredictability of actual market swings. Accordingly, the study can evaluate the effectiveness of the optimization strategies under various data kinds and risk profiles by comparing the performance of stocks in the two conditions.

Table 2

Descriptive Statistics of Stock Return

Stock	Min	Max	Mean	Variance	Standard Deviation	Skewness	Kurtosis
ADRO-Real	-0.07257	0.15251	0.00164	6.813e-04	0.02610	0.45166	2.16879
AKRA-Real	-0.08100	0.09411	0.00156	6.132e-04	0.02476	0.36181	1.73067
ICBP-Real	-0.09293	0.09531	0.00021	2.760e-04	0.01661	0.08788	4.37845
KLBF-Real	-0.07091	0.16319	0.00013	3.730e-04	0.01931	0.82107	6.07436
TLKM-Real	-0.06946	0.09042	0.00013	2.882e-04	0.01698	0.37268	2.76873
UNTR-Real	-0.07232	0.14938	0.00089	5.322e-04	0.02307	0.49378	3.09934
UNVR-Real	-0.11321	0.12323	-0.00101	3.922e-04	0.01980	0.77224	5.86068
ADRO-Sim	-0.09129	0.09160	-0.00081	7.487e-04	0.02736	0.01509	-0.06638
AKRA-Sim	-0.07947	0.08265	-0.00097	6.945e-04	0.02635	-0.01094	-0.01361
ICBP-Sim	-0.04963	0.04708	0.00057	2.773e-04	0.01665	-0.04383	-0.17790
KLBF-Sim	-0.05864	0.06663	0.00046	3.820e-04	0.01955	0.04267	-0.00408
TLKM-Sim	-0.05571	0.06590	-0.00042	3.207e-04	0.01791	-0.05346	-0.04833
UNTR-Sim	-7.702e-02	6.343e-02	-2.415e-04	5.642e-04	0.02375	-0.13560	0.05632
UNVR-Sim	-6.894e-02	7.341e-02	9.348e-05	4.256e-04	0.02069	0.00472	0.10356

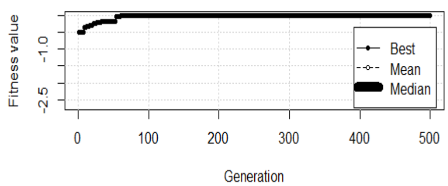
Figure 2 illustrates the fitness value convergence for the long-only and long-short non-convex RP portfolios using GA on both real and simulated datasets. GA-LO-Real and GA-LO-Sim refer to GA for long-only non-convex RP portfolios on real and simulated datasets, respectively. After approximately 50 generations, the fitness value stabilizes for the long-only RP portfolio and improves further for the long-short RP.

Tables 3 and 4 summarize the risk contributions and asset allocations for long-only and long-short non-convex RP (3) using GA and SCRIP. The results reveal that GA effectively optimizes both portfolios, achieving nearly equal risk contributions across assets. In the long-only RP portfolio, all stocks except UNTR have similar risk contributions, while the long-short RP portfolio demonstrates minimal variation, with AKRA, TLKM, and UNTR as exceptions. Using the same datasets, SCRIP for both portfolio types (with $\lambda = 0$) yields identical results, with each asset's risk contribution equal to 0.14286. Despite ICBP having the lowest risk, its allocation contributes nearly the same risk as ADRO, the highest-risk stock.

Figure 2

Convergence Plot of Fitness Value of The RP Portfolio with GA (a) GA-LO-Real; (b) GA-LS-Real; (c) GA-LO-Sim; (d) GA-LS-Sim.

A.



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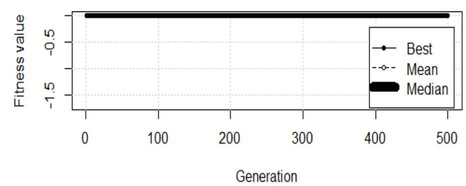
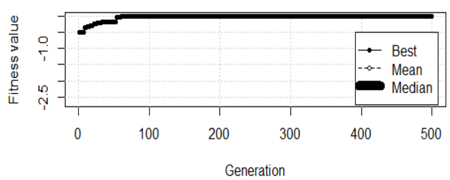


C.



D.

