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Designing Bayesian Modified Group Chain Sampling Plan for Quality Regions Based on Average Number of Defectives

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Abstract

Acceptance sampling provides a mid-way between zero and 100% inspection, which can provide a statistically reliable inference on deciding whether or not to accept the entire lot based on a sample inspection. If the prior information is available then according to experts, Bayesian approach is the best approach to reach a correct decision. Based on the Bayesian approach, this study proposes a Bayesian modified group chain sampling plan (BMGChSP) to estimate the average number of defectives. The Poisson distribution is used to estimate the average number of defectives and gamma as a prior distribution for the average probability of acceptance. Instead of basing a sampling plan one point-wise description of quality like the conventional plans, the proposed plan uses quality range for a wider coverage and offers better protection to both consumer's and producer's. In this paper by considering consumer's risk and producer's risk, quality regions are estimated for the average probability of acceptance. For all quality regions, acceptable quality level (AQL) and limiting quality level (LQL) are used to find design parameters for BMGChSP. Where AQL is associated with consumer's risk and LQL is associated with producer's risk. The values based on all possible combinations of design parameters for BMGChSP are tabulated and inflection points are found. Based on the minimum number of defective the finding exposes that BMGChSP is a better substitute for industrial practitioners. In comparison study by OC curves, it is concluded that the proposed BMGChSP plan gives a smaller number of defective than the existing BGChSP.

Keywords: Acceptance sampling, gamma, Poisson, prior distribution

1. Introduction

Quality is not just an option or aim for companies, but also a necessity for businesses in the world market. There are two important techniques for quality assurance: one is statistical process control and the other is acceptance sampling. Acceptance sampling is a very common technique in which a decision is made about a lot under inspection to either accept or reject based on

representative samples (Montgomery 2009). During World War II, the US military introduced acceptance sampling to test the reliability of bullets. If no bullets were tested in advance (zero inspection), then there may be a possibility of providing malfunctioned bullets to the army. However, such testing is destructive, thus making it impossible to test each bullet (100% inspection) and owing to the urgency in supplying the bullets, only a representative sample of the bullets were tested. Acceptance sampling provides a mid-way between zero and 100% inspection, which can provide a statistically reliable inference on deciding whether or not to accept the entire lot based on a sample inspection.

Experimenters cannot carry out an almost 100 percent inspection because it is more expensive and time consuming than inspecting several units (sample). Thus, acceptance sampling provides the tools to deal with such situations. For instance, it is extremely destructive to test each object when electro-product is tested (bulbs, fans, tube lamps and mobiles). It is therefore not feasible for the manufacturer to check the quality and reliability of all products. In such situation, acceptance sampling is helpful in inspection or testing of some products. Over the decades multiple sampling plans have been proposed in literature. The development of these plans is driven by three factors, the first being a smaller sample size, n. The smaller sample size reduces the inspection time and hence minimizes the cost. The second factor is the probability of lot acceptance (PA). The sampling plan with a lower PA for bad lots is always the consumer's priority as it reduces their risk of receiving defective products. The third factor is a platform for multiple inspections, where the number of products is placed into groups and inspections for each group are constructed simultaneously. This technique of group chain reduces the cost and inspection time.

Some studies consider another factor in developing a sampling plan called product quality variation. Their studies employed prior information to construct Bayesian sampling plans for SSP and ChSP only, without considering the platform for multiple inspections. If past information about the product is available, then Bayesian plans can be used to make a decision. An efficient quality improvement program can increase productivity at a reduced cost (Latha and Arivazhagan 2015).

The first time Dodge (1955) introduced a chain sampling plan for inspection by considering cost as a linear function of p. To reduce the average cost, Hald (1964) developed a single sampling plan (SSP) for attribute. Latha and Suresh (2002) defined the plan for construction and performance measure by using gamma prior in advance for the BChSP. For construction and performance assessment for the quality region, Latha and Arivazhagan (2015) addressed the Bayesian double sampling plan by using beta prior.

For the Pareto distribution, the efficient group acceptance sampling plan (GASP) was introduced by Mughal and Aslam (2011). If for inspection multiple testers are available, then group acceptance sampling plans are used to inspect more than one product at once. In group sampling plans, total sample size is divided into equal groups. The evaluation of the design parameters for the economic reliability group acceptance sampling plan was carried out by Mughal and Aslam (2011). For the given sample size, producer's risk and acceptance numbers, they obtained a small termination time. It showed that the less test termination time required in the proposed plan was shorter than that of the plan established by Rao (2009). Various combinations of design parameters were used to make comparisons between Poisson and the weighted Poisson distributions. A more suitable selection of OC curve tables was also presented. Moreover, Aslam et al. (2011) proposed group sampling plan for the Rayleigh distribution. The findings in all these studies, group sampling plans perform much better than the old SSP in aspects of reducing inspection time and cost.

Jamaludin et al. (2016) developed a modified group chain sampling plan (MGChSP) for truncated life test when the lifetime of a product follows Rayleigh distribution. Small numbers of

groups and OC curve values are found for pre-specified consumer's risk, mean ratio and test termination time. Mughal (2018) worked on MGChSP by considering several values of the proportion of defectives and for pre-specified consumer's risk. He reports the PA and the small number of groups to reduce sample size. Aziz et al. (2017) considered the generalized MGChSP for Pareto distribution of second kind based on non-symmetrical data and compare the performance with Mughal and Aslam (2011).

Based on Mughal and Aslam (2011), a BGChSP was proposed for binomial distribution for the average PA with beta as a prior distribution (Hafeez and Aziz 2019). With the growing demands on customer quality and new product technology innovation, many current quality assurance methods and strategies need to be updated. Many researchers work on MGChSP for the proportion of defectives, but they consider only binomial distribution and estimate pointwise quality measures based on current information. Hafeez and Aziz (2019) plan was extended for BMGChSP by Hafeez and Aziz (2022). For same design parameters, their proposed plan gives less proportion of defectives then existing plan. This study is limited to the average number of defectives and a BMGChSP is developed to estimate the average proportion of defectives. By using quality region approach four quality regions are estimated in this paper that satisfy both risks. For each quality region acceptable quality level (AQL) and limiting quality level (LQL) are defined. Where AQL is associated with consumer's risk and LQL is associated with producer's risk. To designs BMGChSP, indexed parameters are consumer's risk (α), producer's risk (β), prior shape parameter (s), preceding lots (i), and available testers (r).

2. Methodology

2.1. Operating procedure

The modified group chain sampling plan (MGChSP) procedure is based on the following steps:

1. Inspect a sample of size n from current lot and divide it to an optimal number of g groups and allocate r items to each group, that is the required sample size $n = rg$.

2. Count the number of defective d.

3. Accept the lot if $d = 0$ in current sample and immediately preceding i samples have no defective, i.e., $d_i = 0$.

4. Accept the lot if $d = 0$ in current sample and preceding i samples have only one defective, i.e., $d_i = 1$.

5. Reject the lot if more than zero defectives are found in the current lot $d \geq 1$.

All the above steps can be summarized in a flow chart, as shown in Figure 1.

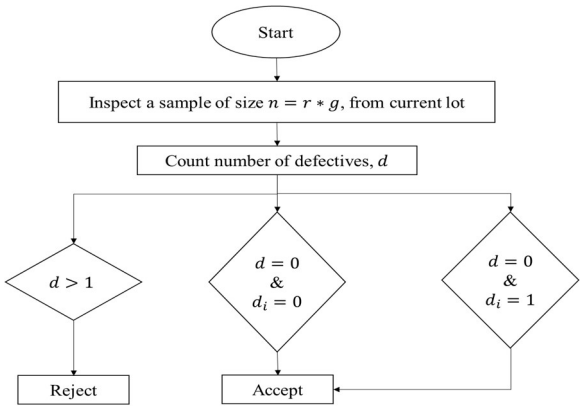


Figure 1. Operating procedure of MGChSP for $i = 2$

For MGChSP, the procedure can be illustrated through a tree diagram for $i = 2$ in Figure 2, where defective and non-defective products are denoted by D and $\bar{D}$, respectively.

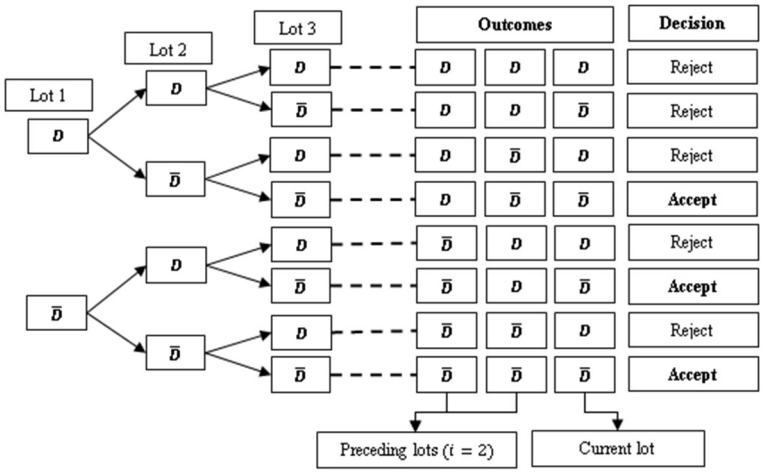


Figure 2. Tree diagram of MGChSP for $i = 2$

From Figure 2, the outcomes that meet acceptance criteria for $i = 2$ in chain sampling are $\{D\bar{D}\bar{D}, \bar{D}D\bar{D}, \bar{D}\bar{D}\bar{D}\}$. From the tree diagram, we can observe that MGChSP has three acceptance criteria (AC) that is tighter than the existing BGChSP. The PA for MGChSP can be written in the following form.

$$L(p)_{MGChSP} = P_{1,(r*g)}P_{0,(r*g)}P_{0,(r*g)} + P_{0,(r*g)}P_{1,(r*g)}P_{0,(r*g)} + P_{0,(r*g)}P_{0,(r*g)}P_{0,(r*g)}, \quad (1)$$

$$L(p)_{MGChSP} = \left(P_{0,(r*g)}\right)^3 + 2P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2. \quad (2)$$

The usual expression of the PA for MGChSP based on (2) for $i = 2$ is

$$L(p)_{MGChSP} = \left(P_{0,(r*g)}\right)^{i+1} + iP_{1,(r*g)}\left(P_{0,(r*g)}\right)^i. \quad (3)$$

When developing the procedures, $L(p)$ can be calculated for the chain acceptance sampling plans, with the assumption that the underlying distribution for the plan is following either binomial or Poisson distribution (Latha and Arivazhagan 2015, Hafeez and Aziz 2019, Rosaiah and Kantam 2005, Suresh and Sangeetha 2005). The Poisson distribution is applicable for the proposed plans with the following conditions:

- The experiment has a smaller probability of defectives, i.e., $np < 0.10$.
- The sample fraction is less than 10%.
- The outcomes of the lot consist of identical and independent products.