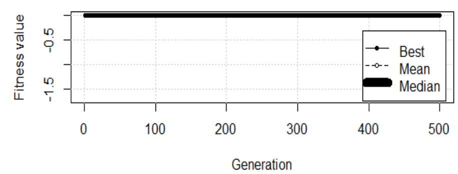


Table 3

Relative Risk Contribution

Relative Risk Contribution	ADRO	AKRA	ICBP	KLBF	TLKM	UNTR	UNVR
GA-LO-Real	0.03256	0.14155	0.01853	0.11536	0.14786	0.41949	0.12466
GA-LS-Real	-0.04259	0.41795	-0.00532	0.04591	0.34338	0.20948	0.03119
SCRIP-Real	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286
GA-LO-Sim	0.03126	0.14645	0.01542	0.10468	0.14204	0.42853	0.13163
GA-LS-Sim	-0.03871	0.42568	-0.00518	0.03917	0.34009	0.20647	0.03248
SCRIP-Sim	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286

Table 4

Asset Allocation

Allocation	ADRO	AKRA	ICBP	KLBF	TLKM	UNTR	UNVR
GA-LO-Real	0.02718	0.12865	0.04453	0.14965	0.19568	0.27903	0.17529
GA-LS-Real	-0.05774	0.29915	-0.02528	0.09053	0.38944	0.20801	0.09589
SCRIP-Real	0.10155	0.12029	0.18148	0.15223	0.16833	0.11612	0.16000
GA-LO-Sim	0.02718	0.12865	0.04453	0.14965	0.19568	0.27903	0.17529
GA-LS-Sim	-0.05774	0.29915	-0.02528	0.09053	0.38944	0.20801	0.09589
SCRIP-Sim	0.10130	0.11255	0.18968	0.16107	0.16707	0.11140	0.15693

Discussion

Table 5 summarizes the performance comparison between GA and SCRIP on real and simulated datasets. The effectiveness of an algorithm is often measured by how well it achieves desired outcomes, such as equal risk contributions or minimizing MSE. In this study, the goal is to achieve an HHI value of 0.14286, representing equal risk distribution across seven assets. While GA produces a slightly higher MSE than SCRIP, its HHI value is close to the target, making it a viable alternative for RP portfolio optimization. This indicates that GA effectively distributes risk, even if it does not achieve the exact MSE reduction as SCRIP.

SCRIP allocates assets to achieve equal risk contributions, resulting in a perfect HHI of 0.14286 for both long-only and long-short RP portfolios, effectively balancing risk across assets. However, SCRIP struggles with non-convex constraints, which are common in real-world financial problems, complicates the optimization process and limits its ability to find the global optimum (Feng & Palomar, 2015). In such cases, the optimization process becomes more complex, and the algorithm's capacity to find the true global optimum may be hindered by multiple local minima. Considering these constraints, GA offers a valuable alternative by approximating RP solutions while also being more adaptable to non-convex problems.

Despite its slightly higher MSE, GA's performance remains competitive, and it demonstrates greater flexibility in overseeing non-convex constraints, which are common in practical portfolio optimization scenarios. For future research, a deeper evaluation of GA's capabilities in optimizing non-convex RP portfolios with real-world constraints would be beneficial. Additionally, comparing GA's performance against other meta-heuristic algorithms could provide further insights into the strengths and weaknesses of GA in more complex portfolio optimization tasks. Moreover, the application of GA to larger datasets could also offer a better understanding of its scalability and performance in real-world portfolio management scenarios.

Table 5

The Mean Square Error (MSE) and Herfindahl-Hirschman Index (HHI) of Relative Risk Contribution

Portfolio	MSE	HHI
GA-LO	0.01504	0.24812
GA-LS	0.02836	0.34140
SCRIP	2.558e-18	0.14286
S-GA-LO	0.01599	0.25476
S-GA-LS	0.02868	0.34361
S-SCRIP	1.609e-18	0.14286

Table 6 indicates that the RP portfolio generally outperforms others, though not consistently, across all metrics. In both real and simulated datasets, the RP portfolio has a slightly lower SR than the GMV portfolio and a higher MD ratio than the EW portfolio. However, it excels in VaR and TO ratios, which are the lowest among the portfolios, indicating better risk management and operational efficiency. In particular, the low VaR suggests that the RP portfolio carries less risk of significant losses, making it a more conservative investment strategy that is resilient to market shocks by limiting potential losses during adverse conditions. This risk reduction is key to the RP portfolio's design, balancing risk contributions and avoiding overexposure to any single asset.

The RP portfolio's low TO ratio emphasizes stability and reduced trading activity. A lower TO ratio indicates steady holdings, lowering transaction costs and minimizing the impact of frequent rebalancing. This conservative approach, along with its low VaR and minimal TO, makes the RP portfolio a safer, more stable choice for risk-averse investors seeking long-term returns without the volatility of the EW or GMV portfolios. While the RP portfolio may not outperform the GMV or EW portfolios in metrics like SR and MD ratios, its lower VaR and TO ratios highlight its strong risk management and stability, offering a well-rounded strategy focused on minimizing risk and delivering consistent returns.