The Poisson distribution has sample size n, and parameter $\mu = np$ be the average number of defectives. In group chain $n = rg$, therefore Poisson distribution function can be written as

$$P(d) = \frac{e^{-rgp}(rgp)^d}{d!}. \tag{4}$$

By substituting $d = 0$ and $d = 1$, the PA from (4) is obtained as follows:

$$P_0 = e^{-rgp}, \tag{5}$$

$$P_1 = rgpe^{-rgp}. \tag{6}$$

Therefore $n = rg$ and $\mu = np = rgp$, PA can be written by putting (5) and (6) in (3), after solving we obtain

$$L(p)_{MGChSP} = e^{-rgp(i+1)} + irgpe^{-rgp(i+1)}. \tag{7}$$

As Poisson distribution and gamma distribution belongs to exponential family therefore, we can use gamma as a prior distribution. Hence the PDF of the gamma distribution for unknown parameter p is

$$f(p) = \frac{t^s}{\Gamma(s)} p^{s-1} e^{-tp}, \tag{8}$$

with $s > 0$ shape parameter, $t > 0$ rate parameter and mean $\mu = s/t$. For BMGChSP, the general equation used in Bayesian approach is

$$P = \int_{-\infty}^{+\infty} L(p)_{MGChSP} f(p) dp. \tag{9}$$

After replacing (7) and (8) in (9), we obtain

$$P = \int_0^{\infty} \left[e^{-rgp(i+1)} + irgpe^{-rgp(i+1)} \right] \frac{t^s}{\Gamma(s)} p^{s-1} e^{-tp} dp \tag{10}$$

$$P = \frac{t^s}{\Gamma(s)} \left[\frac{\Gamma(s)}{(rg(i+1)+t)^s} + irg \frac{\Gamma(s+1)}{(rg(i+1)+t)^{s+1}} \right] \tag{11}$$

$$P = \left(\frac{t}{rg(i+1)+t} \right)^s + irg \frac{st^s}{(rg(i+1)+t)^{s+1}}. \tag{12}$$

Replace mean $\mu = s/t$ in (12) then, we get

$$P = \left(\frac{s}{rg\mu(i+1)+s} \right)^s + irg\mu \left(\frac{s}{rg\mu(i+1)+s} \right)^{s+1}. \tag{13}$$

After simplifying (13) for $s = 1, 2, 3,$ we get

$$P = \frac{1}{rg\mu(i+1)+1} + irg\mu \frac{1}{\left(rg\mu(i+1)+1\right)^2} \tag{14}$$

$$P = \frac{4}{\left(rg\mu(i+1)+2\right)^2} + irg\mu \frac{8}{\left(rg\mu(i+1)+2\right)^3} \tag{15}$$

$$P = \frac{27}{\left(rg\mu(i+1)+3\right)^3} + irg\mu \frac{81}{\left(rg\mu(i+1)+3\right)^4}. \tag{16}$$

By using Newton's approximation, Equations (14)-(16) are used in simulation of BMGChSP, where reducing P to the limited values and μ is used as a point of control. Then the generated average number of defectives are shown in Table 1.

Table 1. The generated average number of defectives for specified values of P certain $g\mu$ values in BMGChSP

s	r	i	0.99	0.95	0.90	0.50	0.25	0.10
1	2	1	0.0050	0.0252	0.0515	0.4045	1.1615	3.4147
		2	0.0049	0.0229	0.0446	0.3114	0.8732	2.5428
		3	0.0047	0.0204	0.0383	0.2500	0.6927	2.0076
		4	0.0045	0.0182	0.0333	0.2081	0.5724	1.6544
	3	1	0.0033	0.0168	0.0343	0.2697	0.7743	2.2765
		2	0.0032	0.0153	0.0297	0.2076	0.5822	1.6952
		3	0.0031	0.0136	0.0256	0.1667	0.4618	1.3384
		4	0.0030	0.0122	0.0222	0.1387	0.3816	1.1029
	4	1	0.0025	0.0126	0.0257	0.2022	0.5807	1.7073
		2	0.0024	0.0115	0.0223	0.1557	0.4366	1.2714
		3	0.0023	0.0102	0.0192	0.1250	0.3463	1.0038
		4	0.0023	0.0091	0.0167	0.1040	0.2862	0.8272
2	2	1	0.0050	0.0251	0.0508	0.3376	0.7670	1.5987
		2	0.0049	0.0231	0.0447	0.2600	0.5715	1.1714
		3	0.0047	0.0210	0.0389	0.2086	0.4509	0.9169
		4	0.0046	0.0189	0.0341	0.1734	0.3713	0.7516
	3	1	0.0033	0.0168	0.0339	0.2250	0.5113	1.0658
		2	0.0033	0.0154	0.0298	0.1733	0.3810	0.781
		3	0.0032	0.0140	0.0260	0.1390	0.3006	0.6113
		4	0.0030	0.0126	0.0227	0.1156	0.2475	0.501
	4	1	0.0025	0.0126	0.0254	0.1688	0.3835	0.7993
		2	0.0025	0.0116	0.0224	0.1300	0.2858	0.5857
		3	0.0024	0.0105	0.0195	0.1043	0.2255	0.4585
		4	0.0023	0.0095	0.0170	0.0867	0.1857	0.3758

Table 1. (Continued)

s	r	i	0.99	0.95	0.90	0.50	0.25	0.10
3	2	1	0.0050	0.0251	0.0506	0.3189	0.6736	1.2659
		2	0.0049	0.0233	0.0449	0.2458	0.5000	0.9212
		3	0.0047	0.0212	0.0392	0.1971	0.3936	0.7183
		4	0.0046	0.0192	0.0345	0.1638	0.3236	0.5874
	3	1	0.0033	0.0167	0.0338	0.2126	0.4491	0.8440
		2	0.0033	0.0155	0.0299	0.1638	0.3333	0.6141
		3	0.0032	0.0141	0.0262	0.1314	0.2624	0.4789
		4	0.0031	0.0128	0.023	0.1092	0.2158	0.3916
	4	1	0.0025	0.0126	0.0253	0.1595	0.3368	0.6330
		2	0.0025	0.0117	0.0224	0.1229	0.2500	0.4606
		3	0.0024	0.0106	0.0196	0.0985	0.1968	0.3592
		4	0.0023	0.0096	0.0172	0.0819	0.1618	0.2937

2.2. Designing of quality regions for BMGChSP

2.2.1 Quality decision region (QDR)

In QDR, the product is accepted between two probabilities that are a maximum 0.95 associated with AQL represented by μ_1 and a minimum 0.90 associated with LQL represented by μ_*. Figure 3 explains that QDR is defined as $\mu_1 < \mu < \mu_*$ and range is denoted by $d_1 = \mu_* - \mu_1$.

$$P(\mu_1 < \mu < \mu_*) = \left(\frac{s}{rg\mu(i+1)+s}\right)^s + irg\mu\left(\frac{s}{rg\mu(i+1)+s}\right)^{s+1}.$$

The mean of gamma $\mu = s/t$ is the approximate average quality of product.

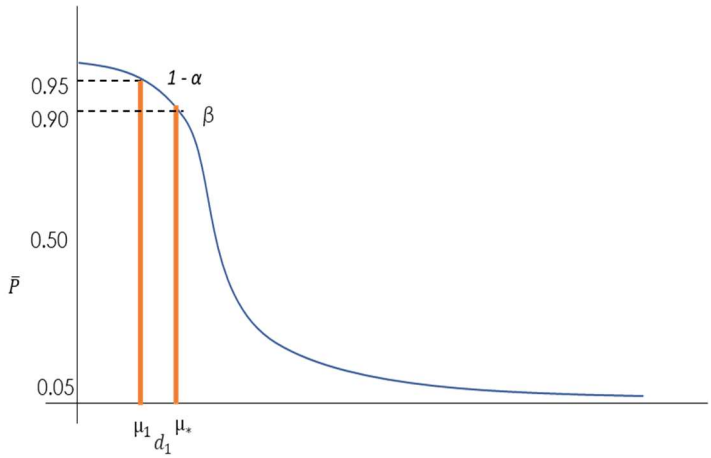


Figure 3. OC curve with the corresponding points for QDR

2.2.2 Probabilistic quality region (PQR)

In PQR, the product is accepted between two probabilities that are a minimum probability of 0.10 and a maximum probability of 0.95. Figure 4 explains that PQR is defined as $\mu_1 < \mu < \mu_2$ and its range is denoted by $d_2 = \mu_2 - \mu_1$.

$$P(\mu_1 < \mu < \mu_2) = \left(\frac{s}{rg\mu(i+1)+s}\right)^s + irg\mu\left(\frac{s}{rg\mu(i+1)+s}\right)^{s+1}.$$

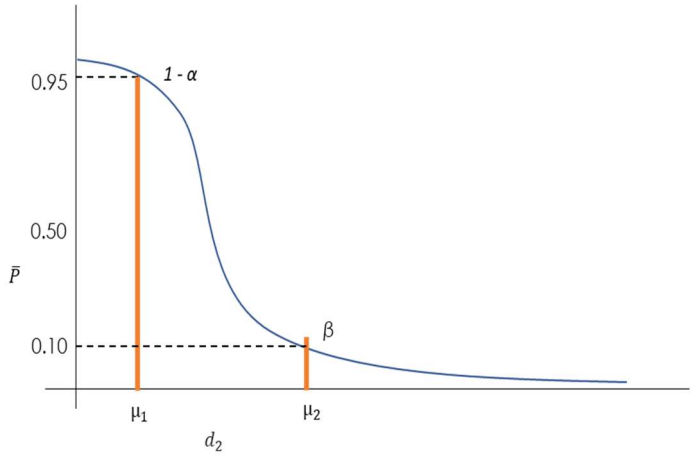


Figure 4. OC curve with the corresponding points for PQR

2.2.3 Limiting quality region (LQR)

In LQR, the product is accepted between two probabilities that are a minimum probability of 0.1 and a maximum of 0.9. Figure 5 explains that LQR is defined as $\mu_* < \mu < \mu_2$ and range is denoted by $d_3 = \mu_2 - \mu_*$.