Table 6

*The Portfolio Performance with Look-Back Rolling Window (A) 12*20 Days; (B) 24*20 Days*

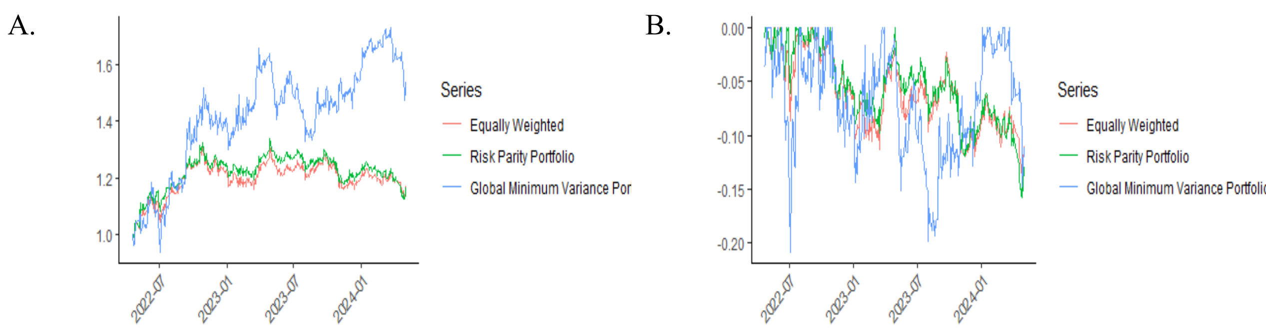
	EW		RP		GMV	
Metric	JII	S-JII	JII	S-JII	JII	S-JII
SR[a]	1.053e+00	1.053e+00	8.220e-01	8.220e-01	1.481e+00	1.481e+00
MD[a]	1.396e-01	1.396e-01	1.5682e-01	1.568e-01	1.941e-01	1.941e-01
VaR (0.95)[a]	1.426e-02	1.426e-02	1.401e-02	1.401e-02	2.723e-02	2.723e-02
TO[a]	1.683e-03	1.683e-03	1.635e-03	1.635e-03	1.131e-02	1.131e-02
SR[b]	6.069e-01	6.069e-01	6.016e-01	6.016e-01	8.833e-01	8.833e-01
MD[b]	1.396e-01	1.396e-01	1.580e-01	1.580e-01	2.081e-01	2.081e-01
VaR (0.95)[b]	1.405e-02	1.405e-02	1.376e-02	1.376e-02	2.950e-02	2.950e-02
TO[b]	1.533e-03	1.533e-03	1.532e-03	1.533e-03	7.905e-03	7.905e-03

The MD and cumulative returns charts in Figure 3 highlight the RP portfolio's superior risk management compared to the GMV and EW portfolios. While the GMV portfolio presents higher total returns, its greater volatility exposes it to sharp declines, making it riskier for conservative investors. In contrast, the RP portfolio outperforms the EW portfolio and offers greater stability over time, with consistent growth and lower volatility. This makes the RP portfolio more suitable for investors seeking a balanced, less risky strategy, as its design minimizes the chances of large declines by ensuring equal risk distribution across assets.

The MD chart tracks the largest peak-to-trough loss during a period, highlighting each portfolio's risk exposure. A larger negative value indicates a greater loss, which can concern risk-sensitive investors. The RP portfolio exhibits a stable MD, remaining less volatile than the GMV and EW portfolios, which experience steep declines at times. This stability suggests that the RP portfolio is less vulnerable to significant losses during market downturns. Overall, the RP portfolio offers better long-term stability, consistent returns, and more effective risk management, making it a preferable choice for risk-averse investors.

Figure 1

The Portfolio Performance over Time (A) Cumulative Return; (B) Drawdown



CONCLUSION

In this paper, the long-only and long-short non-convex RP portfolio optimization problems are solved by comparing the effectiveness of GA and SCRIP. GA offers a nearly ideal RP solution with minor variations, but SCRIP delivers an optimal solution by spreading risk contributions equally. Furthermore, GA proves to be an efficient alternative for optimizing both long-only and long-short RP portfolios, making it a valuable addition to existing approaches. It is particularly suited for portfolios with non-convex real-world constraints, surpassing SCRIP's capabilities, even with large datasets. In addition, the rolling-window analysis suggests that the RP portfolio, with lower VaR and TO ratios, outperforms the EW and GMV portfolios, though not across all metrics. However, the study's reliance on historical data may not fully reflect dynamic market conditions, and the methods' performance could vary under varying conditions. Therefore, future research should address these limitations and evaluate GA on more complex problems with varying objectives and constraints, comparing it with other hybrid or meta-heuristic methods.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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