$$P(\mu_* < \mu < \mu_2) = \left(\frac{s}{rg\mu(i+1)+s}\right)^s + irg\mu\left(\frac{s}{rg\mu(i+1)+s}\right)^{s+1}.$$

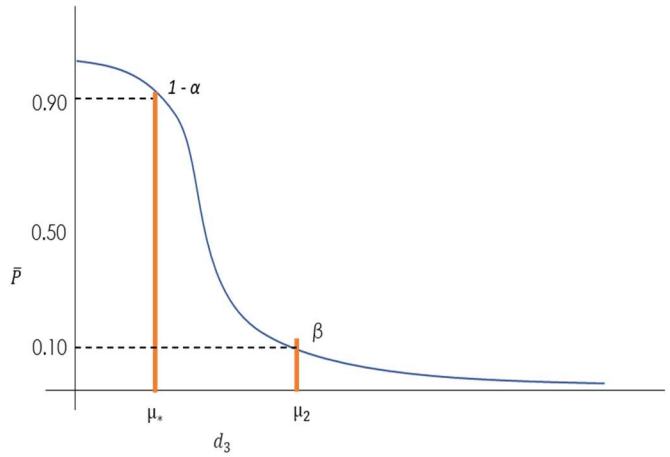


Figure 5. OC curve with the corresponding points for LQR

2.2.4 Indifference quality region (IQR)

In IQR, the product is accepted between two probabilities that are a minimum probability of 0.50 and a maximum of 0.9. Figure 6 explains that IQR is defined as $\mu_1 < \mu < \mu_0$ and range is denoted by $d_0 = \mu_0 - \mu_1$.

$$P(\mu_1 < \mu < \mu_0) = \left(\frac{s}{rg\mu(i+1)+s}\right)^s + irg\mu\left(\frac{s}{rg\mu(i+1)+s}\right)^{s+1}$$

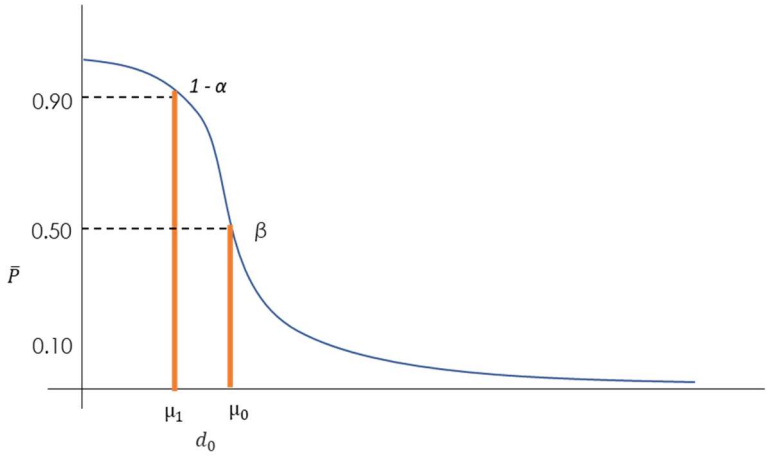


Figure 6. OC curve with the corresponding points for IQR

2.3. Selection of sampling plan

In Table 2, the ranges of QDR (gd_1), PQR (gd_2), LQR (gd_3) and IQR (gd_0), are shown with corresponding design parameters s, r and i. Where the operating ratios are $T = \frac{\mu_* - \mu_1}{\mu_2 - \mu_1} = \frac{g\mu_* - g\mu_1}{g\mu_2 - g\mu_1}$, $T_1 = \frac{\mu_* - \mu_1}{\mu_2 - \mu_*}$ and $T_2 = \frac{\mu_* - \mu_1}{\mu_0 - \mu_1}$, which are used to characterize the sampling plan. For any given values of QDR (d_1), PQR (d_2), LQR (d_3) and IQR (d_0), We can find the operating ratio $T = \frac{d_1}{d_2}$, $T_1 = \frac{d_1}{d_3}$ and $T_2 = \frac{d_1}{d_0}$. In Table 2, find the value under the columns T, T_1 and T_2, which is approximately equal to the specified ratio and note the value of design parameters. From this ratio, we can determine the small value of g and other design parameters for the BMGChSP.

2.3.1 Numerical examples

Given that $\mu_1 = 0.01, r = 2, s = 2$ and $i = 4$ compute the respective values of QDR, PQR, LQR, IQR, T, T_1 and T_2 from Table 2. The nearest values are $gd_1 = 0.0152$, $gd_2 = 0.7326$, $gd_3 = 0.7175$, $gd_0 = 0.1545$ with operating ratios $T = 0.02071$, $T_1 = 0.02115$ and $T_2 = 0.09822$. From Table 1, the corresponding value of $g\mu_1 = 0.0189$ from which the required smallest number of groups can obtain $g = g\mu_1 / \mu_1 = 0.0189 / 0.01 = 1.89 \cong 2$. Thus, the selected parameters for BMGChSP are $g = 2, r = 2, s = 2$ and $i = 4$. Also, the values of QDR $d_1 = 0.0080$, PQR $d_2 = 0.3876$, LQR $d_3 = 0.3796$, IQR $d_0 = 0.0817$, with the operating ratios $T = 0.02071, T_1 = 0.02115$ and $T_2 = 0.09822$.

Table 2. Range of quality regions and operating ratios for specified s, r and i

s	r	i	$g\mu_1$	$g\mu_*$	$g\mu_0$	$g\mu_2$	gd_1	gd_2	gd_3	gd_0	T	T_1	T_2
1	2	1	0.0252	0.0515	0.4045	3.4147	0.0263	3.3895	3.3632	0.3793	0.00776	0.00782	0.06932
		2	0.0229	0.0446	0.3114	2.5428	0.0217	2.5199	2.4982	0.2885	0.0086	0.00867	0.07510
		3	0.0204	0.0383	0.2500	2.0076	0.0179	1.9871	1.9692	0.2296	0.0090	0.00909	0.07794
		4	0.0182	0.0333	0.2081	1.6544	0.0151	1.6362	1.6211	0.1898	0.00922	0.00931	0.07947
	3	1	0.0168	0.0343	0.2697	2.2765	0.0175	2.2596	2.2421	0.2529	0.00776	0.00782	0.06932
		2	0.0153	0.0297	0.2076	1.6952	0.0144	1.6799	1.6655	0.1924	0.00860	0.00867	0.07509
		3	0.0136	0.0256	0.1667	1.3384	0.0119	1.3247	1.3128	0.1530	0.00901	0.00910	0.07802
		4	0.0122	0.0222	0.1387	1.1029	0.010	1.0908	1.0807	0.1265	0.00920	0.00929	0.07935
	4	1	0.0126	0.0257	0.2022	1.7073	0.0131	1.6947	1.6816	0.1896	0.00775	0.00781	0.06929
		2	0.0115	0.0223	0.1557	1.2714	0.0108	1.2599	1.2491	0.1442	0.00859	0.00867	0.07504
		3	0.0102	0.0192	0.1250	1.0038	0.0090	0.9936	0.9846	0.1148	0.00902	0.00910	0.07808
		4	0.0091	0.0167	0.1040	0.8272	0.0075	0.8181	0.8106	0.0949	0.00921	0.00929	0.07936
2	2	1	0.0251	0.0508	0.3376	1.5987	0.0257	1.5735	1.5478	0.3124	0.01633	0.01660	0.08224
		2	0.0231	0.0447	0.2600	1.1714	0.0216	1.1483	1.1267	0.2368	0.01881	0.01917	0.09120
		3	0.0210	0.0389	0.2086	0.9169	0.0179	0.8959	0.8780	0.1876	0.02003	0.02044	0.09569
		4	0.0189	0.0341	0.1734	0.7516	0.0152	0.7326	0.7175	0.1545	0.02071	0.02115	0.09822
	3	1	0.0168	0.0339	0.2250	1.0658	0.0171	1.0490	1.0319	0.2083	0.01632	0.01659	0.08220
		2	0.0154	0.0298	0.1733	0.7810	0.0144	0.7656	0.7511	0.1579	0.01883	0.01919	0.09126
		3	0.0140	0.026	0.1390	0.6113	0.0120	0.5973	0.5853	0.1251	0.02006	0.02047	0.09579
		4	0.0126	0.0227	0.1156	0.5010	0.0101	0.4884	0.4783	0.1030	0.02069	0.02112	0.09809
	4	1	0.0126	0.0254	0.1688	0.7993	0.0128	0.7867	0.7739	0.1562	0.01631	0.01658	0.08214
		2	0.0116	0.0224	0.1300	0.5857	0.0108	0.5741	0.5634	0.1184	0.01878	0.01914	0.09103
		3	0.0105	0.0195	0.1043	0.4585	0.0090	0.4480	0.4390	0.0938	0.02003	0.02044	0.09568
		4	0.0095	0.0170	0.0867	0.3758	0.0076	0.3663	0.3587	0.0772	0.02071	0.02114	0.09819
3	2	1	0.0251	0.0506	0.3189	1.2659	0.0255	1.2408	1.2153	0.2938	0.02057	0.02101	0.08688
		2	0.0233	0.0449	0.2458	0.9212	0.0216	0.8979	0.8763	0.2225	0.02403	0.02463	0.09699
		3	0.0212	0.0392	0.1971	0.7183	0.0180	0.6971	0.6791	0.1759	0.02589	0.02658	0.10262
		4	0.0192	0.0345	0.1638	0.5874	0.0153	0.5682	0.5529	0.1446	0.02691	0.02765	0.10573
	3	1	0.0167	0.0338	0.2126	0.8440	0.0170	0.8272	0.8102	0.1959	0.02057	0.02101	0.08688
		2	0.0155	0.0299	0.1638	0.6141	0.0144	0.5986	0.5842	0.1483	0.02403	0.02462	0.09698
		3	0.0141	0.0262	0.1314	0.4789	0.0120	0.4647	0.4527	0.1172	0.02589	0.02658	0.10261
		4	0.0128	0.023	0.1092	0.3916	0.0102	0.3788	0.3686	0.0964	0.0269	0.02764	0.10569
	4	1	0.0126	0.0253	0.1595	0.6330	0.0128	0.6204	0.6076	0.1469	0.02056	0.02100	0.08684
		2	0.0117	0.0224	0.1229	0.4606	0.0108	0.4489	0.4382	0.1112	0.0240	0.02459	0.09687
		3	0.0106	0.0196	0.0985	0.3592	0.0090	0.3486	0.3396	0.0879	0.02579	0.02647	0.10227
		4	0.0096	0.0172	0.0819	0.2937	0.0076	0.2841	0.2765	0.0723	0.02672	0.02745	0.10503

2.3.2 For specified QDR and PQR

When QDR and PQR are specified, then Table 2 is used to construct the plan for any values of d_1 and d_2 we can find ratio, $T = d_1 / d_2$. Find the value which is approximately equal to the specified ratio under column T in Table 2 and note the corresponding values of s, r and i. By this procedure, we can find the parameter values for BMGChSP.

Let in a manufacturer company required QDR $d_1 = 0.002$ and PQR $d_2 = 0.075$, then the calculated operating ratio is $T = 0.02667$. The value from Table 2, is obtained to be $T = 0.02589$, with design parameters $s = 3, r = 3$ and $i = 3$. So, for this operating ratio $gd_1 = 0.0120$ and $gd_2 = 0.4647$, then the value of $g = gd_1 / d_1 = 0.0120 / 0.002 = 6$. Hence for the required QDR $d_1 = 0.002$ and PQR $d_2 = 0.075$, design parameters of BMGChSP are $s = 3, g = 6, r = 3$ and $i = 3$.

2.3.3 For specified QDR and LQR

Let a manufacturer required QDR $d_1 = 0.001$ and LQR $d_3 = 0.09$, then the calculated operating ratio is $T_1 = 0.0222$. From Table 2, the value is found to be $T_1 = 0.01919$, with design parameters $s = 2, r = 3$ and $i = 2$. So, for this operating ratio $gd_1 = 0.0144$ and $gd_3 = 0.7511$, then the value of $g = gd_1 / d_1 = 0.0144 / 0.001 = 14.4 \cong 15$. Hence for the required QDR $d_1 = 0.001$ and LQR $d_3 = 0.05$, design parameters of BMGChSP are $s = 2, g = 15, r = 3$ and $i = 2$.

2.3.4 For specified QDR and IQR

Let in a manufacturer company required QDR $d_1 = 0.01$ and IQR $d_0 = 0.09$, then the calculated operating ratio is $T_2 = 0.1111$. From Table 2, the value is found to be $T_2 = 0.10573$, with design parameters $s = 3, r = 2$ and $i = 4$. So, for this operating ratio $gd_1 = 0.0153$ and $gd_0 = 0.1446$, then the value of $g = gd_1 / d_1 = 0.0153 / 0.01 = 1.53 \cong 2$. Hence for the required QDR $d_1 = 0.01$ and IQR $d_0 = 0.09$ design parameters of BMGChSP are $s = 3, g = 2, r = 2$ and $i = 4$.

3. Results

3.1. Graphs and discussion

Consider shape parameter $s = 2$ and number of testers $r = 3$, then for changed value of $i = 1, 2, 3, 4$ the OC curves are presented in Figure 7.

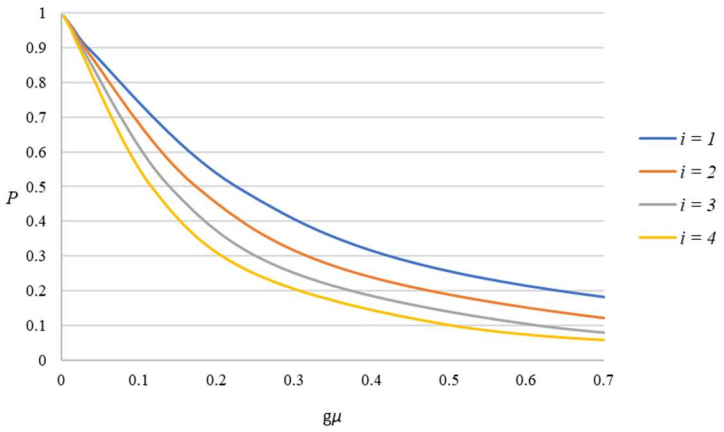


Figure 7. OC curves of each value of i for $s = 2$ and $r = 3$.

OC curves for $s = 2, i = 3$ and more than one number of testers $r = 2, 3, 4$, are presented in Figure 8.

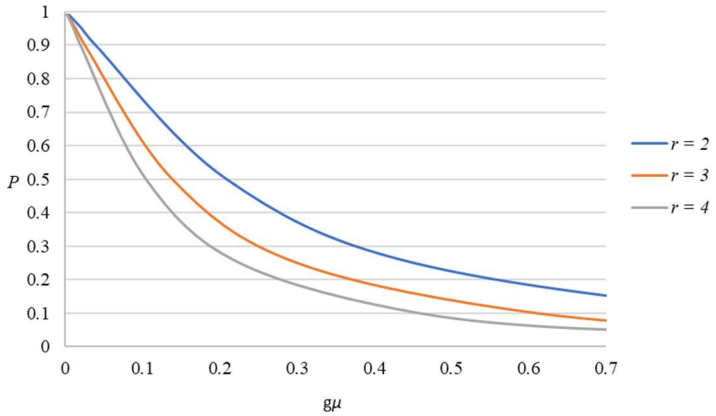


Figure 8. OC curves of each value of r for $s = 2$ and $i = 3$.

OC curves for $r = 4, i = 3$ and changed in shape parameters $s = 1, 2, 3$ are shown in Figure 9.

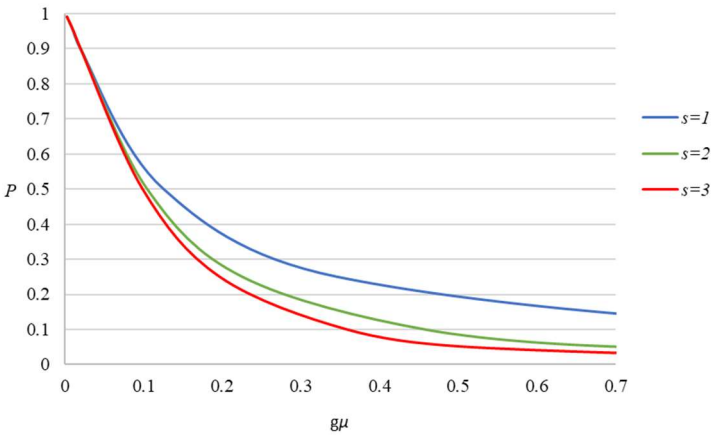


Figure 9. OC curves of each value of s for $r = 4$ and $i = 3$.

It can be noted from Figures 7 to 9, that as the values of i, r and s increase, the ideal OC curve can be achieved and approach to the less proportion of defectives for the same value of PA.

3.2. Comparison study

For comparison purposes, we consider a plan BGChSP proposed by (Hafeez et al. 2022). The proposed BMGChSP and existing BGChSP both consider Poisson distribution for the average number of defective with gamma prior. For the same design parameters in Figure 10, OC curves for both plans are represented. The average number of defectives for both plans is represented for $s = 2, r = 3$ and $i = 2$.

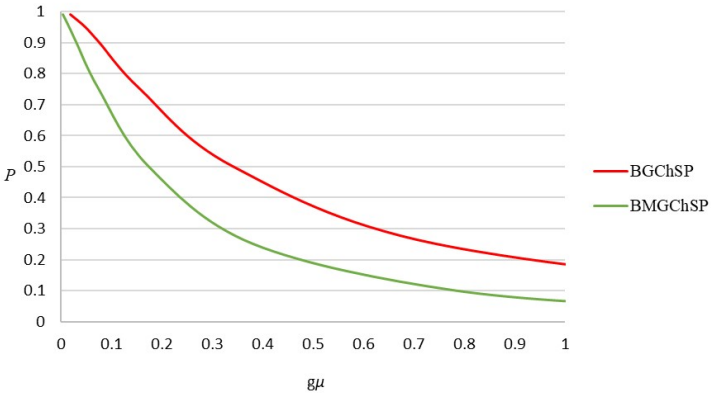


Figure 10. OC curves of BMGChSP and BGChSP for $s = 2, r = 3$ and $i = 2$.

From Figure 10, it can be observed that the proposed BMGChSP gives more ideal OC curve than the existing BGChSP. This explains that for the same design parameters, BMGhSP gives a smaller number of defectives than BGChSP. When values of design parameters increase, their effect on AQL, LQL, range of quality region and the probability of acceptance of a defective item is indicated in Table 3.

Table 3. The effect of increasing parameter on AQL, LQL, range of quality region, and the probability of acceptance of a defective product

Parameter	AQL	LQL	Range of quality region	Probability of acceptance defective product
s	decrease	decrease	-	decrease
g	decrease	decrease	-	decrease
r	decrease	decrease	-	decrease
i	decrease	decrease	-	decrease
α	increase	no effect	decrease	decrease
β	no effect	decrease	decrease	decrease

We can observe from Table 3, that as values of s, g, r and i are increase, the AQL, LQL, range of quality region and the probability of acceptance of a defective item are decreased. As the value of α increases, the value of AQL increase but it does not affect LQL because LQL does not depend on α. AQL is the left-hand limit of quality region, hence the increase in the value of AQL is in fact decrease in the range of quality region. As the value of β increase, the value of LQL decrease but it does not affect the value of AQL because AQL does not depend on β.

4. Conclusions

The presented work in this paper is limited to the construction and selection of BMGChSP for quality regions. For the specified consumer's and producer's risks, four quality regions are estimated. This research presents the idea to estimate quality regions that are acceptable for both parties. By considering the quality level of the lots and uses a criterion to reduce the risk for

consumer and producer. All four quality regions are estimated for the possible combinations of design parameters s, r, i, α and β. From OC curves, as the values of design parameters s, r and i increase, the average number of defective decreases. The effect of increase the value of α and β cause to decrease the range of quality region because AQL and LQL become close to each other. By considering both risks this plan provide the acceptance regions for a lot. With small sample and for different mean ratio, this plan has the ability to provide a more precise PA. Hence this study extends the knowledge boundary in this area of research and gives benefit to both researchers and practitioners. In future many other quality reliability characteristics with other distributions can be discovered.

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